Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.7-Miscellaneous/140-4.7.6-f^-a+b-x+c-x^2-trig-d+e-x+f-x^2-^n

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September 27, 2022 Compiled on September 27, 2022 at 11:00pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [142]. This is test number [140].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (142)	0.00 (0)
Rubi	98.59 (140)	1.41 (2)
Fricas	80.99 (115)	19.01 (27)
Maple	80.28 (114)	19.72 (28)
Maxima	80.28 (114)	19.72 (28)
Giac	44.37 (63)	55.63 (79)
Mupad	35.21 (50)	64.79 (92)
Sympy	28.87 (41)	71.13 (101)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

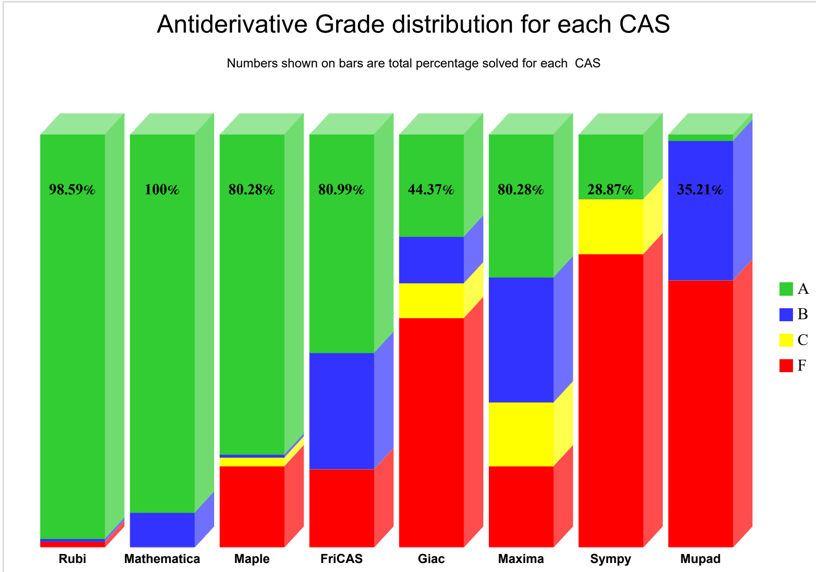
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

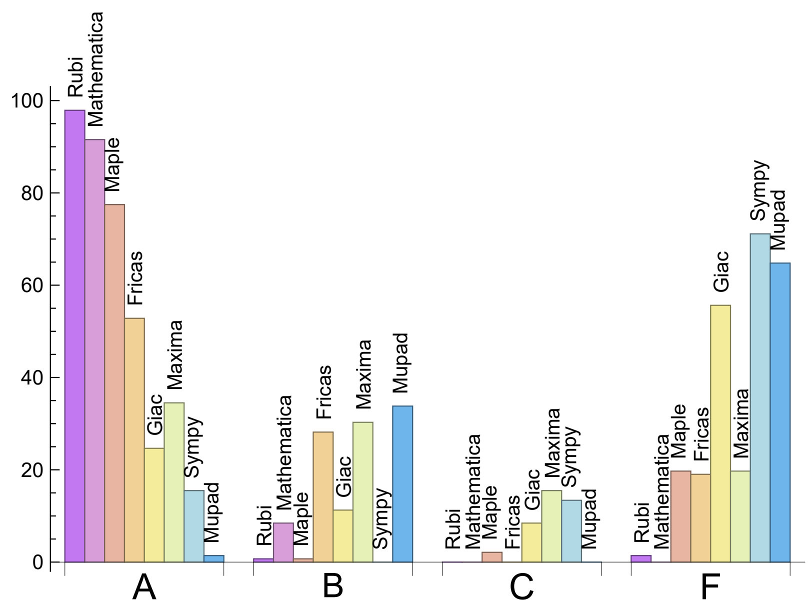
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.89	0.70	0.00	1.41
Mathematica	91.55	8.45	0.00	0.00
Maple	77.46	0.70	2.11	19.72
Fricas	52.82	28.17	0.00	19.01
Maxima	34.51	30.28	15.49	19.72
Giac	24.65	11.27	8.45	55.63
Sympy	15.49	0.00	13.38	71.13
Mupad	N/A	33.80	0.00	64.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	28	100.00 %	0.00 %	0.00 %
Fricas	27	100.00 %	0.00 %	0.00 %
Giac	79	100.00 %	0.00 %	0.00 %
Maxima	28	100.00 %	0.00 %	0.00 %
Sympy	101	90.10 %	8.91 %	0.99 %
Mupad	92	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

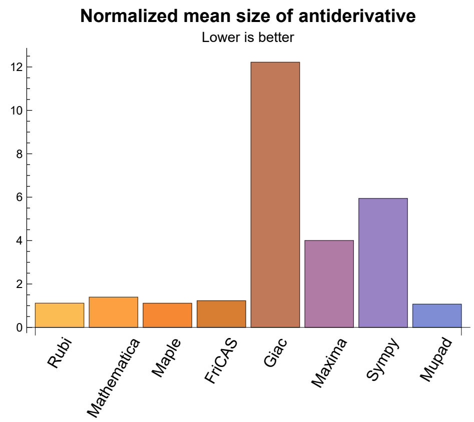
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

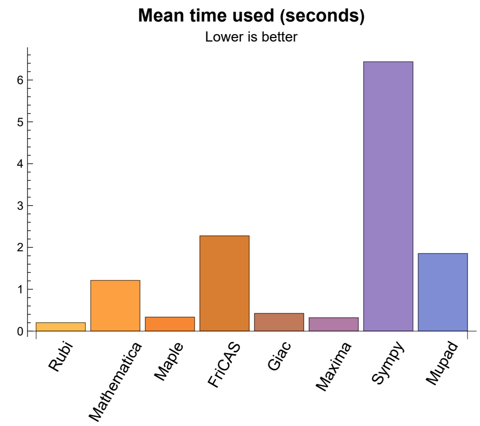
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.20	140.37	1.12	129.00	1.00
Mathematica	1.21	281.51	1.39	117.50	1.00
Maple	0.33	136.06	1.11	118.00	0.91
Maxima	0.32	497.75	4.00	238.00	1.66
Fricas	2.28	200.89	1.23	156.00	1.05
Sympy	6.44	875.39	5.94	70.00	1.43
Giac	0.42	521.38	12.22	127.00	1.10
Mupad	1.85	67.46	1.07	25.50	0.84

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{29, 30\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{92, 93, 98, 100, 101, 102, 123, 124, 129, 131, 132, 133\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

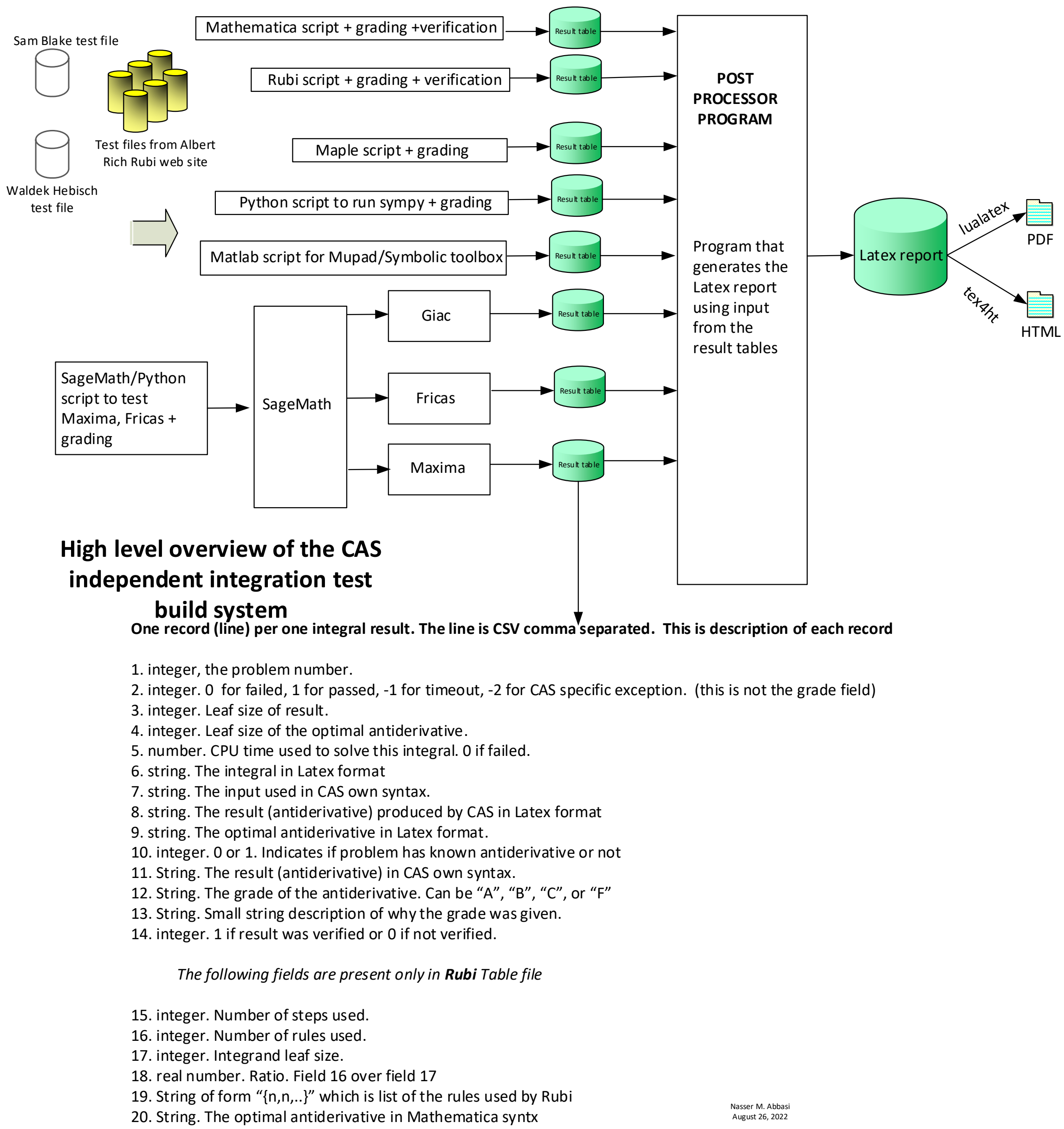
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade: { 34 }

C grade: { }

F grade: { 28, 32 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade: { 7, 21, 22, 63, 93, 99, 101, 102, 124, 130, 132, 133 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 9, 11, 12, 13, 18, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140 }

B grade: { 51 }

C grade: { 31, 32, 33 }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

2.1.4 Maxima

A grade: { 9, 18, 29, 30, 31, 32, 33, 38, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115 }

B grade: { 2, 3, 4, 11, 12, 13, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 63, 77, 78, 88, 90, 91, 93, 97, 99, 100, 102, 103, 108, 109, 119, 121, 122, 124, 128, 130, 131, 133, 134, 135, 136, 139, 140 }

C grade: { 36, 37, 85, 86, 87, 89, 92, 94, 95, 96, 98, 101, 116, 117, 118, 120, 123, 125, 126, 127, 129, 132 }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 9, 11, 12, 13, 18, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 76, 77, 78, 85, 86, 87, 88, 89, 94, 95, 96, 104, 107, 108, 109, 116, 117, 118, 119, 120, 125, 126, 127, 135, 136, 139, 140 }

B grade: { 63, 70, 74, 75, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 105, 106, 110, 111, 112, 113, 114, 115, 121, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134 }

C grade: { }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 53, 54, 55, 56, 137, 138, 141, 142 }

2.1.6 Sympy

A grade: { 9, 18, 29, 30, 34, 35, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 69, 70, 71, 72 }

B grade: { }

C grade: { 3, 4, 12, 13, 38, 39, 40, 41, 42, 43, 44, 45, 46, 73, 104, 135, 136, 139, 140 }

F grade: { 1, 2, 5, 6, 7, 8, 10, 11, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 63, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

2.1.7 Giac

A grade: { 9, 18, 29, 30, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 104, 105, 106 }

B grade: { 31, 32, 63, 70, 79, 80, 81, 82, 83, 84, 110, 111, 112, 113, 114, 115 }

C grade: { 2, 3, 4, 11, 12, 13, 34, 35, 135, 136, 139, 140 }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 76, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

2.1.8 Mupad

A grade: { 29, 30 }

B grade: { 2, 3, 4, 9, 11, 12, 13, 18, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 104, 135, 136, 139, 140 }

C grade: { }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 57, 58, 59, 60, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.071	0.101	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	154	377	761	174	0	1275	190
N.S.	1	1.00	0.77	1.89	3.82	0.87	0.00	6.41	0.95
time (sec)	N/A	0.051	0.701	0.636	0.337	1.976	0.000	0.460	3.217

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	86	106	348	91	1117	915	95
N.S.	1	1.00	0.67	0.83	2.72	0.71	8.73	7.15	0.74
time (sec)	N/A	0.035	0.209	0.284	0.292	2.772	15.468	0.454	3.023

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	74	197	51	462	634	50
N.S.	1	1.00	0.66	1.01	2.70	0.70	6.33	8.68	0.68
time (sec)	N/A	0.012	0.120	0.078	0.281	2.241	2.643	0.411	2.395

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	114	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	1.442	0.050	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	101	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	1.315	0.063	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	334	0	0	0	0	0	-1
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	7.137	0.098	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	173	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	3.090	0.100	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	33	34	37	36	70	35	41
N.S.	1	1.00	0.61	0.63	0.69	0.67	1.30	0.65	0.76
time (sec)	N/A	0.018	0.043	0.089	0.277	2.874	0.627	0.412	0.046

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.063	0.133	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	155	158	759	143	0	1271	191
N.S.	1	1.00	0.78	0.79	3.81	0.72	0.00	6.39	0.96
time (sec)	N/A	0.038	0.698	0.526	0.316	2.997	0.000	0.442	3.143

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	85	105	348	80	1117	915	98
N.S.	1	1.00	0.66	0.82	2.72	0.62	8.73	7.15	0.77
time (sec)	N/A	0.027	0.202	0.240	0.305	2.855	14.967	0.427	2.976

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	73	195	50	437	631	48
N.S.	1	1.00	0.65	1.01	2.71	0.69	6.07	8.76	0.67
time (sec)	N/A	0.011	0.103	0.102	0.283	2.793	2.518	0.418	2.357

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.025	0.057	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.019	0.077	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.335	0.154	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	111	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.243	0.214	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	33	34	37	36	70	35	41
N.S.	1	1.00	0.61	0.63	0.69	0.67	1.30	0.65	0.76
time (sec)	N/A	0.018	0.032	0.076	0.267	2.591	0.645	0.403	0.042

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	212	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	2.174	0.039	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	174	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	1.716	0.033	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	166	0	0	0	0	0	-1
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.458	0.032	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	163	0	0	0	0	0	-1
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	1.051	0.045	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	170	0	0	0	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	1.670	0.036	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	210	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.230	0.076	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	133	0	0	0	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.278	0.092	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.101	0.184	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.115	0.142	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	A	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	143	0	0	135	0	0	-1
N.S.	1	0.00	1.03	0.00	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.364	0.620	0.028	0.000	1.038	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.446	9.173	0.038	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.647	10.407	0.069	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	213	32	27	0	6346	27
N.S.	1	1.00	1.08	8.88	1.33	1.12	0.00	264.42	1.12
time (sec)	N/A	2.854	0.973	0.382	0.522	2.303	0.000	0.669	2.902

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	F	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	23	201	31	25	0	4746	23
N.S.	1	0.00	1.00	8.74	1.35	1.09	0.00	206.35	1.00
time (sec)	N/A	1.750	0.582	0.274	0.510	2.493	0.000	0.585	2.861

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	199	28	23	0	0	21
N.S.	1	1.00	1.00	9.05	1.27	1.05	0.00	0.00	0.95
time (sec)	N/A	1.859	0.621	0.257	0.525	3.324	0.000	0.000	2.803

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	327	17	18	1358	19	19	1941	17
N.S.	1	19.24	1.00	1.06	79.88	1.12	1.12	114.18	1.00
time (sec)	N/A	0.539	0.248	0.152	0.377	2.402	3.573	0.450	2.872

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	399	18	17	639	16
N.S.	1	1.00	1.00	1.06	24.94	1.12	1.06	39.94	1.00
time (sec)	N/A	0.020	0.033	0.114	0.297	1.869	0.597	0.435	2.348

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	20	613	21	0	0	19
N.S.	1	1.00	0.95	1.00	30.65	1.05	0.00	0.00	0.95
time (sec)	N/A	1.259	0.437	0.170	0.743	2.477	0.000	0.000	2.736

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	20	1189	21	0	0	19
N.S.	1	1.00	0.95	1.00	59.45	1.05	0.00	0.00	0.95
time (sec)	N/A	1.389	0.428	0.174	0.973	2.619	0.000	0.000	2.751

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	60	44	56	325	55	46
N.S.	1	1.00	0.70	0.95	0.70	0.89	5.16	0.87	0.73
time (sec)	N/A	0.032	0.142	0.150	0.274	5.036	0.965	0.396	0.502

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	74	108	538	109	1040	98	166
N.S.	1	1.00	0.62	0.91	4.52	0.92	8.74	0.82	1.39
time (sec)	N/A	0.062	0.696	0.259	0.302	2.941	3.201	0.402	3.012

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	82	118	550	135	1357	111	178
N.S.	1	1.00	0.64	0.91	4.26	1.05	10.52	0.86	1.38
time (sec)	N/A	0.060	0.987	0.280	0.287	1.996	8.844	0.401	3.029

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	74	108	538	98	1030	100	167
N.S.	1	1.00	0.62	0.91	4.52	0.82	8.66	0.84	1.40
time (sec)	N/A	0.057	0.708	0.197	0.288	0.942	3.213	0.433	0.857

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	71	236	90	850	66	58
N.S.	1	1.00	0.72	0.90	2.99	1.14	10.76	0.84	0.73
time (sec)	N/A	0.053	0.367	0.308	0.310	1.837	6.978	0.399	0.375

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1148	201	2751	155	255
N.S.	1	1.00	0.60	0.91	6.27	1.10	15.03	0.85	1.39
time (sec)	N/A	0.083	0.947	0.209	0.332	1.703	30.193	0.407	3.741

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	81	118	550	114	1353	111	179
N.S.	1	1.00	0.63	0.91	4.26	0.88	10.49	0.86	1.39
time (sec)	N/A	0.057	0.704	0.264	0.296	1.779	8.896	0.396	0.822

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1144	200	2958	152	255
N.S.	1	1.00	0.60	0.91	6.25	1.09	16.16	0.83	1.39
time (sec)	N/A	0.083	0.797	0.236	0.327	1.437	30.047	0.416	3.713

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	118	550	156	1991	111	178
N.S.	1	1.00	0.86	0.91	4.26	1.21	15.43	0.86	1.38
time (sec)	N/A	0.068	0.957	0.269	0.298	1.327	66.438	0.420	1.006

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	19	19	17	17	27	16	16
N.S.	1	1.00	0.63	0.63	0.57	0.57	0.90	0.53	0.53
time (sec)	N/A	0.027	0.042	0.062	0.273	1.498	0.157	0.393	0.076

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	25	27	26	26	48	25	21
N.S.	1	1.00	0.50	0.54	0.52	0.52	0.96	0.50	0.42
time (sec)	N/A	0.080	0.041	0.059	0.264	1.999	0.306	0.417	2.377

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	17	17	27	15	17
N.S.	1	1.00	0.60	0.67	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.030	0.030	0.053	0.276	1.460	0.157	0.397	2.349

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	23	28	26	26	48	24	22
N.S.	1	1.00	0.45	0.55	0.51	0.51	0.94	0.47	0.43
time (sec)	N/A	0.079	0.040	0.061	0.276	1.523	0.322	0.381	2.364

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	103	39	21	27	39	19
N.S.	1	1.00	0.81	3.81	1.44	0.78	1.00	1.44	0.70
time (sec)	N/A	0.056	0.079	0.102	0.267	1.190	0.096	0.419	0.059

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	30	23	26	32	23	31
N.S.	1	1.00	0.80	0.73	0.56	0.63	0.78	0.56	0.76
time (sec)	N/A	0.014	0.065	0.082	0.273	1.474	0.177	0.394	2.411

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	1.605	0.242	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	1.917	0.251	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.678	0.102	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.630	0.114	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	43	42	37	30	0	0	-1
N.S.	1	1.00	0.62	0.61	0.54	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.030	0.115	0.263	1.863	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	44	38	32	0	0	-1
N.S.	1	1.00	0.72	0.68	0.58	0.49	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.022	0.098	0.273	1.553	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	51	45	0	0	-1
N.S.	1	1.00	1.00	0.64	0.63	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.088	0.096	0.273	1.306	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	52	46	0	0	-1
N.S.	1	1.00	1.06	0.70	0.68	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.090	0.095	0.278	1.667	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	30	29	29	29	21	21
N.S.	1	1.00	0.69	0.86	0.83	0.83	0.83	0.60	0.60
time (sec)	N/A	0.056	0.048	0.101	0.271	1.149	1.228	0.397	0.084

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.006	0.013	0.030	0.269	1.720	0.095	0.395	2.211

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	21	5	19	21	0	17	43
N.S.	1	1.00	4.20	1.00	3.80	4.20	0.00	3.40	8.60
time (sec)	N/A	0.012	0.023	0.135	0.278	1.112	0.000	0.383	2.508

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	3	3	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.005	0.014	0.054	0.275	1.515	0.095	0.397	0.037

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.006	0.014	0.061	0.273	1.374	0.102	0.404	0.049

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	10	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	1.00	0.70	0.70
time (sec)	N/A	0.006	0.015	0.063	0.269	1.408	0.181	0.393	2.204

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	10	10	12	10	10
N.S.	1	1.00	1.00	0.85	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.012	0.023	0.067	0.275	1.194	6.962	0.393	2.231

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	42	0	24	24
N.S.	1	1.00	1.00	0.83	0.80	1.40	0.00	0.80	0.80
time (sec)	N/A	0.025	0.085	0.335	0.279	1.480	0.000	0.440	0.313

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	4	12	10	7	10
N.S.	1	1.00	1.00	1.00	0.57	1.71	1.43	1.00	1.43
time (sec)	N/A	0.007	0.011	0.027	0.272	1.745	0.080	0.388	2.654

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	9	8	19	10	29	10
N.S.	1	1.00	1.00	1.80	1.60	3.80	2.00	5.80	2.00
time (sec)	N/A	0.006	0.008	0.029	0.278	1.540	0.602	0.399	2.790

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	5	5	3	5	5
N.S.	1	1.00	1.00	1.00	1.25	1.25	0.75	1.25	1.25
time (sec)	N/A	0.013	0.013	0.049	0.271	1.713	0.174	0.406	0.085

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	10	5	7	13
N.S.	1	1.00	1.00	1.00	1.17	1.67	0.83	1.17	2.17
time (sec)	N/A	0.011	0.021	0.045	0.265	3.282	0.485	0.402	2.271

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	36	28	30	116	35	26
N.S.	1	1.00	0.73	0.97	0.76	0.81	3.14	0.95	0.70
time (sec)	N/A	0.009	0.068	0.059	0.269	2.997	0.251	0.403	0.101

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	108	88	100	193	0	127	-1
N.S.	1	1.00	0.94	0.77	0.87	1.68	0.00	1.10	-0.01
time (sec)	N/A	0.086	0.176	0.188	0.286	2.912	0.000	0.423	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	134	119	131	229	0	147	-1
N.S.	1	1.00	0.93	0.83	0.91	1.59	0.00	1.02	-0.01
time (sec)	N/A	0.156	0.250	0.188	0.279	2.440	0.000	0.422	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	51	45	0	0	-1
N.S.	1	1.00	1.00	0.64	0.63	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.025	0.001	0.276	2.291	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	129	62	137	66	0	0	-1
N.S.	1	1.00	1.48	0.71	1.57	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.264	0.107	0.294	2.512	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	165	129	475	161	0	0	-1
N.S.	1	1.00	1.06	0.83	3.06	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.621	0.243	0.301	2.508	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	116	147	265	0	300	-1
N.S.	1	1.00	0.93	0.82	1.04	1.87	0.00	2.11	-0.01
time (sec)	N/A	0.148	0.248	0.431	0.284	3.209	0.000	0.449	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	156	139	186	270	0	521	-1
N.S.	1	1.00	0.99	0.89	1.18	1.72	0.00	3.32	-0.01
time (sec)	N/A	0.140	1.149	0.683	0.504	2.172	0.000	0.455	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	268	239	302	525	0	595	-1
N.S.	1	1.00	0.90	0.80	1.01	1.76	0.00	2.00	-0.00
time (sec)	N/A	0.254	0.900	0.955	0.516	3.039	0.000	0.505	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	152	189	321	0	378	-1
N.S.	1	1.00	1.00	0.94	1.17	1.98	0.00	2.33	-0.01
time (sec)	N/A	0.227	0.388	0.409	0.289	2.187	0.000	0.463	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	244	175	240	334	0	599	-1
N.S.	1	1.00	1.36	0.98	1.34	1.87	0.00	3.35	-0.01
time (sec)	N/A	0.238	1.192	0.730	0.494	2.707	0.000	0.481	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	323	311	374	645	0	751	-1
N.S.	1	1.00	0.95	0.91	1.10	1.90	0.00	2.21	-0.00
time (sec)	N/A	0.441	1.616	1.120	0.529	2.115	0.000	0.548	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	119	123	206	144	0	0	-1
N.S.	1	1.00	0.79	0.81	1.36	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.165	0.394	0.290	2.341	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	132	145	236	161	0	0	-1
N.S.	1	1.00	0.77	0.85	1.38	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.276	0.417	0.292	2.662	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	224	246	412	282	0	0	-1
N.S.	1	1.00	0.74	0.82	1.37	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.463	0.763	0.301	2.016	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	170	84	209	107	0	0	-1
N.S.	1	1.00	1.59	0.79	1.95	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.516	0.307	0.279	2.791	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	188	107	315	169	0	0	-1
N.S.	1	1.00	1.34	0.76	2.25	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.879	0.412	0.304	2.948	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	386	166	661	317	0	0	-1
N.S.	1	1.00	1.81	0.78	3.10	1.49	0.00	0.00	-0.00
time (sec)	N/A	0.251	2.533	0.754	0.310	1.840	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	216	169	748	299	0	0	-1
N.S.	1	1.00	1.16	0.90	4.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.989	0.630	0.330	1.469	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	251	191	851	363	0	0	-1
N.S.	1	1.00	1.19	0.91	4.03	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.299	2.383	0.708	0.298	2.352	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	3003	338	2135	713	0	0	-1
N.S.	1	1.00	7.97	0.90	5.66	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.449	7.064	1.220	0.327	2.444	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	155	170	362	180	0	0	-1
N.S.	1	1.00	0.88	0.97	2.06	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.335	0.453	0.301	2.154	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	204	217	399	226	0	0	-1
N.S.	1	1.00	0.88	0.94	1.73	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.721	0.577	0.320	2.653	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	391	338	696	350	0	0	-1
N.S.	1	1.00	1.10	0.95	1.97	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.345	1.032	0.898	0.323	2.651	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	230	180	647	309	0	0	-1
N.S.	1	1.00	1.19	0.93	3.35	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.268	1.043	0.552	0.302	2.195	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	299	227	997	402	0	0	-1
N.S.	1	1.00	1.22	0.93	4.07	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.331	3.301	0.661	0.339	2.312	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	3291	358	2451	731	0	0	-1
N.S.	1	1.00	8.53	0.93	6.35	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.391	7.032	1.093	0.328	3.040	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	347	216	1005	379	0	0	-1
N.S.	1	1.00	1.64	1.02	4.74	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.400	2.297	0.583	0.310	4.081	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	C	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	1120	263	1481	474	0	0	-1
N.S.	1	1.00	4.18	0.98	5.53	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.460	6.684	0.767	0.323	2.183	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	3835	430	4349	873	0	0	-1
N.S.	1	1.00	8.92	1.00	10.11	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.754	7.236	1.341	0.364	2.853	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	324	217	1012	384	0	0	-1
N.S.	1	1.00	1.52	1.02	4.75	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.563	1.950	0.908	0.302	3.308	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	35	25	28	114	33	25
N.S.	1	1.00	0.72	0.97	0.69	0.78	3.17	0.92	0.69
time (sec)	N/A	0.010	0.061	0.077	0.285	2.177	0.238	0.424	0.093

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	109	86	100	193	0	127	-1
N.S.	1	1.00	0.95	0.75	0.87	1.68	0.00	1.10	-0.01
time (sec)	N/A	0.074	0.179	0.142	0.286	2.346	0.000	0.417	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	135	117	131	229	0	147	-1
N.S.	1	1.00	0.94	0.81	0.91	1.59	0.00	1.02	-0.01
time (sec)	N/A	0.138	0.250	0.142	0.280	3.244	0.000	0.422	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	52	46	0	0	-1
N.S.	1	1.00	1.06	0.70	0.68	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.088	0.000	0.266	2.254	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	107	60	133	70	0	0	-1
N.S.	1	1.00	1.29	0.72	1.60	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.218	0.107	0.291	2.628	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	166	127	474	164	0	0	-1
N.S.	1	1.00	1.10	0.84	3.14	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.617	0.159	0.284	2.620	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	133	114	147	265	0	300	-1
N.S.	1	1.00	0.94	0.80	1.04	1.87	0.00	2.11	-0.01
time (sec)	N/A	0.135	0.251	0.168	0.273	2.069	0.000	0.455	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	158	139	186	270	0	521	-1
N.S.	1	1.00	1.01	0.89	1.18	1.72	0.00	3.32	-0.01
time (sec)	N/A	0.132	1.129	0.381	0.491	2.298	0.000	0.458	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	267	235	302	525	0	595	-1
N.S.	1	1.00	0.90	0.79	1.01	1.76	0.00	2.00	-0.00
time (sec)	N/A	0.230	0.956	0.601	0.490	2.897	0.000	0.492	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	163	150	189	321	0	378	-1
N.S.	1	1.00	1.01	0.93	1.17	1.98	0.00	2.33	-0.01
time (sec)	N/A	0.203	0.404	0.188	0.285	2.400	0.000	0.461	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	245	175	240	334	0	599	-1
N.S.	1	1.00	1.37	0.98	1.34	1.87	0.00	3.35	-0.01
time (sec)	N/A	0.225	1.157	0.398	0.485	3.043	0.000	0.476	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	322	307	374	645	0	751	-1
N.S.	1	1.00	0.95	0.90	1.10	1.90	0.00	2.21	-0.00
time (sec)	N/A	0.409	1.635	0.808	0.513	1.992	0.000	0.560	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	116	121	204	142	0	0	-1
N.S.	1	1.00	0.79	0.82	1.39	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.161	0.162	0.288	1.853	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	131	145	236	159	0	0	-1
N.S.	1	1.00	0.77	0.85	1.38	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.272	0.225	0.312	2.132	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	218	242	406	280	0	0	-1
N.S.	1	1.00	0.74	0.83	1.39	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.445	0.565	0.293	3.313	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	170	82	205	109	0	0	-1
N.S.	1	1.00	1.65	0.80	1.99	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.514	0.155	0.280	2.239	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	189	107	315	167	0	0	-1
N.S.	1	1.00	1.35	0.76	2.25	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.893	0.263	0.275	2.269	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	389	162	667	311	0	0	-1
N.S.	1	1.00	1.90	0.79	3.25	1.52	0.00	0.00	-0.00
time (sec)	N/A	0.204	2.363	0.562	0.298	2.020	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	217	167	749	301	0	0	-1
N.S.	1	1.00	1.19	0.91	4.09	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.961	0.189	0.291	1.773	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	252	191	851	361	0	0	-1
N.S.	1	1.00	1.19	0.91	4.03	1.71	0.00	0.00	-0.00
time (sec)	N/A	0.282	2.436	0.349	0.292	2.328	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	2997	334	2140	707	0	0	-1
N.S.	1	1.00	8.12	0.91	5.80	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.425	7.012	0.809	0.333	2.593	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	151	168	362	178	0	0	-1
N.S.	1	1.00	0.88	0.98	2.10	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.317	0.176	0.296	2.425	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	204	217	399	226	0	0	-1
N.S.	1	1.00	0.88	0.94	1.73	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.598	0.276	0.298	2.394	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	386	334	696	348	0	0	-1
N.S.	1	1.00	1.12	0.97	2.01	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.969	0.617	0.313	2.660	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	231	178	648	311	0	0	-1
N.S.	1	1.00	1.22	0.94	3.43	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.240	1.042	0.154	0.285	2.705	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	301	227	997	402	0	0	-1
N.S.	1	1.00	1.23	0.93	4.07	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.298	3.300	0.276	0.311	2.720	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	3285	354	2456	725	0	0	-1
N.S.	1	1.00	8.69	0.94	6.50	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.368	6.999	0.731	0.335	2.951	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	348	214	1006	381	0	0	-1
N.S.	1	1.00	1.67	1.03	4.84	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.334	2.208	0.230	0.304	3.130	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	C	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	1118	263	1481	474	0	0	-1
N.S.	1	1.00	4.17	0.98	5.53	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.414	6.698	0.365	0.321	2.696	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	3829	426	4354	867	0	0	-1
N.S.	1	1.00	9.07	1.01	10.32	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.709	6.825	1.128	0.361	3.582	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	325	215	1013	386	0	0	-1
N.S.	1	1.00	1.56	1.03	4.85	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.402	1.278	0.198	0.312	3.580	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	271	576	257	8277	1738	248
N.S.	1	1.00	0.73	1.11	2.35	1.05	33.78	7.09	1.01
time (sec)	N/A	0.248	0.784	0.565	0.299	2.892	23.712	0.463	3.420

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	97	221	84	920	923	84
N.S.	1	1.00	0.84	0.98	2.23	0.85	9.29	9.32	0.85
time (sec)	N/A	0.110	0.226	0.124	0.286	2.648	2.492	0.451	2.590

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	128	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	1.056	0.105	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	240	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.821	0.786	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	228	274	573	243	8277	1736	247
N.S.	1	1.00	0.93	1.12	2.34	0.99	33.78	7.09	1.01
time (sec)	N/A	0.232	0.391	0.431	0.304	2.389	23.622	0.503	3.311

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	82	96	219	81	920	920	83
N.S.	1	1.00	0.84	0.98	2.23	0.83	9.39	9.39	0.85
time (sec)	N/A	0.110	0.133	0.167	0.292	1.682	2.502	0.444	2.557

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.037	0.082	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.278	0.233	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [31] had the largest ratio of [44]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	18	0.111
2	A	2	2	1.00	18	0.111
3	A	2	2	1.00	18	0.111
4	A	1	1	1.00	16	0.062
5	A	1	1	1.00	16	0.062
6	A	1	1	1.00	18	0.056
7	A	2	2	1.00	18	0.111
8	A	2	2	1.00	18	0.111
9	A	3	2	1.00	8	0.250
10	A	2	2	1.00	18	0.111
11	A	2	2	1.00	18	0.111
12	A	2	2	1.00	18	0.111
13	A	1	1	1.00	16	0.062
14	A	1	1	1.00	16	0.062
15	A	1	1	1.00	18	0.056
16	A	2	2	1.00	18	0.111
17	A	2	2	1.00	18	0.111
18	A	3	2	1.00	8	0.250
19	A	6	3	1.00	18	0.167
20	A	5	3	1.00	18	0.167
21	A	4	3	1.00	16	0.188
22	A	4	3	1.00	16	0.188
23	A	5	3	1.00	18	0.167
24	A	6	3	1.00	18	0.167
25	A	5	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	18	0.111
27	A	2	2	1.00	18	0.111
28	F	0	0	N/A	0.	N/A
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	11	5	1.00	44	0.114
32	F	0	0	N/A	0.	N/A
33	A	7	4	1.00	43	0.093
34	B	14	6	19.24	35	0.171
35	A	1	1	1.00	30	0.033
36	A	6	3	1.00	38	0.079
37	A	10	4	1.00	38	0.105
38	A	3	3	1.00	20	0.150
39	A	4	2	1.00	22	0.091
40	A	4	2	1.00	22	0.091
41	A	4	2	1.00	22	0.091
42	A	4	3	1.00	24	0.125
43	A	5	2	1.00	24	0.083
44	A	4	2	1.00	22	0.091
45	A	5	2	1.00	24	0.083
46	A	4	2	1.00	24	0.083
47	A	4	3	1.00	7	0.429
48	A	11	5	1.00	9	0.556
49	A	4	3	1.00	7	0.429
50	A	11	5	1.00	9	0.556
51	A	4	3	1.00	19	0.158
52	A	3	1	1.00	15	0.067
53	A	5	4	1.00	26	0.154
54	A	5	4	1.00	27	0.148
55	A	5	4	1.00	26	0.154
56	A	5	4	1.00	27	0.148
57	A	6	3	1.00	10	0.300
58	A	6	3	1.00	10	0.300
59	A	6	3	1.00	12	0.250
60	A	6	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	15	0.133
62	A	2	2	1.00	8	0.250
63	A	3	3	1.00	12	0.250
64	A	2	2	1.00	8	0.250
65	A	2	2	1.00	12	0.167
66	A	2	2	1.00	12	0.167
67	A	3	3	1.00	10	0.300
68	A	4	3	1.00	26	0.115
69	A	2	2	1.00	8	0.250
70	A	2	2	1.00	8	0.250
71	A	3	3	1.00	12	0.250
72	A	3	3	1.00	10	0.300
73	A	1	1	1.00	10	0.100
74	A	6	4	1.00	12	0.333
75	A	6	4	1.00	15	0.267
76	A	6	3	1.00	12	0.250
77	A	4	2	1.00	14	0.143
78	A	6	3	1.00	17	0.176
79	A	8	5	1.00	16	0.312
80	A	9	6	1.00	18	0.333
81	A	14	5	1.00	18	0.278
82	A	8	5	1.00	19	0.263
83	A	9	6	1.00	21	0.286
84	A	14	5	1.00	21	0.238
85	A	8	4	1.00	16	0.250
86	A	9	4	1.00	18	0.222
87	A	14	4	1.00	18	0.222
88	A	6	4	1.00	18	0.222
89	A	7	4	1.00	20	0.200
90	A	10	4	1.00	20	0.200
91	A	8	5	1.00	21	0.238
92	A	9	5	1.00	23	0.217
93	A	14	5	1.00	23	0.217
94	A	8	4	1.00	19	0.210
95	A	10	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	14	4	1.00	21	0.190
97	A	8	5	1.00	21	0.238
98	A	10	5	1.00	23	0.217
99	A	14	5	1.00	23	0.217
100	A	8	5	1.00	24	0.208
101	A	10	5	1.00	26	0.192
102	A	14	5	1.00	26	0.192
103	A	8	5	1.00	24	0.208
104	A	1	1	1.00	10	0.100
105	A	6	4	1.00	12	0.333
106	A	6	4	1.00	15	0.267
107	A	6	3	1.00	12	0.250
108	A	4	2	1.00	14	0.143
109	A	6	3	1.00	17	0.176
110	A	8	5	1.00	16	0.312
111	A	9	6	1.00	18	0.333
112	A	14	5	1.00	18	0.278
113	A	8	5	1.00	19	0.263
114	A	9	6	1.00	21	0.286
115	A	14	5	1.00	21	0.238
116	A	8	4	1.00	16	0.250
117	A	9	4	1.00	18	0.222
118	A	14	4	1.00	18	0.222
119	A	6	4	1.00	18	0.222
120	A	7	4	1.00	20	0.200
121	A	10	4	1.00	20	0.200
122	A	8	5	1.00	21	0.238
123	A	9	5	1.00	23	0.217
124	A	14	5	1.00	23	0.217
125	A	8	4	1.00	19	0.210
126	A	10	4	1.00	21	0.190
127	A	14	4	1.00	21	0.190
128	A	8	5	1.00	21	0.238
129	A	10	5	1.00	23	0.217
130	A	14	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	8	5	1.00	24	0.208
132	A	10	5	1.00	26	0.192
133	A	14	5	1.00	26	0.192
134	A	8	5	1.00	24	0.208
135	A	8	6	1.00	22	0.273
136	A	6	5	1.00	20	0.250
137	A	2	2	1.00	22	0.091
138	A	3	3	1.00	22	0.136
139	A	8	6	1.00	22	0.273
140	A	6	5	1.00	20	0.250
141	A	2	2	1.00	22	0.091
142	A	3	3	1.00	22	0.136

Chapter 3

Listing of integrals

Local contents

3.1 $\int F^{c(a+bx)}\sin^n(d+ex)\,dx$

Optimal. Leaf size=107

$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)\sin^n(d+ex)}{ien-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/2*n-1/2*I*b*c*
ln(F)/e],exp(2*I*(e*x+d)))*sin(e*x+d)^n/((1-exp(2*I*(e*x+d)))^n)/(I*e*n-b*c
*ln(F))
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4525, 2291}

$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\sin^n(d+ex)\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right);e^{2i(d+ex)}\right)}{-bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*Log[F])/e, (2 -
n - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))]*Sin[d + e*x]^n)/((1 - E^((2*I
)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4525

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Dist[E^(I*n*(d + e*x))*(Sin[d + e*x]^n/(-1 + E^(2*I*(d + e*x)))^n), I
nt[F^(c*(a + b*x))*((-1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /
; FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \left(e^{in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^{-n}\sin^n(d+ex)\right)\int e^{-in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^n F^{c(a+b}$$

$$= -\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right);e^{2i(d+e}\right.}{ien-bc\log(F)}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 1.03

$$\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\,{}_2F_1\left(-n,-\frac{i(-ien+bc\log(F))}{2e};1-\frac{i(-ien+bc\log(F))}{2e};e^{2i(d+ex)}\right)\sin^n(d+ex)}{-ien+bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n + b*c*Log[F]))/e
, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, E^((2*I)*(d + e*x))]*Sin[d + e*x]^
n)/((1 - E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\sin^n(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sin(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sin(x*e + d)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\sin^n(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sin(d + e*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\sin\left(d+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)^n,x)
```

```
[Out] int(F^(c*(a + b*x))*sin(d + e*x)^n, x)
```

3.2 $\int F^{c(a+bx)} \sin^3(d+ex)\,dx$

Optimal. Leaf size=199

$$-\frac{6e^3F^{c(a+bx)}\cos(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{6bce^2F^{c(a+bx)}\log(F)\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}-\frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(}{9e^2+b^2c^2\log^2(F)}$$

```
[Out] -6*e^3*F^(c*(b*x+a))*cos(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)
^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sin(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+
b^4*c^4*ln(F)^4)-3*e*F^(c*(b*x+a))*cos(e*x+d)*sin(e*x+d)^2/(9*e^2+b^2*c^2*l
n(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*sin(e*x+d)^3/(9*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4519, 4517}

$$\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}-\frac{3e\sin^2(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6bce^2\log(F)\sin(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4}-\frac{6e^3\cos(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cos[d + e*x])/(9*e^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^
4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sin[d + e*x])/(9*e^4 +
10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) - (3*e*F^(c*(a + b*x))*Cos[d +
e*x]*Sin[d + e*x]^2)/(9*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(c*(a + b*x))*Log[
F]*Sin[d + e*x]^3)/(9*e^2 + b^2*c^2*Log[F]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx = -\frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(d+ex)}{9e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+bx)}\log(F)\sin^3(d+ex)}{9e^2+b^2c^2\log^2(F)}+\frac{(6e^2}{}$$

$$= -\frac{6e^3F^{c(a+bx)}\cos(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{6bce^2F^{c(a+bx)}\log(F)\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(}$$

Mathematica [A]
time = 0.70, size = 154, normalized size = 0.77

$$\frac{F^{c(a+bx)}\left(-3e\cos(d+ex)\left(9e^2+b^2c^2\log^2(F)\right)+3\cos(3(d+ex))\left(e^3+b^2c^2e\log^2(F)\right)-2bc\log(F)\left(-13e^2-b^2c^2\log^2(F)+\cos(2(d+ex))\left(e^2+b^2c^2\log^2(F)\right)\right)\sin(d+ex)\right)}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-3*e*Cos[d + e*x]*(9*e^2 + b^2*c^2*Log[F]^2) + 3*Cos[3*(d
 + e*x)]*(e^3 + b^2*c^2*e*Log[F]^2) - 2*b*c*Log[F]*(-13*e^2 - b^2*c^2*Log[F
]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A]
time = 0.64, size = 377, normalized size = 1.89

method	result
risch	$-\frac{3e\,F^{c(bx+a)}\cos(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3bc\,F^{c(bx+a)}\ln(F)\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(bx+a)}\cos(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}-\frac{cb\ln(F)F^{c(bx+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
default	$-\frac{F^{ac}\left(\frac{\frac{4e\,\mathrm{e}^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}-\frac{4e\,\mathrm{e}^{bcx\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e^2+b^2c^2\ln(F)^2}-\frac{8bc\ln(F)\mathrm{e}^{bcx\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}+\frac{-\frac{3e\,\mathrm{e}^{bcx\ln(F)}}{9e^2+b^2c^2\ln(F)^2}+\frac{3e\,\mathrm{e}^{bcx\ln(F)}\left(\tan^2\left(\frac{3ex}{2}+\frac{3d}{2}\right.\right.}{9e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{3ex}{2}+\right.}\right.}{4}$
norman	$\frac{-\frac{6e^3\mathrm{e}^{c(bx+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e^3\mathrm{e}^{c(bx+a)\ln(F)}\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{6e\left(2b^2c^2\ln(F)^2+3e^2\right)\mathrm{e}^{c(bx+a)\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(2b^2c^2\ln\right.}{9e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*F^(a*c)*((4/(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))-4/(e^2+b^2*c^2*ln
(F)^2)*e*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)^2-8*b*c*ln(F)/(e^2+b^2*c^2*ln(
F)^2)*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x))/(1+tan(1/2*d+1/2*e*x)^2)+(-3/(9*
e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))+3/(9*e^2+b^2*c^2*ln(F)^2)*e*exp(b*c
*x*ln(F))*tan(3/2*e*x+3/2*d)^2+2*b*c*ln(F)/(9*e^2+b^2*c^2*ln(F)^2)*exp(b*c*
x*ln(F))*tan(3/2*e*x+3/2*d))/(1+tan(3/2*e*x+3/2*d)^2)+(1/(e^2+b^2*c^2*ln(F)
```

```
^2)*e*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)^2-1/(e^2+b^2*c^2*ln(F)^2)*e*exp(b
*c*x*ln(F))+2*b*c*ln(F)/(e^2+b^2*c^2*ln(F)^2)*exp(b*c*x*ln(F))*tan(1/2*d+1/
2*e*x))/(1+tan(1/2*d+1/2*e*x)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(197) = 394$.
time = 0.34, size = 761, normalized size = 3.82

$$\frac{\left(3\left(F^{ac}b^2c^2e\log\left(F\right)^2+F^{ac}e^3\right)\cos\left(3d\right)-\left(F^{ac}b^3c^3\log\left(F\right)^3+F^{ac}bce^2\log\left(F\right)\right)\sin\left(3d\right)\right)F^{bcx}\cos\left(3xe\right)+\left(3\left(F^{ac}b^2c^2e\log\left(F\right)^2+F^{ac}e^3\right)\cos\left(3d\right)+\left(F^{ac}b^3c^3\log\left(F\right)^3+F^{ac}bce^2\log\left(F\right)\right)\sin\left(3d\right)\right)F^{bcx}\cos\left(3xe+6d\right)-3\left(\left(F^{ac}b^2c^2e\log\left(F\right)^2+9F^{ac}e^3\right)\cos\left(3d\right)+\left(F^{ac}b^3c^3\log\left(F\right)^3+9F^{ac}bce^2\log\left(F\right)\right)\sin\left(3d\right)\right)F^{bcx}\cos\left(xe+4d\right)-3\left(\left(F^{ac}b^2c^2e\log\left(F\right)^2+9F^{ac}e^3\right)\cos\left(3d\right)-\left(F^{ac}b^3c^3\log\left(F\right)^3+9F^{ac}bce^2\log\left(F\right)\right)\sin\left(3d\right)\right)F^{bcx}\cos\left(xe-2d\right)-\left(\left(F^{ac}b^3c^3\log\left(F\right)^3+F^{ac}bce^2\log\left(F\right)\right)\cos\left(3d\right)+3\left(F^{ac}b^2c^2e\log\left(F\right)^2+F^{ac}e^3\right)\sin\left(3d\right)\right)F^{bcx}\sin\left(3xe\right)-\left(\left(F^{ac}b^3c^3\log\left(F\right)^3+F^{ac}bce^2\log\left(F\right)\right)\cos\left(3d\right)-3\left(F^{ac}b^2c^2e\log\left(F\right)^2+F^{ac}e^3\right)\sin\left(3d\right)\right)F^{bcx}\sin\left(3xe+6d\right)+3\left(\left(F^{ac}b^3c^3\log\left(F\right)^3+9F^{ac}bce^2\log\left(F\right)\right)\cos\left(3d\right)-\left(F^{ac}b^2c^2e\log\left(F\right)^2+9F^{ac}e^3\right)\sin\left(3d\right)\right)F^{bcx}\sin\left(xe+4d\right)+3\left(\left(F^{ac}b^3c^3\log\left(F\right)^3+9F^{ac}bce^2\log\left(F\right)\right)\cos\left(3d\right)+\left(F^{ac}b^2c^2e\log\left(F\right)^2+9F^{ac}e^3\right)\sin\left(3d\right)\right)F^{bcx}\sin\left(xe-2d\right)}{8\left(\left(b^4c^4\log\left(F\right)^4+10b^2c^2e^2\log\left(F\right)^2+9e^4\right)\cos\left(3d\right)^2+\left(b^4c^4\log\left(F\right)^4+10b^2c^2e^2\log\left(F\right)^2+9e^4\right)\sin\left(3d\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(3*d) - (F^(a*c)*b^3*
c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*cos(3*x*e) + (3*
(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(3*d) + (F^(a*c)*b^3*c^3*log(
F)^3 + F^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*cos(3*x*e + 6*d) - 3*((F
^(a*c)*b^2*c^2*e*log(F)^2 + 9*F^(a*c)*e^3)*cos(3*d) + (F^(a*c)*b^3*c^3*log(
F)^3 + 9*F^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*cos(x*e + 4*d) - 3*((F
^(a*c)*b^2*c^2*e*log(F)^2 + 9*F^(a*c)*e^3)*cos(3*d) - (F^(a*c)*b^3*c^3*log(
F)^3 + 9*F^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*cos(x*e - 2*d) - ((F^(
a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*cos(3*d) + 3*(F^(a*c)*b^2*c
^2*e*log(F)^2 + F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*sin(3*x*e) - ((F^(a*c)*b^3
*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*cos(3*d) - 3*(F^(a*c)*b^2*c^2*e*log
(F)^2 + F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*sin(3*x*e + 6*d) + 3*((F^(a*c)*b^3
*c^3*log(F)^3 + 9*F^(a*c)*b*c*e^2*log(F))*cos(3*d) - (F^(a*c)*b^2*c^2*e*log
(F)^2 + 9*F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*sin(x*e + 4*d) + 3*((F^(a*c)*b^3
*c^3*log(F)^3 + 9*F^(a*c)*b*c*e^2*log(F))*cos(3*d) + (F^(a*c)*b^2*c^2*e*log
(F)^2 + 9*F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*sin(x*e - 2*d))/((b^4*c^4*log(F)
^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)*cos(3*d)^2 + (b^4*c^4*log(F)^4 + 10*b
^2*c^2*e^2*log(F)^2 + 9*e^4)*sin(3*d)^2)
```

Fricas [A]
time = 1.98, size = 174, normalized size = 0.87

$$\frac{\left(3\,\cos\left(xe+d\right)^3e^3+3\left(b^2c^2\cos\left(xe+d\right)^3e-b^2c^2\cos\left(xe+d\right)e\right)\log\left(F\right)^2-9\,\cos\left(xe+d\right)e^3-\left(\left(b^3c^3\cos\left(xe+d\right)^2-b^3c^3\right)\log\left(F\right)^3+\left(bc\cos\left(xe+d\right)^2e^2-7\,bce^2\right)\log\left(F\right)\right)\sin\left(xe+d\right)\right)F^{bcx+ac}}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(F\right)^2+9\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] (3*cos(x*e + d)^3*e^3 + 3*(b^2*c^2*cos(x*e + d)^3*e - b^2*c^2*cos(x*e + d)*
e)*log(F)^2 - 9*cos(x*e + d)*e^3 - ((b^3*c^3*cos(x*e + d)^2 - b^3*c^3)*log(
F)^3 + (b*c*cos(x*e + d)^2*e^2 - 7*b*c*e^2)*log(F))*sin(x*e + d))*F^(b*c*x
+ a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.46, size = 1275, normalized size = 6.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c
*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 6*e)*cos(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c
^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) + 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*p
i*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^
2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(
1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x +
 d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*
log(abs(F)) + a*c*log(abs(F))) - 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sg
n(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*
log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c
 - 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2
))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*log(abs(F))*sin(1/2
*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x -
3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 6*e)^2) - (pi*b*c
*sgn(F) - pi*b*c - 6*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c
*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c - 6*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (I*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + 3
*I*e*x + 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*I
*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c - 3*I*e*x - 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*l
og(abs(F)) - 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 3*(-I*e^(1/
2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c +
```

```
 I*e*x + I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*I*e
) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/
2*I*pi*a*c - I*e*x - I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs
(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 3*(I*e^(1/2*I*pi*
b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x
- I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*I*e) + I*e
^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*
a*c + I*e*x + I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) +
16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (-I*e^(1/2*I*pi*b*c*x*sg
n(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 3*I*e*x - 3*I*
d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*I*e) - I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c
+ 3*I*e*x + 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) +
48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 3.22, size = 190, normalized size = 0.95

$$-\frac{3\,F^{c\,(a+b\,x)}\,(\cos(e\,x)-\sin(e\,x)\,1\mathrm{i})\,(\cos(d)-\sin(d)\,1\mathrm{i})}{8\,(e+b\,c\,\ln(F)\,1\mathrm{i})}+\frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)+\sin(3\,e\,x)\,1\mathrm{i})\,(\cos(3\,d)+\sin(3\,d)\,1\mathrm{i})\,1\mathrm{i}}{8\,(b\,c\,\ln(F)+e\,3\mathrm{i})}+\frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)-\sin(3\,e\,x)\,1\mathrm{i})\,(\cos(3\,d)-\sin(3\,d)\,1\mathrm{i})}{8\,(3\,e+b\,c\,\ln(F)\,1\mathrm{i})}-\frac{F^{c\,(a+b\,x)}\,(\cos(e\,x)+\sin(e\,x)\,1\mathrm{i})\,(\cos(d)+\sin(d)\,1\mathrm{i})\,3\mathrm{i}}{8\,(b\,c\,\ln(F)+e\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)^3,x)
```

```
[Out] (F^(c*(a + b*x))*(cos(3*e*x) + sin(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/
(8*(e*3i + b*c*log(F))) - (3*F^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(cos(
d) - sin(d)*1i))/(8*(e + b*c*log(F)*1i)) + (F^(c*(a + b*x))*(cos(3*e*x) - s
in(3*e*x)*1i)*(cos(3*d) - sin(3*d)*1i))/(8*(3*e + b*c*log(F)*1i)) - (F^(c*(
a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e*1i + b*c*
log(F)))
```

3.3 $\int F^{c(a+bx)} \sin^2(d+ex)\,dx$

Optimal. Leaf size=128

$$\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}-\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

```
[Out] 2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)-2*e*F^(c*(b*x+a))*cos
(e*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*sin(e*x+
d)^2/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4519, 2225}

$$\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) - (2*e*F^(c
*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^
(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx=-\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{(2e^2)}{4e^2+}$$

$$=\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}-\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+}}{4}$$

Mathematica [A]
time = 0.21, size = 86, normalized size = 0.67

$$\frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)-b^2c^2\cos(2(d+ex))\log^2(F)-2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 - b^2*c^2*Cos[2*(d + e*x)]*Log[F
]^2 - 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F
]^3)
```

Maple [A]
time = 0.28, size = 106, normalized size = 0.83

method	result
risch	$\frac{F^{c(bx+a)}}{2bc\ln(F)}-\frac{\ln(F)cb\,F^{c(bx+a)}\cos(2ex+2d)}{2\left(4e^2+b^2c^2\ln(F)^2\right)}-\frac{e\,F^{c(bx+a)}\sin(2ex+2d)}{4e^2+b^2c^2\ln(F)^2}$
norman	$\frac{-\frac{4e\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{4e^2+b^2c^2\ln(F)^2}+\frac{4e\,\mathrm{e}^{c(bx+a)\ln(F)}\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4e^2+b^2c^2\ln(F)^2}+\frac{2e^2\mathrm{e}^{c(bx+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{2e^2\mathrm{e}^{c(bx+a)\ln(F)}\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4\left(e^2+b^2\right.}{}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*F^(c*(b*x+a))/b/c/ln(F)-1/2/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*b*F^(c*(b*x
+a))*cos(2*e*x+2*d)-e*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(128) = 256.
time = 0.29, size = 348, normalized size = 2.72

$$-\frac{\left(F^{ac}b^2c^2\cos(2d)\log(F)^2+2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe)+\left(F^{ac}b^2c^2\cos(2d)\log(F)^2-2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe+4d)-\left(F^{ac}b^2c^2\log(F)^2\sin(2d)-2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe)+\left(F^{ac}b^2c^2\log(F)^2\sin(2d)+2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe+4d)-2\left(\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\cos(2d)^2+\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\sin(2d)^2\right)F^{bcx}}{4\left(\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\cos(2d)^2+\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\sin(2d)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*x*e) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*
e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*x*e + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*s
in(2*d) - 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(2*x*e) + (F^(a*c)*
b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(
2*x*e + 4*d) - 2*((F^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*cos(2*d)^2 + (
F^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*sin(2*d)^2)*F^(b*c*x))/((b^3*c^3*
```

```
log(F)^3 + 4*b*c*e^2*log(F))*cos(2*d)^2 + (b^3*c^3*log(F)^3 + 4*b*c*e^2*log
(F))*sin(2*d)^2)
```

Fricas [A]
time = 2.77, size = 91, normalized size = 0.71

$$-\frac{\left(2\,bc\cos\left(xe+d\right)e\log\left(F\right)\sin\left(xe+d\right)+\left(b^2c^2\cos\left(xe+d\right)^2-b^2c^2\right)\log\left(F\right)^2-2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*cos(x*e + d)*e*log(F)*sin(x*e + d) + (b^2*c^2*cos(x*e + d)^2 - b^2*
c^2)*log(F)^2 - 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F)
)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.47, size = 1117, normalized size = 8.73

$$\begin{cases} \frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}-\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} & \text{for}\: F=1 \\ \frac{b^2c^2\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{2ie}{bc}}\right)^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)}-\frac{2bce\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{2ie}{bc}}\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\sin^2\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\cos^2\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)} & \text{for}\: F=e^{-\frac{2ie}{bc}} \\ \frac{b^2c^2\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{\frac{2ie}{bc}}\right)^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)}-\frac{2bce\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{\frac{2ie}{bc}}\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\sin^2\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\cos^2\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)} & \text{for}\: F=e^{\frac{2ie}{bc}} \\ F^{ac}\left(\frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}-\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}\right) & \text{for}\: b=0 \\ \frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}-\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} & \text{for}\: c=0 \\ \frac{F^{ac}F^{bcx}b^2c^2\log\left(F\right)^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}-\frac{2F^{ac}F^{bcx}bce\log\left(F\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac}F^{bcx}e^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac}F^{bcx}e^2\cos^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d +
 e*x)/(2*e), Eq(F, 1)), (b**2*c**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c
))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*sin(d + e*x)**2/(b**3*c**3*log(exp(-2
*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-2*I*e/(b*c)))) - 2*b*c*e*exp(-2*I*e/(
b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))*sin(d + e*x)
*cos(d + e*x)/(b**3*c**3*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-2*
I*e/(b*c)))) + 2*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*s
in(d + e*x)**2/(b**3*c**3*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-2
*I*e/(b*c)))) + 2*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*
cos(d + e*x)**2/(b**3*c**3*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-
2*I*e/(b*c)))), Eq(F, exp(-2*I*e/(b*c)))), (b**2*c**2*exp(2*I*e/(b*c))**(a*
c)*exp(2*I*e/(b*c))**(b*c*x)*log(exp(2*I*e/(b*c)))**2*sin(d + e*x)**2/(b**3
*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(2*I*e/(b*c)))) - 2*b*c*
e*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*log(exp(2*I*e/(b*c)))*s
in(d + e*x)*cos(d + e*x)/(b**3*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*l
```

```
og(exp(2*I*e/(b*c))) + 2*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b
*c*x)*sin(d + e*x)**2/(b**3*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*log(
exp(2*I*e/(b*c)))) + 2*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*
x)*cos(d + e*x)**2/(b**3*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp
(2*I*e/(b*c)))), Eq(F, exp(2*I*e/(b*c)))), (F**(a*c)*(x*sin(d + e*x)**2/2 +
 x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(
d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c
, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*sin(d + e*x)**2/(b**3*c**3*
log(F)**3 + 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sin(d +
 e*x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c)*F
**(b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) +
2*F**(a*c)*F**(b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**
2*log(F)), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.45, size = 915, normalized size = 7.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b^2*c
^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/
2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(
F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*
sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2
*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e
^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(a
bs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*
pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2
*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c +
 8*b*c*log(abs(F)) + 16*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
 - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F
) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c +
 8*b*c*log(abs(F)) - 16*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
```

```
 - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F
) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)))
 - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2
*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*
log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 3.02, size = 95, normalized size = 0.74

$$\frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2+\frac{b^2\,c^2\,\ln(F)^2}{2}-\frac{b^2\,c^2\,\ln(F)^2\,\cos(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sin\left(2\,d+2\,e\,x\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\,\ln\left(F\right)^2+4\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)^2,x)
```

```
[Out] (F^(a*c + b*c*x)*(2*e^2 + (b^2*c^2*log(F)^2)/2 - (b^2*c^2*log(F)^2*cos(2*d
+ 2*e*x))/2 - b*c*e*log(F)*sin(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 + b^2*c^2*
log(F)^2))
```

3.4 $\int F^{c(a+bx)} \sin(d+ex)\,dx$

Optimal. Leaf size=73

$$-\frac{eF^{c(a+bx)}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+bx)}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] -e*F^(c*(b*x+a))*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*s
in(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4517}

$$\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x],x]
```

```
[Out] -((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sin(d+ex)\,dx=-\frac{eF^{c(a+bx)}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+bx)}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

Mathematica [A]
time = 0.12, size = 48, normalized size = 0.66

$$\frac{F^{c(a+bx)}(-e\cos(d+ex)+bc\log(F)\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Cos[d + e*x]) + b*c*Log[F]*Sin[d + e*x]))/(e^2 + b^2*
c^2*Log[F]^2)
```

Maple [A]
time = 0.08, size = 74, normalized size = 1.01

method	result	size
risch	$-\frac{e\,F^{c(bx+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{bc\,F^{c(bx+a)}\ln(F)\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	74
norman	$\frac{\frac{e\,\mathrm{e}^{c(bx+a)\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e^2+b^2c^2\ln(F)^2}-\frac{e\,\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2bc\ln(F)\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e*F^(c*(b*x+a))*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*s
in(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(74) = 148.
time = 0.28, size = 197, normalized size = 2.70

$$-\frac{\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)+F^{ac}\cos\left(d\right)e\right)F^{bcx}\cos\left(xe+2\,d\right)-\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)-F^{ac}\cos\left(d\right)e\right)F^{bcx}\cos\left(xe\right)-\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)-F^{ac}e\sin\left(d\right)\right)F^{bcx}\sin\left(xe+2\,d\right)-\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)+F^{ac}e\sin\left(d\right)\right)F^{bcx}\sin\left(xe\right)}{2\left(\left(b^2c^2\log\left(F\right)^2+e^2\right)\cos\left(d\right)^2+\left(b^2c^2\log\left(F\right)^2+e^2\right)\sin\left(d\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*cos(x*e + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*cos(x*e) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(x*e + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(x*e))/((b^2*c^2*l
og(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(d)^2)
```

Fricas [A]
time = 2.24, size = 51, normalized size = 0.70

$$\frac{\left(bc\log\left(F\right)\sin\left(xe+d\right)-\cos\left(xe+d\right)e\right)F^{bcx+ac}}{b^2c^2\log\left(F\right)^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] (b*c*log(F)*sin(x*e + d) - cos(x*e + d)*e)*F^(b*c*x + a*c)/(b^2*c^2*log(F)^
2 + e^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.64, size = 462, normalized size = 6.33

$$\begin{cases} \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}} x \sin(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}} ix \cos(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}} i \sin(d+ex)}{2e} - \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}} \cos(d+ex)}{e} & \text{for } F=-1 \wedge b=\frac{e}{\pi c} \\ x\sin(d) & \text{for } F=1 \wedge e=0 \\ \frac{bc\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)\sin(d+ex)}{b^2c^2\log\left(e^{-\frac{ie}{bc}}\right)^2+e^2} - \frac{e\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\cos(d+ex)}{b^2c^2\log\left(e^{-\frac{ie}{bc}}\right)^2+e^2} & \text{for } F=e^{-\frac{ie}{bc}} \\ \frac{bc\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)\sin(d+ex)}{b^2c^2\log\left(e^{\frac{ie}{bc}}\right)^2+e^2} - \frac{e\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\cos(d+ex)}{b^2c^2\log\left(e^{\frac{ie}{bc}}\right)^2+e^2} & \text{for } F=e^{\frac{ie}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\sin(d+ex)}{b^2c^2\log(F)^2+e^2} - \frac{F^{ac}F^{bcx}e\cos(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d),x)
```

```
[Out] Piecewise(((-1)**(a*c)*(-1)**(e*x/pi)*x*sin(d + e*x)/2 + (-1)**(a*c)*(-1)**
(e*x/pi)*I*x*cos(d + e*x)/2 + (-1)**(a*c)*(-1)**(e*x/pi)*I*sin(d + e*x)/(2*
e) - (-1)**(a*c)*(-1)**(e*x/pi)*cos(d + e*x)/e, Eq(F, -1) & Eq(b, e/(pi*c))
), (x*sin(d), Eq(F, 1) & Eq(e, 0)), (b*c*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b
*c))**(b*c*x)*log(exp(-I*e/(b*c)))*sin(d + e*x)/(b**2*c**2*log(exp(-I*e/(b*
c)))**2 + e**2) - e*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*cos(d +
 e*x)/(b**2*c**2*log(exp(-I*e/(b*c)))**2 + e**2), Eq(F, exp(-I*e/(b*c)))),
(b*c*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*log(exp(I*e/(b*c)))*sin(
d + e*x)/(b**2*c**2*log(exp(I*e/(b*c)))**2 + e**2) - e*exp(I*e/(b*c))**(a*c
)*exp(I*e/(b*c))**(b*c*x)*cos(d + e*x)/(b**2*c**2*log(exp(I*e/(b*c)))**2 +
e**2), Eq(F, exp(I*e/(b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log(F)*sin(d + e*x)
/(b**2*c**2*log(F)**2 + e**2) - F**(a*c)*F**(b*c*x)*e*cos(d + e*x)/(b**2*c*
*2*log(F)**2 + e**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 634, normalized size = 8.68

$$\left(\frac{2\,bc\log\left(|F|\right)\sin\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac+ex+d\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc+2\,e\right)^2}-\frac{\left(\pi bc\mathrm{sgn}(F)-\pi bc+2\,e\right)\cos\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac+ex+d\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc+2\,e\right)^2}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}-\left(\frac{2\,bc\log\left(|F|\right)\sin\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac-ex-d\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc-2\,e\right)^2}-\frac{\left(\pi bc\mathrm{sgn}(F)-\pi bc-2\,e\right)\cos\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac-ex-d\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc-2\,e\right)^2}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}-\left(-\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac+i\,ex+i\,d\right)}}{2i\,\pi bc\mathrm{sgn}\left(F\right)-2i\,\pi bc+4\,bc\log\left(|F|\right)+4i\,e}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac-i\,ex-i\,d\right)}}{-2i\,\pi bc\mathrm{sgn}\left(F\right)+2i\,\pi bc+4\,bc\log\left(|F|\right)-4i\,e}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}-\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac-i\,ex-i\,d\right)}}{2i\,\pi bc\mathrm{sgn}\left(F\right)-2i\,\pi bc+4\,bc\log\left(|F|\right)-4i\,e}+\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac+i\,ex+i\,d\right)}}{-2i\,\pi bc\mathrm{sgn}\left(F\right)+2i\,\pi bc+4\,bc\log\left(|F|\right)+4i\,e}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="giac")
```

```
[Out] (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(
F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) - (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
```

```
) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*cos(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F))) - (-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*
c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*
b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*
pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))
) - (I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F
)) - 4*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*s
gn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*
c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 2.40, size = 50, normalized size = 0.68

$$-\frac{F^{a\,c+b\,c\,x}\,\left(e\,\cos\left(d+e\,x\right)-b\,c\,\sin\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b^2\,c^2\,{\ln\left(F\right)}^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sin(d + e*x),x)
```

```
[Out] -(F^(a*c + b*c*x)*(e*cos(d + e*x) - b*c*sin(d + e*x)*log(F)))/(e^2 + b^2*c^
2*log(F)^2)
```

3.5 $\int F^{c(a+bx)} \csc(d+ex)\,dx$

Optimal. Leaf size=81

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

```
[Out] -2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1
/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(e-I*b*c*ln(F))
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{4538\}$

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x],x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))])/(e - I*b*c*Log[F])
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

Mathematica [A]
time = 1.44, size = 114, normalized size = 1.41

$$\frac{iF^{c(a+bx)}\left({}_2F_1\left(1,-\frac{ibc\log(F)}{e};1-\frac{ibc\log(F)}{e};-\cos(d+ex)-i\sin(d+ex)\right)-{}_2F_1\left(1,-\frac{ibc\log(F)}{e};1-\frac{ibc\log(F)}{e};\cos(d+ex)+i\sin(d+ex)\right)\right)}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x],x]
```

```
[Out] (I*F^(c*(a + b*x))*(Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Lo
g[F])/e, -Cos[d + e*x] - I*Sin[d + e*x]] - Hypergeometric2F1[1, ((-I)*b*c*L
og[F])/e, 1 - (I*b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]))/(b*c*Log[F
])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)} \csc \left(ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(x*e + d) + F^(a*c)*cos(x*e + d)*e^(b*c*
x*log(F) + 1) - (F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(x*e + d) + F^(a*c)*cos(x*
e + d)*e^(b*c*x*log(F) + 1))*cos(2*x*e + 2*d) - 2*(F^(a*c)*b^2*c^2*log(F)^2
 + (F^(a*c)*b^2*c^2*log(F)^2 + F^(a*c)*e^2)*cos(2*x*e + 2*d)^2 + (F^(a*c)*b
^2*c^2*log(F)^2 + F^(a*c)*e^2)*sin(2*x*e + 2*d)^2 - 2*(F^(a*c)*b^2*c^2*log(
F)^2 + F^(a*c)*e^2)*cos(2*x*e + 2*d) + F^(a*c)*e^2)*integrate((b*c*cos(x*e
+ d)*e^(b*c*x*log(F) + 1)*log(F) + (b*c*cos(x*e + d)*e^(b*c*x*log(F) + 1)*l
og(F) - e^(b*c*x*log(F) + 2)*sin(x*e + d))*cos(4*x*e + 4*d) - 2*(b*c*cos(x*
e + d)*e^(b*c*x*log(F) + 1)*log(F) - e^(b*c*x*log(F) + 2)*sin(x*e + d))*cos
(2*x*e + 2*d) + (b*c*e^(b*c*x*log(F) + 1)*log(F)*sin(x*e + d) + cos(x*e + d
)*e^(b*c*x*log(F) + 2))*sin(4*x*e + 4*d) - 2*(b*c*e^(b*c*x*log(F) + 1)*log(
F)*sin(x*e + d) + cos(x*e + d)*e^(b*c*x*log(F) + 2))*sin(2*x*e + 2*d) - e^(
b*c*x*log(F) + 2)*sin(x*e + d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2
)*cos(4*x*e + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d)^2 + (b^2
*c^2*log(F)^2 + e^2)*sin(4*x*e + 4*d)^2 - 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*
x*e + 4*d)*sin(2*x*e + 2*d) + 4*(b^2*c^2*log(F)^2 + e^2)*sin(2*x*e + 2*d)^2
 + 2*(b^2*c^2*log(F)^2 - 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d) + e^2)
*cos(4*x*e + 4*d) - 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d) + e^2), x)
+ (F^(b*c*x)*F^(a*c)*b*c*cos(x*e + d)*log(F) - F^(a*c)*e^(b*c*x*log(F) + 1)
```

```
*sin(x*e + d))*sin(2*x*e + 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^
2)*cos(2*x*e + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(2*x*e + 2*d)^2 - 2*(b^
2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d) + e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\csc\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d),x)
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,\left(a+b\,x\right)}}{\sin\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sin(d + e*x),x)
[Out] int(F^(c*(a + b*x))/sin(d + e*x), x)
```

3.6 $\int F^{c(a+bx)} \csc^2(d+ex)\,dx$

Optimal. Leaf size=78

$$-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

```
[Out] -4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/2
*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4538}

$$-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

```
[Out] (-4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = -\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Mathematica [A]
time = 1.31, size = 101, normalized size = 1.29

$$-\frac{2iF^{c(a+bx)}\left(\left(-1+e^{2id}\right)\,{}_2F_1\left(1,-\frac{ibc\log(F)}{2e};1-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)+\csc(d+ex)\sin(d)(\cos(ex)-i\sin(ex))\right)}{e\left(-1+e^{2id}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

```
[Out] ((-2*I)*F^(c*(a + b*x))*((-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)*
b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))] + Csc[d + e*x
]*Sin[d]*(Cos[e*x] - I*Sin[e*x])))/(e*(-1 + E^((2*I)*d)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\csc^2\left(ex+d\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 4*(24*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3
 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*x*e + 2*d)^2 + 2*(F^(a*c)*b^3
*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*x*e + 2*d)^2 - (
F^(a*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*x*e +
 2*d) + 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*x
*e + 2*d) + (24*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F) - (F^(a*c)*b^3*c^3*
log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*x*e + 2*d) + 2*(F^(a*
c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*x*e + 2*d))*cos(4*x
*e + 4*d) - 4*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 +
 64*F^(a*c)*b*c*e^4*log(F) + (F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3
*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*cos(4*x*e + 4*d)^2 + 4*(F^(a*c)*
b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log
(F))*cos(2*x*e + 2*d)^2 + (F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^
2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*sin(4*x*e + 4*d)^2 - 4*(F^(a*c)*b^5
*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F)
)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a
*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*sin(2*x*e + 2*d)^2 +
2*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*
b*c*e^4*log(F) - 2*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F
)^3 + 64*F^(a*c)*b*c*e^4*log(F))*cos(2*x*e + 2*d))*cos(4*x*e + 4*d) - 4*(F^
```

```
(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e
^4*log(F))*cos(2*x*e + 2*d))*integrate(-(6*b*c*cos(6*x*e + 6*d)*e^(b*c*x*lo
g(F) + 2)*log(F) - 18*b*c*cos(4*x*e + 4*d)*e^(b*c*x*log(F) + 2)*log(F) + 18
*b*c*cos(2*x*e + 2*d)*e^(b*c*x*log(F) + 2)*log(F) - 6*b*c*e^(b*c*x*log(F) +
 2)*log(F) - (b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(6*x*e + 6*d) + 3*(b
^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(4*x*e + 4*d) - 3*(b^2*c^2*e*log(F)
^2 - 8*e^3)*F^(b*c*x)*sin(2*x*e + 2*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*
log(F)^2 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(6*x*e
+ 6*d)^2 + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*x*
e + 4*d)^2 + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*
x*e + 2*d)^2 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(6*
x*e + 6*d)^2 + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(
4*x*e + 4*d)^2 - 18*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*s
in(4*x*e + 4*d)*sin(2*x*e + 2*d) + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log
(F)^2 + 64*e^4)*sin(2*x*e + 2*d)^2 - 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*l
og(F)^2 + 3*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*x*e
 + 4*d) - 3*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e
 + 2*d) + 64*e^4)*cos(6*x*e + 6*d) + 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*l
og(F)^2 - 3*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e
 + 2*d) + 64*e^4)*cos(4*x*e + 4*d) - 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*l
og(F)^2 + 64*e^4)*cos(2*x*e + 2*d) - 6*((b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*
log(F)^2 + 64*e^4)*sin(4*x*e + 4*d) - (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*lo
g(F)^2 + 64*e^4)*sin(2*x*e + 2*d))*sin(6*x*e + 6*d) + 64*e^4), x) - (2*(F^(
a*c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*cos(2*x*e + 2*d) + (F^(
a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*x*e + 2*
d) + 4*(F^(a*c)*b^2*c^2*e*log(F)^2 - 8*F^(a*c)*e^3)*F^(b*c*x))*sin(4*x*e +
4*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + (b^4*c^4*log(F)^4 + 20*
b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*x*e + 4*d)^2 + 4*(b^4*c^4*log(F)^4 + 2
0*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e + 2*d)^2 + (b^4*c^4*log(F)^4 + 2
0*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*x*e + 4*d)^2 - 4*(b^4*c^4*log(F)^4 +
 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 4*(b
^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(2*x*e + 2*d)^2 + 2*
(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 - 2*(b^4*c^4*log(F)^4 + 20*b^2*
c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e + 2*d) + 64*e^4)*cos(4*x*e + 4*d) - 4*
(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e + 2*d) + 64
*e^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(x*e + d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} \csc^2(d+ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sin(d + e*x)^2,x)
```

```
[Out] int(F^(c*(a + b*x))/sin(d + e*x)^2, x)
```

3.7 $\int F^{c(a+bx)} \csc^3(d+ex)\,dx$

Optimal. Leaf size=137

$$-\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e}-\frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2}-\frac{e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\right.\right.}{e^2}$$

```
[Out] -1/2*F^(c*(b*x+a))*cot(e*x+d)*csc(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csc(e*x+d)
*ln(F)/e^2-exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e
],[3/2-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(e+I*b*c*ln(F))/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4534, 4538}

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e^2}-\frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\cot(d+ex)\csc(d+ex)F^{c(a+bx)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

```
[Out] -1/2*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x])/e - (b*c*F^(c*(a + b*x))*C
sc[d + e*x]*Log[F])/(2*e^2) - (E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometr
ic2F1[1, (e - I*b*c*Log[F])/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d +
e*x))]*(e + I*b*c*Log[F]))/e^2
```

Rule 4534

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a + b*x
))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} + \frac{1}{2}\Bigg(1 + \frac{b^2}{}$$
$$= -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} - \frac{e^{i(d+ex)}F^{c}}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 334 vs. 2(137) = 274.
time = 7.14, size = 334, normalized size = 2.44

$$\frac{F^{c(a+bx)}\left(-e\csc^2\left(\frac{1}{2}(d+ex)\right) - 4bc\csc(d)\log(F) + \csc(d)\left(\frac{4e^2}{bc\log(F)} + 4bc\log(F)\right) + e\sec^2\left(\frac{1}{2}(d+ex)\right) - \frac{4i(e^2+b^2c^2\log^2(F))\left(1+{}_2F_1\left(1,-\frac{ibc\log(F)}{e};1-\frac{ibc\log(F)}{e};\cos(d+ex)+i\sin(d+ex)\right)(-1+\cos(d)+i\sin(d))\right)}{bc\log(F)(-1+\cos(d)+i\sin(d))} - \frac{4i(e^2+b^2c^2\log^2(F))\left(1-{}_2F_1\left(1,-\frac{ibc\log(F)}{e};1-\frac{ibc\log(F)}{e};-\cos(d+ex)-i\sin(d+ex)\right)(1+\cos(d)+i\sin(d))\right)}{bc\log(F)(1+\cos(d)+i\sin(d))} + 2bc\csc\left(\frac{d}{2}\right)\csc\left(\frac{1}{2}(d+ex)\right)\log(F)\sin\left(\frac{ex}{2}\right) - 2bc\log(F)\sec\left(\frac{d}{2}\right)\sec\left(\frac{1}{2}(d+ex)\right)\sin\left(\frac{ex}{2}\right)\right)}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csc[(d + e*x)/2]^2) - 4*b*c*Csc[d]*Log[F] + Csc[d]*((
4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + e*Sec[(d + e*x)/2]^2 - ((4*I)*(e^2 +
b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c
*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]*(-1 + Cos[d] + I*Sin[d])))/(b*c*
Log[F]*(-1 + Cos[d] + I*Sin[d])) - ((4*I)*(e^2 + b^2*c^2*Log[F]^2)*(1 - Hyp
ergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Log[F])/e, -Cos[d + e*x]
- I*Sin[d + e*x]]*(1 + Cos[d] + I*Sin[d])))/(b*c*Log[F]*(1 + Cos[d] + I*Sin
[d])) + 2*b*c*Csc[d/2]*Csc[(d + e*x)/2]*Log[F]*Sin[(e*x)/2] - 2*b*c*Log[F]*
Sec[d/2]*Sec[(d + e*x)/2]*Sin[(e*x)/2]))/(8*e^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\csc^3(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 8*(48*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F)*sin(x*e + d) - 6*(F^(a*c)*b^2
*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(x*e + d) - (48*F^(a*c)*b*c*
e^(b*c*x*log(F) + 2)*log(F)*sin(x*e + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 +
25*F^(a*c)*e^3)*F^(b*c*x)*cos(3*x*e + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2
- 15*F^(a*c)*e^3)*F^(b*c*x)*cos(x*e + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F
^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(3*x*e + 3*d))*cos(6*x*e + 6*d) + 3*(48
*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F)*sin(x*e + d) - 3*(F^(a*c)*b^2*c^2*
e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(3*x*e + 3*d) - 6*(F^(a*c)*b^2*c^
2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(x*e + d) - (F^(a*c)*b^3*c^3*lo
g(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(3*x*e + 3*d))*cos(4*x*e +
 4*d) + 3*(3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(2*
x*e + 2*d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*
x)*sin(2*x*e + 2*d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*
x))*cos(3*x*e + 3*d) - 18*(8*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F)*sin(x*
e + d) - (F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(x*e +
d))*cos(2*x*e + 2*d) - 6*(((F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c
^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 3
4*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*cos(6*
x*e + 6*d)^2 + 9*((F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*lo
g(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)
*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*cos(4*x*e + 4*d
)^2 + 9*((F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 +
225*F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*
e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*cos(2*x*e + 2*d)^2 + ((F
^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)
*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^
3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*sin(6*x*e + 6*d)^2 + 9*((F^(a*c)*b^
4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos
(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F
^(a*c)*b*c*e^5*log(F))*sin(d))*sin(4*x*e + 4*d)^2 - 18*((F^(a*c)*b^4*c^4*e^
2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F
^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b
*c*e^5*log(F))*sin(d))*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 9*((F^(a*c)*b^4*
c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d
) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(
a*c)*b*c*e^5*log(F))*sin(d))*sin(2*x*e + 2*d)^2 - 2*(3*((F^(a*c)*b^4*c^4*e^
2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F
^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b
*c*e^5*log(F))*sin(d))*cos(4*x*e + 4*d) - 3*((F^(a*c)*b^4*c^4*e^2*log(F)^4
+ 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*
c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(
F))*sin(d))*cos(2*x*e + 2*d) + (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b
^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5
 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*co
s(6*x*e + 6*d) - 6*(3*((F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e
```

```
^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^
(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*cos(2*x*e
+ 2*d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 +
225*F^(a*c)*e^6)*cos(d) - (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*
e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*cos(4*x*e + 4*d) - 6*((F
^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)
*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^
3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*cos(2*x*e + 2*d) + (F^(a*c)*b^4*c^4
*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*cos(d) -
 6*(((F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*
F^(a*c)*e^6)*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*
log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*sin(d))*sin(4*x*e + 4*d) - ((F^(a*c)
*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*
cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 22
5*F^(a*c)*b*c*e^5*log(F))*sin(d))*sin(2*x*e + 2*d))*sin(6*x*e + 6*d) + (F^(
a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c
*e^5*log(F))*sin(d))*integrate((8*b*c*cos(x*e)*e^(b*c*x*log(F) + 1)*log(F)
+ (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(x*e) + (8*b*c*cos(x*e)*e^(b*c*x
*log(F) + 1)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(x*e))*cos(8
*x*e + 8*d) - 4*(8*b*c*cos(x*e)*e^(b*c*x*log(F) + 1)*log(F) + (b^2*c^2*log(
F)^2 - 15*e^2)*F^(b*c*x)*sin(x*e))*cos(6*x*e + 6*d) + 6*(8*b*c*cos(x*e)*e^(
b*c*x*log(F) + 1)*log(F) + (b^2*c^2*log(F)^2 - ...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(x*e + d)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**3,x)
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sin(d + e*x)^3,x)
```

```
[Out] int(F^(c*(a + b*x))/sin(d + e*x)^3, x)
```

3.8 $\int F^{c(a+bx)} \csc^4(d+ex)\,dx$

Optimal. Leaf size=141

$$-\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e}-\frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2}+\frac{2e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2\right.}{3e^2}$$

```
[Out] -1/3*F^(c*(b*x+a))*cot(e*x+d)*csc(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csc(e*x+
d)^2*ln(F)/e^2+2/3*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c
*ln(F)/e],[2-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(2*I*e-b*c*ln(F))/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4534, 4538}

$$\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\csc^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\cot(d+ex)\csc^2(d+ex)F^{c(a+bx)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

```
[Out] -1/3*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x]^2)/e - (b*c*F^(c*(a + b*x))
*Csc[d + e*x]^2*Log[F])/(6*e^2) + (2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hy
pergeometric2F1[2, 1 - ((I/2)*b*c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((
2*I)*(d + e*x))]*((2*I)*e - b*c*Log[F]))/(3*e^2)
```

Rule 4534

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a + b*x
))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} + \frac{1}{6}\Bigg(4+$$

$$= -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} + \frac{2e^{2i(d+}}{}$$

Mathematica [A]
time = 3.09, size = 173, normalized size = 1.23

$$\frac{F^{c(a+bx)}\left(-e\csc^2(d+ex)(2e\cot(d)+bc\log(F)) - \frac{2i\left(1+(-1+e^{2id})\,{}_2F_1\left(1,-\frac{ibc\log(F)}{2e};1-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)\right)(4e^2+b^2c^2\log^2(F))}{-1+e^{2id}} + 2e^2\csc(d)\csc^3(d+ex)\sin(ex) + \csc(d)\csc(d+ex)\left(4e^2+b^2c^2\log^2(F)\right)\sin(ex)\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csc[d + e*x]^2*(2*e*Cot[d] + b*c*Log[F])) - ((2*I)*(1
 + (-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I
/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])*(4*e^2 + b^2*c^2*Log[F]^2))/(-1 +
E^((2*I)*d)) + 2*e^2*Csc[d]*Csc[d + e*x]^3*Sin[e*x] + Csc[d]*Csc[d + e*x]*(
4*e^2 + b^2*c^2*Log[F]^2)*Sin[e*x]))/(6*e^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\csc^4(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 16*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F
^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*x*e + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*
e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d)^2 + 6*
(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)
```

*b*c*e^4*log(F))*F^(b*c*x)*sin(4*x*e + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*e^2*lo
g(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*x*e + 2*d)^2 + 560*(F^(
a*c)*b^3*c^3*e^2*log(F)^3 - 32*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e
+ 2*d) - 40*(F^(a*c)*b^4*c^4*e*log(F)^4 - 104*F^(a*c)*b^2*c^2*e^3*log(F)^2)
*F^(b*c*x)*sin(2*x*e + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*
c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*x*e + 4*d)
- 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*c
os(2*x*e + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2*c^2*e^3*l
og(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*x*e + 4*d) - 8*(F^(a*c)*b^4*c^4
*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)*F^(b*c*x)
*sin(2*x*e + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c)*b*c*e^4*
log(F))*F^(b*c*x))*cos(8*x*e + 8*d) - 4*((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^
(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*x
*e + 4*d) - 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F
^(b*c*x)*cos(2*x*e + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2
*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*x*e + 4*d) - 8*(F^(a*
c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)
*F^(b*c*x)*sin(2*x*e + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x))*cos(6*x*e + 6*d) - (4*(F^(a*c)*b^5*c^5*log(F)^
5 + 220*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 9984*F^(a*c)*b*c*e^4*log(F))*F^(b*c*
x)*cos(2*x*e + 2*d) + 64*(F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e
^3*log(F)^2 - 576*F^(a*c)*e^5)*F^(b*c*x)*sin(2*x*e + 2*d) - (F^(a*c)*b^5*c^
5*log(F)^5 - 860*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 21504*F^(a*c)*b*c*e^4*log(F
))*F^(b*c*x))*cos(4*x*e + 4*d) - 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(
a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*
c)*b*c*e^8*log(F) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4
*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F
))*cos(8*x*e + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*
c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^
8*log(F))*cos(6*x*e + 6*d)^2 + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)
*b*c*e^8*log(F))*cos(4*x*e + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 11
6*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*
F^(a*c)*b*c*e^8*log(F))*cos(2*x*e + 2*d)^2 + (F^(a*c)*b^7*c^7*e^2*log(F)^7
+ 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36
864*F^(a*c)*b*c*e^8*log(F))*sin(8*x*e + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*lo
g(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)
^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(6*x*e + 6*d)^2 + 36*(F^(a*c)*b^7*c^7
*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6
*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*x*e + 4*d)^2 - 48*(F^(a*c)*
b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*
c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*x*e + 4*d)*sin(2*x*e
+ 2*d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(

```
2*x*e + 2*d)^2 + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*
log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F)
 - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 390
4*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(6*x*e +
6*d) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 +
 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(4*x*
e + 4*d) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(
2*x*e + 2*d))*cos(8*x*e + 8*d) - 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a
*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c
)*b*c*e^8*log(F) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^
4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(
F))*cos(4*x*e + 4*d) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^
5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*
log(F))*cos(2*x*e + 2*d))*cos(6*x*e + 6*d) + 12*(F^(a*c)*b^7*c^7*e^2*log(F)
^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 +
 36864*F^(a*c)*b*c*e^8*log(F) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(x*e + d)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sin(d + e*x)^4,x)
[Out] int(F^(c*(a + b*x))/sin(d + e*x)^4, x)
```

3.9 $\int e^x \sin^4(x)\,dx$

Optimal. Leaf size=54

$$\frac{24e^x}{85} - \frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x)$$

```
[Out] 24/85*exp(x)-24/85*exp(x)*cos(x)*sin(x)+12/85*exp(x)*sin(x)^2-4/17*exp(x)*c
os(x)*sin(x)^3+1/17*exp(x)*sin(x)^4
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4519, 2225}

$$\frac{24e^x}{85} + \frac{1}{17}e^x\sin^4(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\sin^3(x)\cos(x) - \frac{24}{85}e^x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x]^4,x]
```

```
[Out] (24*E^x)/85 - (24*E^x*Cos[x]*Sin[x])/85 + (12*E^x*Sin[x]^2)/85 - (4*E^x*Cos
[x]*Sin[x]^3)/17 + (E^x*Sin[x]^4)/17
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int e^x\sin^4(x)\,dx &= -\frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x) + \frac{12}{17}\int e^x\sin^2(x)\,dx \\
&= -\frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x) + \frac{24\int e^x\,dx}{85} \\
&= \frac{24e^x}{85} - \frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 0.61

$$\frac{1}{680}e^{x}(255-68\cos(2x)+5\cos(4x)-136\sin(2x)+20\sin(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x]^4,x]
```

```
[Out] (E^x*(255 - 68*Cos[2*x] + 5*Cos[4*x] - 136*Sin[2*x] + 20*Sin[4*x]))/680
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.63

method	result
default	$\frac{(\sin(x)-4\cos(x))\mathrm{e}^{x}\left(\sin^{3}(x)\right)}{17}+\frac{12(\sin(x)-2\cos(x))\mathrm{e}^{x}\sin(x)}{85}+\frac{24\,\mathrm{e}^{x}}{85}$
risch	$\frac{3\,\mathrm{e}^{x}}{8}+\frac{\mathrm{e}^{(1+4i)x}}{272}-\frac{i\mathrm{e}^{(1+4i)x}}{68}-\frac{\mathrm{e}^{(1+2i)x}}{20}+\frac{i\mathrm{e}^{(1+2i)x}}{10}-\frac{\mathrm{e}^{(1-2i)x}}{20}-\frac{i\mathrm{e}^{(1-2i)x}}{10}+\frac{\mathrm{e}^{(1-4i)x}}{272}+\frac{i\mathrm{e}^{(1-4i)x}}{68}$
norman	$\frac{-\frac{48\,\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)}{85}+\frac{144\,\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{85}-\frac{208\,\mathrm{e}^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{85}+\frac{64\,\mathrm{e}^{x}\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{17}+\frac{208\,\mathrm{e}^{x}\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{85}+\frac{144\,\mathrm{e}^{x}\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{85}+\frac{48\,\mathrm{e}^{x}\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{85}+\frac{24\,\mathrm{e}^{x}(\mathrm{ta}}{8}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*(sin(x)-4*cos(x))*exp(x)*sin(x)^3+12/85*(sin(x)-2*cos(x))*exp(x)*sin(x
)+24/85*exp(x)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.69

$$\frac{1}{136}\cos\left(4\,x\right)e^{x}-\frac{1}{10}\cos\left(2\,x\right)e^{x}+\frac{1}{34}\,e^{x}\sin\left(4\,x\right)-\frac{1}{5}\,e^{x}\sin\left(2\,x\right)+\frac{3}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/136*cos(4*x)*e^x - 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) - 1/5*e^x*sin(2*
x) + 3/8*e^x
```

Fricas [A]
time = 2.87, size = 36, normalized size = 0.67

$$\frac{4}{85}\left(5\,\cos\left(x\right)^{3}-11\,\cos\left(x\right)\right)e^{x}\sin\left(x\right)+\frac{1}{85}\left(5\,\cos\left(x\right)^{4}-22\,\cos\left(x\right)^{2}+41\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 4/85*(5*cos(x)^3 - 11*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 - 22*cos(x)^2 +
 41)*e^x
```

Sympy [A]
time = 0.63, size = 70, normalized size = 1.30

$$\frac{41e^x \sin^4(x)}{85} - \frac{44e^x \sin^3(x)\cos(x)}{85} + \frac{12e^x \sin^2(x)\cos^2(x)}{17} - \frac{24e^x \sin(x)\cos^3(x)}{85} + \frac{24e^x \cos^4(x)}{85}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)**4,x)
```

```
[Out] 41*exp(x)*sin(x)**4/85 - 44*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)**
2*cos(x)**2/17 - 24*exp(x)*sin(x)*cos(x)**3/85 + 24*exp(x)*cos(x)**4/85
```

Giac [A]
time = 0.41, size = 35, normalized size = 0.65

$$\frac{1}{136}\left(\cos(4x) + 4\sin(4x)\right)e^x - \frac{1}{10}\left(\cos(2x) + 2\sin(2x)\right)e^x + \frac{3}{8}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/136*(cos(4*x) + 4*sin(4*x))*e^x - 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8*
e^x
```

Mupad [B]
time = 0.05, size = 41, normalized size = 0.76

$$\frac{3\,e^x}{8} - \frac{e^x\left(\frac{4\cos(2x)}{5} + \frac{8\sin(2x)}{5} - \frac{2\cos(2x)^2}{17} - \frac{8\cos(2x)\sin(2x)}{17} + \frac{1}{17}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x)^4,x)
```

```
[Out] (3*exp(x))/8 - (exp(x)*((4*cos(2*x))/5 + (8*sin(2*x))/5 - (2*cos(2*x)^2)/17
 - (8*cos(2*x)*sin(2*x))/17 + 1/17))/8
```

3.10 $\int F^{c(a+bx)} \cos^n(d+ex) \, dx$

Optimal. Leaf size=107

$$-\frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \cos^n(d+ex) \, {}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{ien-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*cos(e*x+d)^n*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/2
*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/((1+exp(2*I*(e*x+d)))^n)/(I*e*n-b*
c*ln(F))
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4526, 2291}

$$-\frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \cos^n(d+ex) \, {}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right);-e^{2i(d+ex)}\right)}{-bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*L
og[F])/e, (2 - n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/((1 + E^((2*
I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4526

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Dist[E^(I*n*(d + e*x))*(Cos[d + e*x]^n/(1 + E^(2*I*(d + e*x)))^n), In
t[F^(c*(a + b*x))*((1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /;
FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx = \left(e^{in(d+ex)}\left(1+e^{2i(d+ex)}\right)^{-n}\cos^n(d+ex)\right)\int e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^nF^{c(a+bx)}$$

$$= -\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2-n-\frac{ibc}{}\right.\right.}{ien-bc\log(F)}$$

Mathematica [A]
time = 0.06, size = 110, normalized size = 1.03

$$\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\,{}_2F_1\left(-n,-\frac{i(-ien+bc\log(F))}{2e};1-\frac{i(-ien+bc\log(F))}{2e};-e^{2i(d+ex)}\right)}{-ien+bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n +
 b*c*Log[F]))/e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, -E^((2*I)*(d + e*x)
)])/((1 + E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\cos^n(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*cos(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*cos(x*e + d)^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\cos^{n}(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*cos(d + e*x)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\cos\left(d+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cos(d + e*x)^n,x)
```

```
[Out] int(F^(c*(a + b*x))*cos(d + e*x)^n, x)
```

3.11 $\int F^{c(a+bx)} \cos^3(d+ex)\,dx$

Optimal. Leaf size=199

$$\frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)}+\frac{6bce^2F^{c(a+bx)}\cos(d+ex)\log(F)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)}$$

```
[Out] b*c*F^(c*(b*x+a))*cos(e*x+d)^3*ln(F)/(9*e^2+b^2*c^2*ln(F)^2)+6*b*c*e^2*F^(c
*(b*x+a))*cos(e*x+d)*ln(F)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)+3
*e*F^(c*(b*x+a))*cos(e*x+d)^2*sin(e*x+d)/(9*e^2+b^2*c^2*ln(F)^2)+6*e^3*F^(c
*(b*x+a))*sin(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)
```

Rubi [A]
time = 0.04, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4520, 4518}

$$\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{3e\sin(d+ex)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6bce^2\log(F)\cos(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4}+\frac{6e^3\sin(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]^3*Log[F])/(9*e^2 + b^2*c^2*Log[F]^2) + (6
*b*c*e^2*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(9*e^4 + 10*b^2*c^2*e^2*Log[F
]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cos[d + e*x]^2*Sin[d + e*x])
/(9*e^2 + b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sin[d + e*x])/(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{(6e^2)\int}{9}$$
$$= \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{6bce^2F^{c(a+bx)}\cos(d+ex)\log(F)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c}}{}$$

Mathematica [A]
time = 0.70, size = 155, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(bc\cos(3(d+ex))\log(F)\left(e^2+b^2c^2\log^2(F)\right)+3bc\cos(d+ex)\log(F)\left(9e^2+b^2c^2\log^2(F)\right)+6e\left(5e^2+b^2c^2\log^2(F)+\cos(2(d+ex))\left(e^2+b^2c^2\log^2(F)\right)\right)\sin(d+ex)\right)}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(b*c*Cos[3*(d + e*x)]*Log[F]*(e^2 + b^2*c^2*Log[F]^2) + 3*
b*c*Cos[d + e*x]*Log[F]*(9*e^2 + b^2*c^2*Log[F]^2) + 6*e*(5*e^2 + b^2*c^2*L
og[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e
^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A]
time = 0.53, size = 158, normalized size = 0.79

method	result
risch	$\frac{3bc\,F^{c(bx+a)}\cos(ex+d)\ln(F)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(bx+a)}\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{\ln(F)cb\,F^{c(bx+a)}\cos(3ex+3d)}{4b^2c^2\ln(F)^2+36e^2}+\frac{3e\,F^{c(bx+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{\frac{\ln(F)bc\left(b^2c^2\ln(F)^2+7e^2\right)e^{c(bx+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{12e\left(b^2c^2\ln(F)^2-e^2\right)e^{c(bx+a)\ln(F)}\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(b^2c^2\ln(F)^2+3e^2\right)e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right.}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*b*c*F^(c*(b*x+a))*cos(e*x+d)*ln(F)/(e^2+b^2*c^2*ln(F)^2)+3/4*e*F^(c*(b*
x+a))*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+1/4/(9*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*
b*F^(c*(b*x+a))*cos(3*e*x+3*d)+3/4/(9*e^2+b^2*c^2*ln(F)^2)*e*F^(c*(b*x+a))*
sin(3*e*x+3*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 759 vs. 2(197) = 394.
time = 0.32, size = 759, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(((F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*cos(3*d) + 3*(F^(
a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*cos(3*x*e) + ((F
^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*cos(3*d) - 3*(F^(a*c)*b^2
*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*cos(3*x*e + 6*d) + 3*((F
^(a*c)*b^3*c^3*log(F)^3 + 9*F^(a*c)*b*c*e^2*log(F))*cos(3*d) - (F^(a*c)*b^2
*c^2*e*log(F)^2 + 9*F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*cos(x*e + 4*d) + 3*((F
^(a*c)*b^3*c^3*log(F)^3 + 9*F^(a*c)*b*c*e^2*log(F))*cos(3*d) + (F^(a*c)*b^2
*c^2*e*log(F)^2 + 9*F^(a*c)*e^3)*sin(3*d))*F^(b*c*x)*cos(x*e - 2*d) + (3*(F
^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(3*d) - (F^(a*c)*b^3*c^3*log(F)
^3 + F^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*sin(3*x*e) + (3*(F^(a*c)*b
^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(3*d) + (F^(a*c)*b^3*c^3*log(F)^3 + F^(
a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*sin(3*x*e + 6*d) + 3*((F^(a*c)*b^2
*c^2*e*log(F)^2 + 9*F^(a*c)*e^3)*cos(3*d) + (F^(a*c)*b^3*c^3*log(F)^3 + 9*F
^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*sin(x*e + 4*d) + 3*((F^(a*c)*b^2
*c^2*e*log(F)^2 + 9*F^(a*c)*e^3)*cos(3*d) - (F^(a*c)*b^3*c^3*log(F)^3 + 9*F
^(a*c)*b*c*e^2*log(F))*sin(3*d))*F^(b*c*x)*sin(x*e - 2*d))/((b^4*c^4*log(F)
^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)*cos(3*d)^2 + (b^4*c^4*log(F)^4 + 10*b
^2*c^2*e^2*log(F)^2 + 9*e^4)*sin(3*d)^2)
```

Fricas [A]
time = 3.00, size = 143, normalized size = 0.72

$$\frac{\left(b^3c^3\cos\left(xe+d\right)^3\log\left(F\right)^3+\left(bc\cos\left(xe+d\right)^3e^2+6\,bc\cos\left(xe+d\right)e^2\right)\log\left(F\right)+3\left(b^2c^2\cos\left(xe+d\right)^2e\log\left(F\right)^2+\cos\left(xe+d\right)^2e^3+2\,e^3\right)\sin\left(xe+d\right)\right)F^{bcx+ac}}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(F\right)^2+9\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] (b^3*c^3*cos(x*e + d)^3*log(F)^3 + (b*c*cos(x*e + d)^3*e^2 + 6*b*c*cos(x*e
+ d)*e^2)*log(F) + 3*(b^2*c^2*cos(x*e + d)^2*e*log(F)^2 + cos(x*e + d)^2*e^
3 + 2*e^3)*sin(x*e + d))*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2
*log(F)^2 + 9*e^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 1271, normalized size = 6.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c + 3*e*x + 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c + 6*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 6*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2
*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2
 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1
/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x +
d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*l
og(abs(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi
*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c
- 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2)
)*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)*log(abs(
F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 6*e)^2) + (pi*b*c*
sgn(F) - pi*b*c - 6*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*
sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 6*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2
*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c +
3*I*e*x + 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*
I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
 1/2*I*pi*a*c - 3*I*e*x - 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*
log(abs(F)) - 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*e^(
1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c
 + I*e*x + I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*I
*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c - I*e*x - I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(a
bs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*
e*x - I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*I*e) -
 I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I
*pi*a*c + I*e*x + I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)
) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c
*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 3*I*e*x -
```

```
 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*I*e) - I*
e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi
*a*c + 3*I*e*x + 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F
)) + 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 3.14, size = 191, normalized size = 0.96

$$-\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)+\sin\left(e\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(d\right)+\sin\left(d\right)\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{8\,\left(e-b\,c\,\ln\left(F\right)\,1{}\mathrm{i}\right)}-\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)-\sin\left(3\,e\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,d\right)-\sin\left(3\,d\right)\,1{}\mathrm{i}\right)}{8\,\left(-b\,c\,\ln\left(F\right)+e\,3{}\mathrm{i}\right)}-\frac{F^{c\,(a+b\,x)}\,\left(\cos\left(3\,e\,x\right)+\sin\left(3\,e\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,d\right)+\sin\left(3\,d\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(3\,e-b\,c\,\ln\left(F\right)\,1{}\mathrm{i}\right)}-\frac{3\,F^{c\,(a+b\,x)}\,\left(\cos\left(e\,x\right)-\sin\left(e\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(d\right)-\sin\left(d\right)\,1{}\mathrm{i}\right)}{8\,\left(-b\,c\,\ln\left(F\right)+e\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cos(d + e*x)^3,x)
```

```
[Out] - (F^(c*(a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e
- b*c*log(F)*1i)) - (F^(c*(a + b*x))*(cos(3*e*x) - sin(3*e*x)*1i)*(cos(3*d)
 - sin(3*d)*1i))/(8*(e*3i - b*c*log(F))) - (F^(c*(a + b*x))*(cos(3*e*x) + s
in(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/(8*(3*e - b*c*log(F)*1i)) - (3*F
^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(cos(d) - sin(d)*1i))/(8*(e*1i - b*
c*log(F)))
```

3.12 $\int F^{c(a+bx)} \cos^2(d+ex)\,dx$

Optimal. Leaf size=128

$$\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

```
[Out] 2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*cos
(e*x+d)^2*ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cos(e*x+d)*sin(e*
x+d)/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4520, 2225}

$$\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c
*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*F^(c*(
a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}\cos^2(d+ex)\,dx &= \frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{(2e^2)\int F\ldots}{4e^2+b^2\ldots}\\ &= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\mathrm{co}\ldots}{4e^2\ldots}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 85, normalized size = 0.66

$$\frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)+b^2c^2\cos(2(d+ex))\log^2(F)+2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[2*(d + e*x)]*Log[F
]^2 + 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F
]^3)
```

Maple [A]
time = 0.24, size = 105, normalized size = 0.82

method	result
risch	$\frac{F^{c(bx+a)}}{2bc\ln(F)}+\frac{\ln(F)cb\,F^{c(bx+a)}\cos(2ex+2d)}{2b^2c^2\ln(F)^2+8e^2}+\frac{e\,F^{c(bx+a)}\sin(2ex+2d)}{4e^2+b^2c^2\ln(F)^2}$
norman	$\frac{\frac{\left(b^2c^2\ln(F)^2+2e^2\right)e^{c(bx+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{\left(b^2c^2\ln(F)^2+2e^2\right)e^{c(bx+a)\ln(F)}\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4e\,e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{4e^2+b^2c^2\ln(F)^2}-\frac{4e\,e^{c(bx+a)\ln(F)}\left(\tan\right.}{4e^2+b^2c^2\ln(}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*F^(c*(b*x+a))/b/c/ln(F)+1/2/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*b*F^(c*(b*x
+a))*cos(2*e*x+2*d)+e*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(128) = 256.
time = 0.30, size = 348, normalized size = 2.72

$$\frac{\left(F^{ac}b^2c^2\cos(2d)\log(F)^2+2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe)+\left(F^{ac}b^2c^2\cos(2d)\log(F)^2-2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe+4d)-\left(F^{ac}b^2c^2\log(F)^2\sin(2d)-2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe)+\left(F^{ac}b^2c^2\log(F)^2\sin(2d)+2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe+4d)+2\left(\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\cos(2d)^2+\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\sin(2d)^2\right)F^{bcx}}{4\left(\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\cos(2d)^2+\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\sin(2d)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))*
F^(b*c*x)*cos(2*x*e) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e
*log(F)*sin(2*d))*F^(b*c*x)*cos(2*x*e + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*si
n(2*d) - 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(2*x*e) + (F^(a*c)*b
^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(2
*x*e + 4*d) + 2*((F^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*cos(2*d)^2 + (F
^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*sin(2*d)^2)*F^(b*c*x))/((b^3*c^3*l
```

```
og(F)^3 + 4*b*c*e^2*log(F))*cos(2*d)^2 + (b^3*c^3*log(F)^3 + 4*b*c*e^2*log(
F))*sin(2*d)^2)
```

Fricas [A]
time = 2.85, size = 80, normalized size = 0.62

$$\frac{\left(b^2c^2\cos\left(xe+d\right)^2\log\left(F\right)^2+2\,bc\cos\left(xe+d\right)e\log\left(F\right)\sin\left(xe+d\right)+2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*cos(x*e + d)^2*log(F)^2 + 2*b*c*cos(x*e + d)*e*log(F)*sin(x*e + d)
 + 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F))
```

Sympy [C] Result contains complex when optimal does not.
time = 14.97, size = 1117, normalized size = 8.73

$$\begin{cases} \frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}+\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} & \text{for}\: F=1 \\ \frac{b^2c^2\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{2ie}{bc}}\right)^2\cos^2\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)}+\frac{2bce\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{2ie}{bc}}\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\sin^2\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{-\frac{2ie}{bc}}\right)^{ac}\left(e^{-\frac{2ie}{bc}}\right)^{bcx}\cos^2\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{-\frac{2ie}{bc}}\right)} & \text{for}\: F=e^{-\frac{2ie}{bc}} \\ \frac{b^2c^2\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{\frac{2ie}{bc}}\right)^2\cos^2\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)}+\frac{2bce\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\log\left(e^{\frac{2ie}{bc}}\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\sin^2\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)}+\frac{2e^2\left(e^{\frac{2ie}{bc}}\right)^{ac}\left(e^{\frac{2ie}{bc}}\right)^{bcx}\cos^2\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{2ie}{bc}}\right)^3+4bce^2\log\left(e^{\frac{2ie}{bc}}\right)} & \text{for}\: F=e^{\frac{2ie}{bc}} \\ F^{ac}\left(\frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}+\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}\right) & \text{for}\: b=0 \\ \frac{x\sin^2\left(d+ex\right)}{2}+\frac{x\cos^2\left(d+ex\right)}{2}+\frac{\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} & \text{for}\: c=0 \\ \frac{F^{ac}F^{bcx}b^2c^2\log\left(F\right)^2\cos^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac}F^{bcx}bce\log\left(F\right)\sin\left(d+ex\right)\cos\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac}F^{bcx}e^2\sin^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)}+\frac{2F^{ac}F^{bcx}e^2\cos^2\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+4bce^2\log\left(F\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d +
 e*x)/(2*e), Eq(F, 1)), (b**2*c**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c
))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*cos(d + e*x)**2/(b**3*c**3*log(exp(-2
*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-2*I*e/(b*c)))) + 2*b*c*e*exp(-2*I*e/(
b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))*sin(d + e*x)
*cos(d + e*x)/(b**3*c**3*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-2*
I*e/(b*c)))) + 2*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*s
in(d + e*x)**2/(b**3*c**3*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-2
*I*e/(b*c)))) + 2*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*
cos(d + e*x)**2/(b**3*c**3*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(-
2*I*e/(b*c)))), Eq(F, exp(-2*I*e/(b*c)))), (b**2*c**2*exp(2*I*e/(b*c))**(a*
c)*exp(2*I*e/(b*c))**(b*c*x)*log(exp(2*I*e/(b*c)))**2*cos(d + e*x)**2/(b**3
*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp(2*I*e/(b*c)))) + 2*b*c*
e*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*log(exp(2*I*e/(b*c)))*s
in(d + e*x)*cos(d + e*x)/(b**3*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*l
og(exp(2*I*e/(b*c)))) + 2*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b
```

```
*c*x)*sin(d + e*x)**2/(b**3*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*log(
exp(2*I*e/(b*c)))) + 2*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*
x)*cos(d + e*x)**2/(b**3*c**3*log(exp(2*I*e/(b*c)))**3 + 4*b*c*e**2*log(exp
(2*I*e/(b*c)))), Eq(F, exp(2*I*e/(b*c)))), (F**(a*c)*(x*sin(d + e*x)**2/2 +
 x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(
d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c
, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*cos(d + e*x)**2/(b**3*c**3*
log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sin(d +
 e*x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c)*F
**(b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) +
2*F**(a*c)*F**(b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**
2*log(F)), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 915, normalized size = 7.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2
*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F
))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*s
in(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*
e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^
(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*
log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8
*b*c*log(abs(F)) + 16*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F)
+ 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*
b*c*log(abs(F)) - 16*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F) +
```

```
 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(a
bs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) -
I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log
(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 2.98, size = 98, normalized size = 0.77

$$\frac{2\,F^{a\,c+b\,c\,x}\,e^2+F^{a\,c+b\,c\,x}\,b^2\,c^2\,\cos\left(d+e\,x\right)^2\,\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\cos\left(d+e\,x\right)\,\sin\left(d+e\,x\right)\,\ln\left(F\right)}{b^3\,c^3\,\ln\left(F\right)^3+4\,b\,c\,e^2\,\ln\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cos(d + e*x)^2,x)
```

```
[Out] (2*F^(a*c + b*c*x)*e^2 + F^(a*c + b*c*x)*b^2*c^2*cos(d + e*x)^2*log(F)^2 +
2*F^(a*c + b*c*x)*b*c*e*cos(d + e*x)*sin(d + e*x)*log(F))/(b^3*c^3*log(F)^3
 + 4*b*c*e^2*log(F))
```

3.13 $\int F^{c(a+bx)}\cos(d+ex)\,dx$

Optimal. Leaf size=72

$$\frac{bcF^{c(a+bx)}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] b*c*F^(c*(b*x+a))*cos(e*x+d)*ln(F)/(e^2+b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))*si
n(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{4518\}$

$$\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x],x]
```

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (e*F^(
c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{bcF^{c(a+bx)}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

Mathematica [A]
time = 0.10, size = 47, normalized size = 0.65

$$\frac{F^{c(a+bx)}(bc\cos(d+ex)\log(F)+e\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(b*c*Cos[d + e*x]*Log[F] + e*Sin[d + e*x]))/(e^2 + b^2*c^2
*Log[F]^2)
```

Maple [A]
time = 0.10, size = 73, normalized size = 1.01

method	result	size
risch	$\frac{bc\,F^{c(bx+a)}\cos(ex+d)\ln(F)}{e^2+b^2c^2\ln(F)^2}+\frac{e\,F^{c(bx+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	73
norman	$\frac{\frac{bc\ln(F)e^{c(bx+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2e\,e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{bc\ln(F)e^{c(bx+a)\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] b*c*F^(c*(b*x+a))*cos(e*x+d)*ln(F)/(e^2+b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))*si
n(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(73) = 146.
time = 0.28, size = 195, normalized size = 2.71

$$\frac{(F^{ac}bc\cos(d)\log(F)-F^{ac}e\sin(d))F^{bcx}\cos(xe+2d)+(F^{ac}bc\cos(d)\log(F)+F^{ac}e\sin(d))F^{bcx}\cos(xe)+(F^{ac}bc\log(F)\sin(d)+F^{ac}\cos(d)e)F^{bcx}\sin(xe+2d)-(F^{ac}bc\log(F)\sin(d)-F^{ac}\cos(d)e)F^{bcx}\sin(xe)}{2\left((b^2c^2\log(F)^2+e^2)\cos(d)^2+(b^2c^2\log(F)^2+e^2)\sin(d)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(x*e + 2*d
) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(x*e) + (F^
(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(x*e + 2*d) - (F^(
a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(x*e))/((b^2*c^2*lo
g(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(d)^2)
```

Fricas [A]
time = 2.79, size = 50, normalized size = 0.69

$$\frac{(bc\cos(xe+d)\log(F)+e\sin(xe+d))F^{bcx+ac}}{b^2c^2\log(F)^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] (b*c*cos(x*e + d)*log(F) + e*sin(x*e + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)^
2 + e^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.52, size = 437, normalized size = 6.07

$$\begin{cases} -\frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}ix\sin(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}x\cos(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}\sin(d+ex)}{2e} & \text{for } F=-1 \wedge b=\frac{e}{\pi c} \\ x\cos(d) & \text{for } F=1 \wedge e=0 \\ \frac{bc\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)\cos(d+ex)}{b^2c^2\log\left(e^{-\frac{ie}{bc}}\right)^2+e^2} + \frac{e\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\sin(d+ex)}{b^2c^2\log\left(e^{-\frac{ie}{bc}}\right)^2+e^2} & \text{for } F=e^{-\frac{ie}{bc}} \\ \frac{bc\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)\cos(d+ex)}{b^2c^2\log\left(e^{\frac{ie}{bc}}\right)^2+e^2} + \frac{e\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\sin(d+ex)}{b^2c^2\log\left(e^{\frac{ie}{bc}}\right)^2+e^2} & \text{for } F=e^{\frac{ie}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\cos(d+ex)}{b^2c^2\log(F)^2+e^2} + \frac{F^{ac}F^{bcx}e\sin(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d),x)
```

```
[Out] Piecewise((-(-1)**(a*c)*(-1)**(e*x/pi)*I*x*sin(d + e*x)/2 + (-1)**(a*c)*(-1
)**(e*x/pi)*x*cos(d + e*x)/2 + (-1)**(a*c)*(-1)**(e*x/pi)*sin(d + e*x)/(2*e
), Eq(F, -1) & Eq(b, e/(pi*c))), (x*cos(d), Eq(F, 1) & Eq(e, 0)), (b*c*exp(
-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))*cos(d + e*
x)/(b**2*c**2*log(exp(-I*e/(b*c)))**2 + e**2) + e*exp(-I*e/(b*c))**(a*c)*ex
p(-I*e/(b*c))**(b*c*x)*sin(d + e*x)/(b**2*c**2*log(exp(-I*e/(b*c)))**2 + e*
*2), Eq(F, exp(-I*e/(b*c)))), (b*c*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b
*c*x)*log(exp(I*e/(b*c)))*cos(d + e*x)/(b**2*c**2*log(exp(I*e/(b*c)))**2 +
e**2) + e*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*sin(d + e*x)/(b**2*
c**2*log(exp(I*e/(b*c)))**2 + e**2), Eq(F, exp(I*e/(b*c)))), (F**(a*c)*F**(
b*c*x)*b*c*log(F)*cos(d + e*x)/(b**2*c**2*log(F)**2 + e**2) + F**(a*c)*F**(
b*c*x)*e*sin(d + e*x)/(b**2*c**2*log(F)**2 + e**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 631, normalized size = 8.76

$$\left(\frac{2\,bc\cos\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac+ex+d\right)\log\left(|F|\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc+2\,e\right)^2}+\frac{\left(\pi bc\mathrm{sgn}(F)-\pi bc+2\,e\right)\sin\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac+ex+d\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc+2\,e\right)^2}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}+\left(\frac{2\,bc\cos\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac-ex-d\right)\log\left(|F|\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc-2\,e\right)^2}+\frac{\left(\pi bc\mathrm{sgn}(F)-\pi bc-2\,e\right)\sin\left(\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}\,\pi bcx+\frac{1}{2}\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}\,\pi ac-ex-d\right)}{4\,b^2c^2\log\left(|F|\right)^2+\left(\pi bc\mathrm{sgn}\left(F\right)-\pi bc-2\,e\right)^2}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac+i\,ex+i\,d\right)}}{2i\,\pi bc\mathrm{sgn}\left(F\right)-2i\,\pi bc+4\,bc\log\left(|F|\right)+4i\,e}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac-i\,ex-i\,d\right)}}{-2i\,\pi bc\mathrm{sgn}\left(F\right)+2i\,\pi bc+4\,bc\log\left(|F|\right)-4i\,e}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac-i\,ex-i\,d\right)}}{2i\,\pi bc\mathrm{sgn}\left(F\right)-2i\,\pi bc+4\,bc\log\left(|F|\right)-4i\,e}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac+i\,ex+i\,d\right)}}{-2i\,\pi bc\mathrm{sgn}\left(F\right)+2i\,\pi bc+4\,bc\log\left(|F|\right)+4i\,e}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="giac")
```

```
[Out] (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) + (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) -
```

```
 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4
*b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I
*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)
)) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(ab
s(F)) - 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*
c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4
*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 2.36, size = 48, normalized size = 0.67

$$\frac{F^{a\,c+b\,c\,x}\,\left(e\,\sin\left(d+e\,x\right)+b\,c\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b^2\,c^2\,{\ln\left(F\right)}^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cos(d + e*x),x)
```

```
[Out] (F^(a*c + b*c*x)*(e*sin(d + e*x) + b*c*cos(d + e*x)*log(F)))/(e^2 + b^2*c^2
*log(F)^2)
```

3.14 $\int F^{c(a+bx)}\sec(d+ex)\,dx$

Optimal. Leaf size=84

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

```
[Out] 2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1/
2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(b*c*ln(F)+I*e)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{4536\}$

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x],x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[F])
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec(d+ex)\,dx=\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

Mathematica [A]
time = 0.02, size = 84, normalized size = 1.00

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{1}{2}-\frac{ibc\log(F)}{2e};\frac{3}{2}-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x],x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 - ((I/2)*b*c*Lo
g[F])/e, 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[
F])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)} \sec(ex+d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*c*x)*F^(a*c)*b*c*cos(x*e + d)*log(F) - F^(a*c)*e^(b*c*x*log(F) + 1)
*sin(x*e + d) + (F^(b*c*x)*F^(a*c)*b*c*cos(x*e + d)*log(F) - F^(a*c)*e^(b*c
*x*log(F) + 1)*sin(x*e + d))*cos(2*x*e + 2*d) + 2*(F^(a*c)*b^2*c^2*log(F)^2
 + (F^(a*c)*b^2*c^2*log(F)^2 + F^(a*c)*e^2)*cos(2*x*e + 2*d)^2 + (F^(a*c)*b
^2*c^2*log(F)^2 + F^(a*c)*e^2)*sin(2*x*e + 2*d)^2 + 2*(F^(a*c)*b^2*c^2*log(
F)^2 + F^(a*c)*e^2)*cos(2*x*e + 2*d) + F^(a*c)*e^2)*integrate((b*c*e^(b*c*x
*log(F) + 1)*log(F)*sin(x*e + d) + (b*c*e^(b*c*x*log(F) + 1)*log(F)*sin(x*e
 + d) + cos(x*e + d)*e^(b*c*x*log(F) + 2))*cos(4*x*e + 4*d) + 2*(b*c*e^(b*c
*x*log(F) + 1)*log(F)*sin(x*e + d) + cos(x*e + d)*e^(b*c*x*log(F) + 2))*cos
(2*x*e + 2*d) + cos(x*e + d)*e^(b*c*x*log(F) + 2) - (b*c*cos(x*e + d)*e^(b*
c*x*log(F) + 1)*log(F) - e^(b*c*x*log(F) + 2)*sin(x*e + d))*sin(4*x*e + 4*d
) - 2*(b*c*cos(x*e + d)*e^(b*c*x*log(F) + 1)*log(F) - e^(b*c*x*log(F) + 2)*
sin(x*e + d))*sin(2*x*e + 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2
)*cos(4*x*e + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d)^2 + (b^2
*c^2*log(F)^2 + e^2)*sin(4*x*e + 4*d)^2 + 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*
x*e + 4*d)*sin(2*x*e + 2*d) + 4*(b^2*c^2*log(F)^2 + e^2)*sin(2*x*e + 2*d)^2
 + 2*(b^2*c^2*log(F)^2 + 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d) + e^2)
*cos(4*x*e + 4*d) + 4*(b^2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d) + e^2), x)
+ (F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(x*e + d) + F^(a*c)*cos(x*e + d)*e^(b*c*
x*log(F) + 1))*sin(2*x*e + 2*d))/(b^2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^
```

```
2)*cos(2*x*e + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(2*x*e + 2*d)^2 + 2*(b^
2*c^2*log(F)^2 + e^2)*cos(2*x*e + 2*d) + e^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(x*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\sec{(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d),x)
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cos(d + e*x),x)
[Out] int(F^(c*(a + b*x))/cos(d + e*x), x)
```

3.15 $\int F^{c(a+bx)}\sec^2(d+ex)\,dx$

Optimal. Leaf size=80

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

```
[Out] 4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/2*
I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{4536\}$

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

```
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 1.00

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

```
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\sec^2\left(ex+d\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 4*(24*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3
 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*x*e + 2*d)^2 + 2*(F^(a*c)*b^3
*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*x*e + 2*d)^2 + (
F^(a*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*x*e +
 2*d) - 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*x
*e + 2*d) + (24*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F) + (F^(a*c)*b^3*c^3*
log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*x*e + 2*d) - 2*(F^(a*
c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*x*e + 2*d))*cos(4*x
*e + 4*d) + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 +
 64*F^(a*c)*b*c*e^4*log(F) + (F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3
*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*cos(4*x*e + 4*d)^2 + 4*(F^(a*c)*
b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log
(F))*cos(2*x*e + 2*d)^2 + (F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^
2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*sin(4*x*e + 4*d)^2 + 4*(F^(a*c)*b^5
*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F)
)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a
*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*sin(2*x*e + 2*d)^2 +
2*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*
b*c*e^4*log(F) + 2*(F^(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F
)^3 + 64*F^(a*c)*b*c*e^4*log(F))*cos(2*x*e + 2*d))*cos(4*x*e + 4*d) + 4*(F^
```

```
(a*c)*b^5*c^5*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e
^4*log(F))*cos(2*x*e + 2*d))*integrate(-(6*b*c*cos(6*x*e + 6*d)*e^(b*c*x*lo
g(F) + 2)*log(F) + 18*b*c*cos(4*x*e + 4*d)*e^(b*c*x*log(F) + 2)*log(F) + 18
*b*c*cos(2*x*e + 2*d)*e^(b*c*x*log(F) + 2)*log(F) + 6*b*c*e^(b*c*x*log(F) +
 2)*log(F) - (b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(6*x*e + 6*d) - 3*(b
^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(4*x*e + 4*d) - 3*(b^2*c^2*e*log(F)
^2 - 8*e^3)*F^(b*c*x)*sin(2*x*e + 2*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*
log(F)^2 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(6*x*e
+ 6*d)^2 + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*x*
e + 4*d)^2 + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*
x*e + 2*d)^2 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(6*
x*e + 6*d)^2 + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(
4*x*e + 4*d)^2 + 18*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*s
in(4*x*e + 4*d)*sin(2*x*e + 2*d) + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log
(F)^2 + 64*e^4)*sin(2*x*e + 2*d)^2 + 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*l
og(F)^2 + 3*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*x*e
 + 4*d) + 3*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e
 + 2*d) + 64*e^4)*cos(6*x*e + 6*d) + 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*l
og(F)^2 + 3*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e
 + 2*d) + 64*e^4)*cos(4*x*e + 4*d) + 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*l
og(F)^2 + 64*e^4)*cos(2*x*e + 2*d) + 6*((b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*
log(F)^2 + 64*e^4)*sin(4*x*e + 4*d) + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*lo
g(F)^2 + 64*e^4)*sin(2*x*e + 2*d))*sin(6*x*e + 6*d) + 64*e^4), x) + (2*(F^(
a*c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*cos(2*x*e + 2*d) + (F^(
a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*x*e + 2*
d) - 4*(F^(a*c)*b^2*c^2*e*log(F)^2 - 8*F^(a*c)*e^3)*F^(b*c*x))*sin(4*x*e +
4*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + (b^4*c^4*log(F)^4 + 20*
b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*x*e + 4*d)^2 + 4*(b^4*c^4*log(F)^4 + 2
0*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e + 2*d)^2 + (b^4*c^4*log(F)^4 + 2
0*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*x*e + 4*d)^2 + 4*(b^4*c^4*log(F)^4 +
 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 4*(b
^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(2*x*e + 2*d)^2 + 2*
(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 2*(b^4*c^4*log(F)^4 + 20*b^2*
c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e + 2*d) + 64*e^4)*cos(4*x*e + 4*d) + 4*
(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*x*e + 2*d) + 64
*e^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(x*e + d)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cos(d + e*x)^2,x)
```

```
[Out] int(F^(c*(a + b*x))/cos(d + e*x)^2, x)
```

3.16 $\int F^{c(a+bx)} \sec^3(d+ex)\, dx$

Optimal. Leaf size=141

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)(ie-bc\log(F))}{e^2}-\frac{bcF^{c(a+bx)}\log(F)\sec(d+e}{2e^2}$$

```
[Out] -exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1/2
*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(I*e-b*c*ln(F))/e^2-1/2*b*c*F^(c*(b*x+a)
)*ln(F)*sec(e*x+d)/e^2+1/2*F^(c*(b*x+a))*sec(e*x+d)*tan(e*x+d)/e
```

Rubi [A]
time = 0.03, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4533, 4536}

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{e^2}-\frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tan(d+ex)\sec(d+ex)F^{c(a+bx)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

```
[Out] -((E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*(I*e - b*c*Log[F]))/
e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x])/(2*e^2) + (F^(c*(a + b*x))
*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\log(F)\sec(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\sec(d+ex)\tan(d+ex)}{2e} + \frac{1}{2}\left(1+\right.$$

$$= -\frac{e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)(ie-bc\log(F}{e^2}$$

Mathematica [A]
time = 0.33, size = 112, normalized size = 0.79

$$\frac{F^{c(a+bx)}\left(2e^{i(d+ex)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{3}{2}-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)(-ie+bc\log(F))+\sec(d+ex)(-bc\log(F)+e\tan(d+ex))\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(2*E^(I*(d + e*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-I)*e + b*c*Log
[F]) + Sec[d + e*x]*(-(b*c*Log[F]) + e*Tan[d + e*x])))/(2*e^2)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\sec^3(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 8*(48*F^(a*c)*b*c*cos(x*e + d)*e^(b*c*x*log(F) + 2)*log(F) + 6*(F^(a*c)*b^2
*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(x*e + d) + (48*F^(a*c)*b*c*
cos(x*e + d)*e^(b*c*x*log(F) + 2)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F
^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(3*x*e + 3*d) - 3*(F^(a*c)*b^2*c^2*e*lo
g(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(3*x*e + 3*d) + 6*(F^(a*c)*b^2*c^2*e*
```

log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(x*e + d))*cos(6*x*e + 6*d) + 3*(48
*F^(a*c)*b*c*cos(x*e + d)*e^(b*c*x*log(F) + 2)*log(F) + (F^(a*c)*b^3*c^3*lo
g(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(3*x*e + 3*d) - 3*(F^(a*c)
*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(3*x*e + 3*d) + 6*(F^(a*
c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(x*e + d))*cos(4*x*e +
 4*d) + (3*(F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)
*cos(2*x*e + 2*d) + 9*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*
x)*sin(2*x*e + 2*d) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F)
)*F^(b*c*x))*cos(3*x*e + 3*d) + 18*(8*F^(a*c)*b*c*cos(x*e + d)*e^(b*c*x*log
(F) + 2)*log(F) + (F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*s
in(x*e + d))*cos(2*x*e + 2*d) - 6*(((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c
)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*
c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d
))*cos(6*x*e + 6*d)^2 + 9*((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3
*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*l
og(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*cos(4*
x*e + 4*d)^2 + 9*((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 +
 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*cos(2*x*e + 2*d
)^2 + ((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*
F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)
*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*sin(6*x*e + 6*d)^2 + 9*((F
^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b
*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*
e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*sin(4*x*e + 4*d)^2 + 18*((F^(a*c)*b
^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*l
og(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(
F)^2 + 225*F^(a*c)*e^6)*sin(d))*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 9*((F^(
a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c
*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^
4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*sin(2*x*e + 2*d)^2 + 2*(3*((F^(a*c)*b
^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*l
og(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(
F)^2 + 225*F^(a*c)*e^6)*sin(d))*cos(4*x*e + 4*d) + 3*((F^(a*c)*b^5*c^5*e*lo
g(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(
d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*
F^(a*c)*e^6)*sin(d))*cos(2*x*e + 2*d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^
(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*
b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*s
in(d))*cos(6*x*e + 6*d) + 6*(3*((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^
3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*
e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*c
os(2*x*e + 2*d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 +
 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*cos(4*x*e + 4*d

```
) + 6*((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*
F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)
*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*cos(2*x*e + 2*d) + (F^(a*c
)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 225*F^(a*c)*b*c*e^
5*log(F))*cos(d) + 6*(((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3
*log(F)^3 + 225*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F
)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*sin(4*x*e
+ 4*d) + ((F^(a*c)*b^5*c^5*e*log(F)^5 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 2
25*F^(a*c)*b*c*e^5*log(F))*cos(d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a
*c)*b^2*c^2*e^4*log(F)^2 + 225*F^(a*c)*e^6)*sin(d))*sin(2*x*e + 2*d))*sin(6
*x*e + 6*d) - (F^(a*c)*b^4*c^4*e^2*log(F)^4 + 34*F^(a*c)*b^2*c^2*e^4*log(F)
^2 + 225*F^(a*c)*e^6)*sin(d))*integrate((8*b*c*cos(x*e)*e^(b*c*x*log(F) + 1
)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(x*e) + (8*b*c*cos(x*e)
*e^(b*c*x*log(F) + 1)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(x*
e))*cos(8*x*e + 8*d) + 4*(8*b*c*cos(x*e)*e^(b*c*x*log(F) + 1)*log(F) + (b^2
*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(x*e))*cos(6*x*e + 6*d) + 6*(8*b*c*cos
(x*e)*e^(b*c*x*log(F) + 1)*log(F) + (b^2*c^2*lo...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(x*e + d)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**3,x)
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\cos\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cos(d + e*x)^3,x)
[Out] int(F^(c*(a + b*x))/cos(d + e*x)^3, x)
```

3.17 $\int F^{c(a+bx)} \sec^4(d+ex)\,dx$

Optimal. Leaf size=143

$$-\frac{2e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2}-\frac{bcF^{c(a+bx)}\log(F)\sec^2(d}{6e^2}$$

```
[Out] -2/3*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1
/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(2*I*e-b*c*ln(F))/e^2-1/6*b*c*F^(c*(b*
x+a))*ln(F)*sec(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sec(e*x+d)^2*tan(e*x+d)/e
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4533, 4536}

$$-\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\sec^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tan(d+ex)\sec^2(d+ex)F^{c(a+bx)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

```
[Out] (-2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((2*I)*e - b*c*
Log[F]))/(3*e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x]^2)/(6*e^2) + (F
^(c*(a + b*x))*Sec[d + e*x]^2*Tan[d + e*x])/(3*e)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\log(F)\sec^2(d+ex)}{6e^2}+\frac{F^{c(a+bx)}\sec^2(d+ex)\tan(d+ex)}{3e}+\frac{1}{6}\Big(4+$$

$$= -\frac{2e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)(2ie-bc\log(F}{3e^2}$$

Mathematica [A]
time = 0.24, size = 111, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(4e^{2i(d+ex)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)(-2ie+bc\log(F))+\sec^2(d+ex)(-bc\log(F)+2e\tan(d+ex))\right)}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(4*E^((2*I)*(d + e*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-2*I)*e + b*c
*Log[F]) + Sec[d + e*x]^2*(-(b*c*Log[F]) + 2*e*Tan[d + e*x])))/(6*e^2)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\sec^4(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 16*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F
^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*x*e + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*
e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d)^2 + 6*
(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)
*b*c*e^4*log(F))*F^(b*c*x)*sin(4*x*e + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*e^2*lo
```

g(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*x*e + 2*d)^2 - 560*(F^(
a*c)*b^3*c^3*e^2*log(F)^3 - 32*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e
+ 2*d) + 40*(F^(a*c)*b^4*c^4*e*log(F)^4 - 104*F^(a*c)*b^2*c^2*e^3*log(F)^2)
*F^(b*c*x)*sin(2*x*e + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*
c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*x*e + 4*d)
+ 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*c
os(2*x*e + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2*c^2*e^3*l
og(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*x*e + 4*d) + 8*(F^(a*c)*b^4*c^4
*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)*F^(b*c*x)
*sin(2*x*e + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c)*b*c*e^4*
log(F))*F^(b*c*x))*cos(8*x*e + 8*d) + 4*((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^
(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*x
*e + 4*d) + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F
^(b*c*x)*cos(2*x*e + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2
*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*x*e + 4*d) + 8*(F^(a*
c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)
*F^(b*c*x)*sin(2*x*e + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x))*cos(6*x*e + 6*d) + (4*(F^(a*c)*b^5*c^5*log(F)^
5 + 220*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 9984*F^(a*c)*b*c*e^4*log(F))*F^(b*c*
x)*cos(2*x*e + 2*d) + 64*(F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e
^3*log(F)^2 - 576*F^(a*c)*e^5)*F^(b*c*x)*sin(2*x*e + 2*d) + (F^(a*c)*b^5*c^
5*log(F)^5 - 860*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 21504*F^(a*c)*b*c*e^4*log(F
))*F^(b*c*x))*cos(4*x*e + 4*d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(
a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*
c)*b*c*e^8*log(F) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4
*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F
))*cos(8*x*e + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*
c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^
8*log(F))*cos(6*x*e + 6*d)^2 + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)
*b*c*e^8*log(F))*cos(4*x*e + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 11
6*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*
F^(a*c)*b*c*e^8*log(F))*cos(2*x*e + 2*d)^2 + (F^(a*c)*b^7*c^7*e^2*log(F)^7
+ 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36
864*F^(a*c)*b*c*e^8*log(F))*sin(8*x*e + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*lo
g(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)
^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(6*x*e + 6*d)^2 + 36*(F^(a*c)*b^7*c^7
*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6
*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*x*e + 4*d)^2 + 48*(F^(a*c)*
b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*
c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*x*e + 4*d)*sin(2*x*e
 + 2*d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(
2*x*e + 2*d)^2 + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*

```
log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F)
 + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 390
4*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(6*x*e +
6*d) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 +
 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(4*x*
e + 4*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(
2*x*e + 2*d))*cos(8*x*e + 8*d) + 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a
*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c
)*b*c*e^8*log(F) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^
4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(
F))*cos(4*x*e + 4*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^
5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*
log(F))*cos(2*x*e + 2*d))*cos(6*x*e + 6*d) + 12*(F^(a*c)*b^7*c^7*e^2*log(F)
^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 +
 36864*F^(a*c)*b*c*e^8*log(F) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(x*e + d)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cos(d + e*x)^4,x)
```

```
[Out] int(F^(c*(a + b*x))/cos(d + e*x)^4, x)
```

3.18 $\int e^x \cos^4(x)\,dx$

Optimal. Leaf size=54

$$\frac{24e^x}{85}+\frac{12}{85}e^x\cos^2(x)+\frac{1}{17}e^x\cos^4(x)+\frac{24}{85}e^x\cos(x)\sin(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)$$

```
[Out] 24/85*exp(x)+12/85*exp(x)*cos(x)^2+1/17*exp(x)*cos(x)^4+24/85*exp(x)*cos(x)
*sin(x)+4/17*exp(x)*cos(x)^3*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4520, 2225}

$$\frac{24e^x}{85}+\frac{1}{17}e^x\cos^4(x)+\frac{12}{85}e^x\cos^2(x)+\frac{4}{17}e^x\sin(x)\cos^3(x)+\frac{24}{85}e^x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[x]^4,x]
```

```
[Out] (24*E^x)/85 + (12*E^x*Cos[x]^2)/85 + (E^x*Cos[x]^4)/17 + (24*E^x*Cos[x]*Sin
[x])/85 + (4*E^x*Cos[x]^3*Sin[x])/17
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int e^x\cos^4(x)\,dx&=\frac{1}{17}e^x\cos^4(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)+\frac{12}{17}\int e^x\cos^2(x)\,dx\\&=\frac{12}{85}e^x\cos^2(x)+\frac{1}{17}e^x\cos^4(x)+\frac{24}{85}e^x\cos(x)\sin(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)+\frac{24\int e^x\,dx}{85}\\&=\frac{24e^x}{85}+\frac{12}{85}e^x\cos^2(x)+\frac{1}{17}e^x\cos^4(x)+\frac{24}{85}e^x\cos(x)\sin(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.61

$$\frac{1}{680}e^x(255 + 68\cos(2x) + 5\cos(4x) + 136\sin(2x) + 20\sin(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[x]^4,x]
```

```
[Out] (E^x*(255 + 68*Cos[2*x] + 5*Cos[4*x] + 136*Sin[2*x] + 20*Sin[4*x]))/680
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.63

method	result
default	$\frac{(\cos(x)+4\sin(x))\mathrm{e}^{x}(\cos^{3}(x))}{17}+\frac{12(\cos(x)+2\sin(x))\mathrm{e}^{x}\cos(x)}{85}+\frac{24\,\mathrm{e}^{x}}{85}$
risch	$\frac{3\,\mathrm{e}^{x}}{8}+\frac{\mathrm{e}^{(1+4i)x}}{272}-\frac{i\mathrm{e}^{(1+4i)x}}{68}+\frac{\mathrm{e}^{(1+2i)x}}{20}-\frac{i\mathrm{e}^{(1+2i)x}}{10}+\frac{\mathrm{e}^{(1-2i)x}}{20}+\frac{i\mathrm{e}^{(1-2i)x}}{10}+\frac{\mathrm{e}^{(1-4i)x}}{272}+\frac{i\mathrm{e}^{(1-4i)x}}{68}$
norman	$\frac{\frac{88\,\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)}{85}+\frac{76\,\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{85}-\frac{72\,\mathrm{e}^{x}\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{85}+\frac{30\,\mathrm{e}^{x}\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{17}+\frac{72\,\mathrm{e}^{x}\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{85}+\frac{76\,\mathrm{e}^{x}\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{85}-\frac{88\,\mathrm{e}^{x}\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{85}+\frac{41\,\mathrm{e}^{x}\left(\tan^{8}\left(\cdots\right.\right.}{85}\cdots}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*(cos(x)+4*sin(x))*exp(x)*cos(x)^3+12/85*(cos(x)+2*sin(x))*exp(x)*cos(x
)+24/85*exp(x)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.69

$$\frac{1}{136}\cos\left(4\,x\right)e^{x}+\frac{1}{10}\cos\left(2\,x\right)e^{x}+\frac{1}{34}e^{x}\sin\left(4\,x\right)+\frac{1}{5}e^{x}\sin\left(2\,x\right)+\frac{3}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="maxima")
```

```
[Out] 1/136*cos(4*x)*e^x + 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) + 1/5*e^x*sin(2*
x) + 3/8*e^x
```

Fricas [A]
time = 2.59, size = 36, normalized size = 0.67

$$\frac{4}{85}\left(5\,\cos\left(x\right)^{3}+6\,\cos\left(x\right)\right)e^{x}\sin\left(x\right)+\frac{1}{85}\left(5\,\cos\left(x\right)^{4}+12\,\cos\left(x\right)^{2}+24\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="fricas")
```

```
[Out] 4/85*(5*cos(x)^3 + 6*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 + 12*cos(x)^2 +
24)*e^x
```

Sympy [A]
time = 0.65, size = 70, normalized size = 1.30

$$\frac{24e^{x}\sin^{4}(x)}{85}+\frac{24e^{x}\sin^{3}(x)\cos(x)}{85}+\frac{12e^{x}\sin^{2}(x)\cos^{2}(x)}{17}+\frac{44e^{x}\sin(x)\cos^{3}(x)}{85}+\frac{41e^{x}\cos^{4}(x)}{85}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)**4,x)
```

```
[Out] 24*exp(x)*sin(x)**4/85 + 24*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)**
2*cos(x)**2/17 + 44*exp(x)*sin(x)*cos(x)**3/85 + 41*exp(x)*cos(x)**4/85
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.65

$$\frac{1}{136}\left(\cos(4x)+4\sin(4x)\right)e^{x}+\frac{1}{10}\left(\cos(2x)+2\sin(2x)\right)e^{x}+\frac{3}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/136*(cos(4*x) + 4*sin(4*x))*e^x + 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8*
e^x
```

Mupad [B]
time = 0.04, size = 41, normalized size = 0.76

$$\frac{3\,e^{x}}{8}+\frac{e^{x}\left(\frac{4\cos(2x)}{5}+\frac{8\sin(2x)}{5}+\frac{2\cos(2x)^{2}}{17}+\frac{8\cos(2x)\sin(2x)}{17}-\frac{1}{17}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x)^4,x)
```

```
[Out] (3*exp(x))/8 + (exp(x)*((4*cos(2*x))/5 + (8*sin(2*x))/5 + (2*cos(2*x)^2)/17
 + (8*cos(2*x)*sin(2*x))/17 - 1/17))/8
```

3.19 $\int e^{c(a+bx)} \tan^3(d+ex)\, dx$

Optimal. Leaf size=194

$$\frac{ie^{c(a+bx)}}{bc}-\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{8ie^{c(a+bx)}\,{}_2}{}$$

```
[Out] I*exp(c*(b*x+a))/b/c-6*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*
I*b*c/e],-exp(2*I*(e*x+d)))/b/c+12*I*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*
c/e],[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c-8*I*exp(c*(b*x+a))*hypergeom([3
, -1/2*I*b*c/e],[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4527, 2225, 2283}

$$-\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{8ie^{c(a+bx)}\,{}_2F_1\left(3,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}+\frac{ie^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) + ((12*I)*E^
(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^
((2*I)*(d + e*x))])/(b*c) - ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-
1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
```

```
&& IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)}\tan^3(d+ex)\,dx &= -\left(i\int\left(-e^{c(a+bx)}+\frac{8e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}-\frac{12e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}+\frac{6e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx\right)\\
&= i\int e^{c(a+bx)}\,dx-6i\int\frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx-8i\int\frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}\,dx+12i\int\frac{e^{c}}{(1+e}\\
&= \frac{ie^{c(a+bx)}}{bc}-\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1\right.}{bc}
\end{aligned}
$$

Mathematica [A]
time = 2.17, size = 212, normalized size = 1.09

$$
\frac{1}{2}e^{c(a+bx)}\left(\frac{2(b^2c^2-2e^2)\,e^{2id}\left(bce^{2iex}\,{}_2F_1\left(1,1-\frac{ibc}{2e};2-\frac{ibc}{2e};-e^{2i(d+ex)}\right)-(bc+2ie)\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)\right)}{bc(ibc-2e)e^2\left(1+e^{2id}\right)}+\frac{\sec^2(d+ex)}{e}-\frac{bc\sec(d)\sec(d+ex)\sin(ex)}{e^2}-\frac{2\tan(d)}{bc}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((2*(b^2*c^2 - 2*e^2)*E^((2*I)*d)*(b*c*E^((2*I)*e*x)*Hyper
geometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))]
 - (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e
, -E^((2*I)*(d + e*x))]))/(b*c*(I*b*c - 2*e)*e^2*(1 + E^((2*I)*d))) + Sec[d
 + e*x]^2/e - (b*c*Sec[d]*Sec[d + e*x]*Sin[e*x])/e^2 - (2*Tan[d])/(b*c)))/2
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int e^{c(bx+a)}\left(\tan^3(ex+d)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="maxima")

[Out] 2*(18*(b^4*c^4*e + 52*b^2*c^2*e^3 + 576*e^5)*cos(4*x*e + 4*d)^2*e^(b*c*x +
a*c) - 54*(b^4*c^4*e + 28*b^2*c^2*e^3 - 288*e^5)*cos(2*x*e + 2*d)^2*e^(b*c*
x + a*c) + 18*(b^4*c^4*e + 52*b^2*c^2*e^3 + 576*e^5)*e^(b*c*x + a*c)*sin(4*
x*e + 4*d)^2 - 54*(b^4*c^4*e + 28*b^2*c^2*e^3 - 288*e^5)*e^(b*c*x + a*c)*si
n(2*x*e + 2*d)^2 + 18*(3*b^4*c^4*e - 212*b^2*c^2*e^3 + 640*e^5)*cos(2*x*e +
 2*d)*e^(b*c*x + a*c) - 3*(b^5*c^5 - 268*b^3*c^3*e^2 + 1216*b*c*e^4)*e^(b*c
*x + a*c)*sin(2*x*e + 2*d) + 3*(2*(b^4*c^4*e + 52*b^2*c^2*e^3 + 576*e^5)*co
s(4*x*e + 4*d)*e^(b*c*x + a*c) - 6*(b^4*c^4*e + 28*b^2*c^2*e^3 - 288*e^5)*c
os(2*x*e + 2*d)*e^(b*c*x + a*c) + (b^5*c^5 + 52*b^3*c^3*e^2 + 576*b*c*e^4)*
e^(b*c*x + a*c)*sin(4*x*e + 4*d) + 36*(b^3*c^3*e^2 + 36*b*c*e^4)*e^(b*c*x +
 a*c)*sin(2*x*e + 2*d) + 8*(b^4*c^4*e - 46*b^2*c^2*e^3 + 88*e^5)*e^(b*c*x +
 a*c))*cos(6*x*e + 6*d) - 3*(12*(b^4*c^4*e + 16*b^2*c^2*e^3 - 720*e^5)*cos(
2*x*e + 2*d)*e^(b*c*x + a*c) + 3*(b^5*c^5 + 16*b^3*c^3*e^2 - 720*b*c*e^4)*e
^(b*c*x + a*c)*sin(2*x*e + 2*d) - 2*(13*b^4*c^4*e - 500*b^2*c^2*e^3 + 1632*
e^5)*e^(b*c*x + a*c))*cos(4*x*e + 4*d) + 24*(b^4*c^4*e - 46*b^2*c^2*e^3 + 8
8*e^5)*e^(b*c*x + a*c) - 24*((b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5
+ 736*b^2*c^2*e^7 - 4608*e^9)*cos(6*x*e + 6*d)^2*e^(a*c) + 9*(b^8*c^8*e + 5
4*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(4*x*e + 4
*d)^2*e^(a*c) + 9*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c
^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)^2*e^(a*c) + (b^8*c^8*e + 54*b^6*c^6*e^3
 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c)*sin(6*x*e + 6*d)^2
 + 9*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608
*e^9)*e^(a*c)*sin(4*x*e + 4*d)^2 + 18*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4
*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c)*sin(4*x*e + 4*d)*sin(2*x*e +
 2*d) + 9*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 -
 4608*e^9)*e^(a*c)*sin(2*x*e + 2*d)^2 + 6*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672
*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)*e^(a*c) + 2*(3*
(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)
*cos(4*x*e + 4*d)*e^(a*c) + 3*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5
 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)*e^(a*c) + (b^8*c^8*e + 54*b
^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c))*cos(6*x
*e + 6*d) + 6*(3*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^
2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)*e^(a*c) + (b^8*c^8*e + 54*b^6*c^6*e^3 +
672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c))*cos(4*x*e + 4*d) + (
b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*
e^(a*c) + 6*((b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^
7 - 4608*e^9)*e^(a*c)*sin(4*x*e + 4*d) + (b^8*c^8*e + 54*b^6*c^6*e^3 + 672*
b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c)*sin(2*x*e + 2*d))*sin(6*x
*e + 6*d))*integrate(((b^3*c^3 - 44*b*c*e^2)*cos(8*x*e + 8*d)*e^(b*c*x) + 4
*(b^3*c^3 - 44*b*c*e^2)*cos(6*x*e + 6*d)*e^(b*c*x) + 6*(b^3*c^3 - 44*b*c*e^
2)*cos(4*x*e + 4*d)*e^(b*c*x) + 4*(b^3*c^3 - 44*b*c*e^2)*cos(2*x*e + 2*d)*e
^(b*c*x) + 12*(b^2*c^2*e - 4*e^3)*e^(b*c*x)*sin(8*x*e + 8*d) + 48*(b^2*c^2*
e - 4*e^3)*e^(b*c*x)*sin(6*x*e + 6*d) + 72*(b^2*c^2*e - 4*e^3)*e^(b*c*x)*si
```

```
n(4*x*e + 4*d) + 48*(b^2*c^2*e - 4*e^3)*e^(b*c*x)*sin(2*x*e + 2*d) + (b^3*c
^3 - 44*b*c*e^2)*e^(b*c*x))/(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + (
b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(8*x*e + 8*d)^2 +
 16*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(6*x*e + 6*d
)^2 + 36*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(4*x*e
+ 4*d)^2 + 16*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(2
*x*e + 2*d)^2 + (b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*sin
(8*x*e + 8*d)^2 + 16*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6
)*sin(6*x*e + 6*d)^2 + 36*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 230
4*e^6)*sin(4*x*e + 4*d)^2 + 48*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4
+ 2304*e^6)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 16*(b^6*c^6 + 56*b^4*c^4*e^
2 + 784*b^2*c^2*e^4 + 2304*e^6)*sin(2*x*e + 2*d)^2 + 2*(b^6*c^6 + 56*b^4*c^
4*e^2 + 784*b^2*c^2*e^4 + 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2
304*e^6)*cos(6*x*e + 6*d) + 6*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 +
 2304*e^6)*cos(4*x*e + 4*d) + 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4
 + 2304*e^6)*cos(2*x*e + 2*d) + 2304*e^6)*cos(8*x*e + 8*d) + 8*(b^6*c^6 + 5
6*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 6*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2
*e^4 + 2304*e^6)*cos(4*x*e + 4*d) + 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c
^2*e^4 + 2304*e^6)*cos(2*x*e + 2*d) + 2304*e^6)*cos(6*x*e + 6*d) + 12*(b^6*
c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*
b^2*c^2*e^4 + 2304*e^6)*cos(2*x*e + 2*d) + 2304*e^6)*cos(4*x*e + 4*d) + 8*(
b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(2*x*e + 2*d) + 4
*(2*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*sin(6*x*e + 6*d
) + 3*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tan(x*e + d)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int e^{bcx} \tan^3(d + ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)**3,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{c\,(a+b\,x)}\,\tan(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*tan(d + e*x)^3,x)
[Out] int(exp(c*(a + b*x))*tan(d + e*x)^3, x)
```

3.20 $\int e^{c(a+bx)}\tan^2(d+ex)\,dx$

Optimal. Leaf size=130

$$-\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -exp(c*(b*x+a))/b/c+4*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*I*b
*c/e],-exp(2*I*(e*x+d)))/b/c-4*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*c/e],[
1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4527, 2225, 2283}

$$\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1/2*I
)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*
x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d
+ e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx = -\int\left(e^{c(a+bx)}+\frac{4e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}-\frac{4e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx$$
$$= -\left(4\int\frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}dx\right)+4\int\frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}dx-\int e^{c(a+bx)}\,dx$$
$$= -\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1\right.}{bc}$$

Mathematica [A]
time = 1.72, size = 174, normalized size = 1.34

$$e^{c(a+bx)}\left(-\frac{1}{bc}+\frac{2ie^{2id}\left(bce^{2iex}\,{}_2F_1\left(1,1-\frac{ibc}{2e};2-\frac{ibc}{2e};-e^{2i(d+ex)}\right)-(bc+2ie)\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)\right)}{(bc+2ie)e\left(1+e^{2id}\right)}+\frac{\sec(d)\sec(d+ex)\sin(ex)}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(-(1/(b*c)) + ((2*I)*E^((2*I)*d)*(b*c*E^((2*I)*e*x)*Hyperge
ometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] -
 (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e,
-E^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(1 + E^((2*I)*d))) + (Sec[d]*Sec
[d + e*x]*Sin[e*x])/e)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\left(\tan^2(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -((b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(4*x*e + 4*d)^2*e^(b*c*x + a*c) -
4*(b^4*c^4 + 12*b^2*c^2*e^2 - 64*e^4)*cos(2*x*e + 2*d)^2*e^(b*c*x + a*c) +
(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*e^(b*c*x + a*c)*sin(4*x*e + 4*d)^2 - 4*
(b^4*c^4 + 12*b^2*c^2*e^2 - 64*e^4)*e^(b*c*x + a*c)*sin(2*x*e + 2*d)^2 - 16
*(11*b^2*c^2*e^2 - 16*e^4)*cos(2*x*e + 2*d)*e^(b*c*x + a*c) + 8*(5*b^3*c^3*
e - 16*b*c*e^3)*e^(b*c*x + a*c)*sin(2*x*e + 2*d) + 2*(8*(b^2*c^2*e^2 + 16*e
^4)*cos(2*x*e + 2*d)*e^(b*c*x + a*c) + 4*(b^3*c^3*e + 16*b*c*e^3)*e^(b*c*x
+ a*c)*sin(2*x*e + 2*d) + (b^4*c^4 - 28*b^2*c^2*e^2 + 64*e^4)*e^(b*c*x + a*
c))*cos(4*x*e + 4*d) + (b^4*c^4 - 76*b^2*c^2*e^2 + 64*e^4)*e^(b*c*x + a*c)
- 16*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4 + (b^6*c^6 + 20*b^4*c^4*e^2
 + 64*b^2*c^2*e^4)*cos(4*x*e + 4*d)^2 + 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^
2*c^2*e^4)*cos(2*x*e + 2*d)^2 + (b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)
*sin(4*x*e + 4*d)^2 + 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*sin(4*x
*e + 4*d)*sin(2*x*e + 2*d) + 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*
sin(2*x*e + 2*d)^2 + 2*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4 + 2*(b^6*
c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*cos(2*x*e + 2*d))*cos(4*x*e + 4*d) +
 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*cos(2*x*e + 2*d))*integrate(
-(6*b*c*cos(6*x*e + 6*d)*e^(b*c*x + a*c + 2) + 18*b*c*cos(4*x*e + 4*d)*e^(b
*c*x + a*c + 2) + 18*b*c*cos(2*x*e + 2*d)*e^(b*c*x + a*c + 2) + 6*b*c*e^(b*
c*x + a*c + 2) - (b^2*c^2*e - 8*e^3)*e^(b*c*x + a*c)*sin(6*x*e + 6*d) - 3*(
b^2*c^2*e - 8*e^3)*e^(b*c*x + a*c)*sin(4*x*e + 4*d) - 3*(b^2*c^2*e - 8*e^3)
*e^(b*c*x + a*c)*sin(2*x*e + 2*d))/(b^4*c^4 + 20*b^2*c^2*e^2 + (b^4*c^4 + 2
0*b^2*c^2*e^2 + 64*e^4)*cos(6*x*e + 6*d)^2 + 9*(b^4*c^4 + 20*b^2*c^2*e^2 +
64*e^4)*cos(4*x*e + 4*d)^2 + 9*(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(2*x*
e + 2*d)^2 + (b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*sin(6*x*e + 6*d)^2 + 9*(b^
4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*sin(4*x*e + 4*d)^2 + 18*(b^4*c^4 + 20*b^2*
c^2*e^2 + 64*e^4)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 9*(b^4*c^4 + 20*b^2*c
^2*e^2 + 64*e^4)*sin(2*x*e + 2*d)^2 + 2*(b^4*c^4 + 20*b^2*c^2*e^2 + 3*(b^4*
c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(4*x*e + 4*d) + 3*(b^4*c^4 + 20*b^2*c^2*e
^2 + 64*e^4)*cos(2*x*e + 2*d) + 64*e^4)*cos(6*x*e + 6*d) + 6*(b^4*c^4 + 20*
b^2*c^2*e^2 + 3*(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(2*x*e + 2*d) + 64*e
^4)*cos(4*x*e + 4*d) + 6*(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(2*x*e + 2*
d) + 6*((b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*sin(4*x*e + 4*d) + (b^4*c^4 + 2
0*b^2*c^2*e^2 + 64*e^4)*sin(2*x*e + 2*d))*sin(6*x*e + 6*d) + 64*e^4), x) -
8*((b^3*c^3*e + 16*b*c*e^3)*cos(2*x*e + 2*d)*e^(b*c*x + a*c) - 2*(b^2*c^2*e
^2 + 16*e^4)*e^(b*c*x + a*c)*sin(2*x*e + 2*d) - 2*(b^3*c^3*e - 8*b*c*e^3)*e
^(b*c*x + a*c))*sin(4*x*e + 4*d))/(b^5*c^5 + 20*b^3*c^3*e^2 + (b^5*c^5 + 20
*b^3*c^3*e^2 + 64*b*c*e^4)*cos(4*x*e + 4*d)^2 + 4*(b^5*c^5 + 20*b^3*c^3*e^2
 + 64*b*c*e^4)*cos(2*x*e + 2*d)^2 + 64*b*c*e^4 + (b^5*c^5 + 20*b^3*c^3*e^2
+ 64*b*c*e^4)*sin(4*x*e + 4*d)^2 + 4*(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4
)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 4*(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*
e^4)*sin(2*x*e + 2*d)^2 + 2*(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4 + 2*(b^5
*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4)*cos(2*x*e + 2*d))*cos(4*x*e + 4*d) + 4*
(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4)*cos(2*x*e + 2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="fricas")
[Out] integral(e^(b*c*x + a*c)*tan(x*e + d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int e^{bcx} \tan^2{\left(d + ex \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)**2,x)
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{c\,\left(a+b\,x\right)}\,{\mathrm{tan}\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*tan(d + e*x)^2,x)
[Out] int(exp(c*(a + b*x))*tan(d + e*x)^2, x)
```

3.21 $\int e^{c(a+bx)}\tan(d+ex)\,dx$

Optimal. Leaf size=78

$$-\frac{ie^{c(a+bx)}}{bc}+\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -I*exp(c*(b*x+a))/b/c+2*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2
*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4527, 2225, 2283}

$$\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{ie^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x],x]
```

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\int e^{c(a+bx)}\tan(d+ex)\,dx = i\int\left(-e^{c(a+bx)}+\frac{2e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx$$
$$= -\left(i\int e^{c(a+bx)}\,dx\right)+2i\int\frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx$$
$$= -\frac{ie^{c(a+bx)}}{bc}+\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.
time = 0.46, size = 166, normalized size = 2.13

$$\frac{e^{c(a+bx)}\left(2bce^{2i(d+ex)}\,{}_2F_1\left(1,1-\frac{ibc}{2e};2-\frac{ibc}{2e};-e^{2i(d+ex)}\right)-(bc+2ie)\left(1-e^{2id}+2e^{2id}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)\right)\right)}{bc(ibc-2e)\left(1+e^{2id}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((I/2)
*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] - (b*c + (2*I)*e)*(1 - E^
((2*I)*d) + 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)
*b*c)/e, -E^((2*I)*(d + e*x))])))/(b*c*(I*b*c - 2*e)*(1 + E^((2*I)*d)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\tan(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tan(e*x+d),x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*(b*c*e^(b*c*x + a*c)*sin(2*x*e + 2*d) - 2*cos(2*x*e + 2*d)*e^(b*c*x + a*c
 + 1) + 2*(b^2*c^2 + (b^2*c^2 + 4*e^2)*cos(2*x*e + 2*d)^2 + (b^2*c^2 + 4*e^
```

```
2)*sin(2*x*e + 2*d)^2 + 2*(b^2*c^2 + 4*e^2)*cos(2*x*e + 2*d) + 4*e^2)*integ
rate((b*c*cos(4*x*e + 4*d)*e^(b*c*x + a*c + 1) + 2*b*c*cos(2*x*e + 2*d)*e^(
b*c*x + a*c + 1) + b*c*e^(b*c*x + a*c + 1) + 2*e^(b*c*x + a*c + 2)*sin(4*x*
e + 4*d) + 4*e^(b*c*x + a*c + 2)*sin(2*x*e + 2*d))/(b^2*c^2 + (b^2*c^2 + 4*
e^2)*cos(4*x*e + 4*d)^2 + 4*(b^2*c^2 + 4*e^2)*cos(2*x*e + 2*d)^2 + (b^2*c^2
 + 4*e^2)*sin(4*x*e + 4*d)^2 + 4*(b^2*c^2 + 4*e^2)*sin(4*x*e + 4*d)*sin(2*x
*e + 2*d) + 4*(b^2*c^2 + 4*e^2)*sin(2*x*e + 2*d)^2 + 2*(b^2*c^2 + 2*(b^2*c^
2 + 4*e^2)*cos(2*x*e + 2*d) + 4*e^2)*cos(4*x*e + 4*d) + 4*(b^2*c^2 + 4*e^2)
*cos(2*x*e + 2*d) + 4*e^2), x) - 2*e^(b*c*x + a*c + 1))/(b^2*c^2 + (b^2*c^2
 + 4*e^2)*cos(2*x*e + 2*d)^2 + (b^2*c^2 + 4*e^2)*sin(2*x*e + 2*d)^2 + 2*(b^
2*c^2 + 4*e^2)*cos(2*x*e + 2*d) + 4*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="fricas")
[Out] integral(e^(b*c*x + a*c)*tan(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int e^{bcx} \tan{\left(d + ex \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{c\,\left(a+b\,x\right)}\,\mathrm{tan}\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*tan(d + e*x),x)
```

```
[Out] int(exp(c*(a + b*x))*tan(d + e*x), x)
```

3.22 $\int e^{c(a+bx)} \cot(d+ex)\,dx$

Optimal. Leaf size=76

$$\frac{ie^{c(a+bx)}}{bc} - \frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}$$

```
[Out] I*exp(c*(b*x+a))/b/c-2*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*
I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4528, 2225, 2283}

$$\frac{ie^{c(a+bx)}}{bc} - \frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x],x]
```

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c(a+bx)}\cot(d+ex)\,dx = -\left(i\int\left(-e^{c(a+bx)}-\frac{2e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx\right)$$
$$= i\int e^{c(a+bx)}\,dx+2i\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx$$
$$= \frac{ie^{c(a+bx)}}{bc}-\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 163 vs. $2(76)=152$.
time = 1.05, size = 163, normalized size = 2.14

$$\frac{e^{c(a+bx)}\left(2ibce^{2i(d+ex)}\,{}_2F_1\left(1,1-\frac{ibc}{2e};2-\frac{ibc}{2e};e^{2i(d+ex)}\right)+i(bc+2ie)\left(1+e^{2id}-2e^{2id}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)\right)\right)}{bc(bc+2ie)\left(-1+e^{2id}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*((2*I)*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((
I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + I*(b*c + (2*I)*e)*(1
 + E^((2*I)*d) - 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - (
(I/2)*b*c)/e, E^((2*I)*(d + e*x))])))/(b*c*(b*c + (2*I)*e)*(-1 + E^((2*I)*d
)))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\cot\left(ex+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*cot(e*x+d),x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*(b*c*e^(b*c*x + a*c)*sin(2*x*e + 2*d) - 2*cos(2*x*e + 2*d)*e^(b*c*x + a*c
 + 1) - 2*(b^2*c^2 + (b^2*c^2 + 4*e^2)*cos(2*x*e + 2*d)^2 + (b^2*c^2 + 4*e^
2)*sin(2*x*e + 2*d)^2 - 2*(b^2*c^2 + 4*e^2)*cos(2*x*e + 2*d) + 4*e^2)*integ
rate((b*c*cos(4*x*e + 4*d)*e^(b*c*x + a*c + 1) - 2*b*c*cos(2*x*e + 2*d)*e^(
b*c*x + a*c + 1) + b*c*e^(b*c*x + a*c + 1) + 2*e^(b*c*x + a*c + 2)*sin(4*x*
e + 4*d) - 4*e^(b*c*x + a*c + 2)*sin(2*x*e + 2*d))/(b^2*c^2 + (b^2*c^2 + 4*
e^2)*cos(4*x*e + 4*d)^2 + 4*(b^2*c^2 + 4*e^2)*cos(2*x*e + 2*d)^2 + (b^2*c^2
 + 4*e^2)*sin(4*x*e + 4*d)^2 - 4*(b^2*c^2 + 4*e^2)*sin(4*x*e + 4*d)*sin(2*x
*e + 2*d) + 4*(b^2*c^2 + 4*e^2)*sin(2*x*e + 2*d)^2 + 2*(b^2*c^2 - 2*(b^2*c^
2 + 4*e^2)*cos(2*x*e + 2*d) + 4*e^2)*cos(4*x*e + 4*d) - 4*(b^2*c^2 + 4*e^2)
*cos(2*x*e + 2*d) + 4*e^2), x) + 2*e^(b*c*x + a*c + 1))/(b^2*c^2 + (b^2*c^2
 + 4*e^2)*cos(2*x*e + 2*d)^2 + (b^2*c^2 + 4*e^2)*sin(2*x*e + 2*d)^2 - 2*(b^
2*c^2 + 4*e^2)*cos(2*x*e + 2*d) + 4*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="fricas")
[Out] integral(cot(x*e + d)*e^(b*c*x + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\cot\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="giac")
[Out] integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(d+e\,x)\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d + e*x)*exp(c*(a + b*x)),x)
```

```
[Out] int(cot(d + e*x)*exp(c*(a + b*x)), x)
```

3.23 $\int e^{c(a+bx)} \cot^2(d+ex)\, dx$

Optimal. Leaf size=126

$$-\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -exp(c*(b*x+a))/b/c+4*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*I*b
*c/e],exp(2*I*(e*x+d)))/b/c-4*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*c/e],[1
-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

Rubi [A]
time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4528, 2225, 2283}

$$\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1/2*I
)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*x
))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d +
e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c(a+bx)}\cot^2(d+ex)\,dx = -\int\left(e^{c(a+bx)}+\frac{4e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}+\frac{4e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx$$
$$= -\left(4\int\frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}\,dx\right)-4\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx-\int e^{c(a+bx)}\,dx$$
$$= -\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\right.}{bc}$$

Mathematica [A]
time = 1.67, size = 170, normalized size = 1.35

$$e^{c(a+bx)}\left(-\frac{1}{bc}-\frac{2ie^{2id}\left(-bce^{2iex}\,{}_2F_1\left(1,1-\frac{ibc}{2e};2-\frac{ibc}{2e};e^{2i(d+ex)}\right)+(bc+2ie)\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)\right)}{(bc+2ie)e\left(-1+e^{2id}\right)}+\frac{\csc(d)\csc(d+ex)\sin(ex)}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(-(1/(b*c)) - ((2*I)*E^((2*I)*d)*(-(b*c*E^((2*I)*e*x)*Hyper
geometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])
 + (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e
, E^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(-1 + E^((2*I)*d))) + (Csc[d]*C
sc[d + e*x]*Sin[e*x])/e)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\left(\cot^2\left(ex+d\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -((b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(4*x*e + 4*d)^2*e^(b*c*x + a*c) -
4*(b^4*c^4 + 12*b^2*c^2*e^2 - 64*e^4)*cos(2*x*e + 2*d)^2*e^(b*c*x + a*c) +
(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*e^(b*c*x + a*c)*sin(4*x*e + 4*d)^2 - 4*
(b^4*c^4 + 12*b^2*c^2*e^2 - 64*e^4)*e^(b*c*x + a*c)*sin(2*x*e + 2*d)^2 + 16
*(11*b^2*c^2*e^2 - 16*e^4)*cos(2*x*e + 2*d)*e^(b*c*x + a*c) - 8*(5*b^3*c^3*
e - 16*b*c*e^3)*e^(b*c*x + a*c)*sin(2*x*e + 2*d) - 2*(8*(b^2*c^2*e^2 + 16*e
^4)*cos(2*x*e + 2*d)*e^(b*c*x + a*c) + 4*(b^3*c^3*e + 16*b*c*e^3)*e^(b*c*x
+ a*c)*sin(2*x*e + 2*d) - (b^4*c^4 - 28*b^2*c^2*e^2 + 64*e^4)*e^(b*c*x + a*
c))*cos(4*x*e + 4*d) + (b^4*c^4 - 76*b^2*c^2*e^2 + 64*e^4)*e^(b*c*x + a*c)
+ 16*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4 + (b^6*c^6 + 20*b^4*c^4*e^2
 + 64*b^2*c^2*e^4)*cos(4*x*e + 4*d)^2 + 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^
2*c^2*e^4)*cos(2*x*e + 2*d)^2 + (b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)
*sin(4*x*e + 4*d)^2 - 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*sin(4*x
*e + 4*d)*sin(2*x*e + 2*d) + 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*
sin(2*x*e + 2*d)^2 + 2*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4 - 2*(b^6*
c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*cos(2*x*e + 2*d))*cos(4*x*e + 4*d) -
 4*(b^6*c^6 + 20*b^4*c^4*e^2 + 64*b^2*c^2*e^4)*cos(2*x*e + 2*d))*integrate(
-(6*b*c*cos(6*x*e + 6*d)*e^(b*c*x + a*c + 2) - 18*b*c*cos(4*x*e + 4*d)*e^(b
*c*x + a*c + 2) + 18*b*c*cos(2*x*e + 2*d)*e^(b*c*x + a*c + 2) - 6*b*c*e^(b*
c*x + a*c + 2) - (b^2*c^2*e - 8*e^3)*e^(b*c*x + a*c)*sin(6*x*e + 6*d) + 3*(
b^2*c^2*e - 8*e^3)*e^(b*c*x + a*c)*sin(4*x*e + 4*d) - 3*(b^2*c^2*e - 8*e^3)
*e^(b*c*x + a*c)*sin(2*x*e + 2*d))/(b^4*c^4 + 20*b^2*c^2*e^2 + (b^4*c^4 + 2
0*b^2*c^2*e^2 + 64*e^4)*cos(6*x*e + 6*d)^2 + 9*(b^4*c^4 + 20*b^2*c^2*e^2 +
64*e^4)*cos(4*x*e + 4*d)^2 + 9*(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(2*x*
e + 2*d)^2 + (b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*sin(6*x*e + 6*d)^2 + 9*(b^
4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*sin(4*x*e + 4*d)^2 - 18*(b^4*c^4 + 20*b^2*
c^2*e^2 + 64*e^4)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 9*(b^4*c^4 + 20*b^2*c
^2*e^2 + 64*e^4)*sin(2*x*e + 2*d)^2 - 2*(b^4*c^4 + 20*b^2*c^2*e^2 + 3*(b^4*
c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(4*x*e + 4*d) - 3*(b^4*c^4 + 20*b^2*c^2*e
^2 + 64*e^4)*cos(2*x*e + 2*d) + 64*e^4)*cos(6*x*e + 6*d) + 6*(b^4*c^4 + 20*
b^2*c^2*e^2 - 3*(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(2*x*e + 2*d) + 64*e
^4)*cos(4*x*e + 4*d) - 6*(b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*cos(2*x*e + 2*
d) - 6*((b^4*c^4 + 20*b^2*c^2*e^2 + 64*e^4)*sin(4*x*e + 4*d) - (b^4*c^4 + 2
0*b^2*c^2*e^2 + 64*e^4)*sin(2*x*e + 2*d))*sin(6*x*e + 6*d) + 64*e^4), x) +
8*((b^3*c^3*e + 16*b*c*e^3)*cos(2*x*e + 2*d)*e^(b*c*x + a*c) - 2*(b^2*c^2*e
^2 + 16*e^4)*e^(b*c*x + a*c)*sin(2*x*e + 2*d) + 2*(b^3*c^3*e - 8*b*c*e^3)*e
^(b*c*x + a*c))*sin(4*x*e + 4*d))/(b^5*c^5 + 20*b^3*c^3*e^2 + (b^5*c^5 + 20
*b^3*c^3*e^2 + 64*b*c*e^4)*cos(4*x*e + 4*d)^2 + 4*(b^5*c^5 + 20*b^3*c^3*e^2
 + 64*b*c*e^4)*cos(2*x*e + 2*d)^2 + 64*b*c*e^4 + (b^5*c^5 + 20*b^3*c^3*e^2
+ 64*b*c*e^4)*sin(4*x*e + 4*d)^2 - 4*(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4
)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 4*(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*
e^4)*sin(2*x*e + 2*d)^2 + 2*(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4 - 2*(b^5
*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4)*cos(2*x*e + 2*d))*cos(4*x*e + 4*d) - 4*
(b^5*c^5 + 20*b^3*c^3*e^2 + 64*b*c*e^4)*cos(2*x*e + 2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="fricas")
[Out] integral(cot(x*e + d)^2*e^(b*c*x + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\cot^2\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)**2,x)
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="giac")
[Out] integrate(cot(e*x + d)^2*e^((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(d+e\,x)^2\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d + e*x)^2*exp(c*(a + b*x)),x)
[Out] int(cot(d + e*x)^2*exp(c*(a + b*x)), x)
```

3.24 $\int e^{c(a+bx)} \cot^3(d+ex)\,dx$

Optimal. Leaf size=188

$$-\frac{ie^{c(a+bx)}}{bc}+\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}+\frac{8ie^{c(a+bx)}\,{}_2F_1(}{}$$

```
[Out] -I*exp(c*(b*x+a))/b/c+6*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2
*I*b*c/e],exp(2*I*(e*x+d)))/b/c-12*I*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*
c/e],[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c+8*I*exp(c*(b*x+a))*hypergeom([3,
 -1/2*I*b*c/e],[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4528, 2225, 2283}

$$\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}+\frac{8ie^{c(a+bx)}\,{}_2F_1\left(3,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{ie^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - ((12*I)*
E^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E
^((2*I)*(d + e*x))])/(b*c) + ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((
-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
```

```
x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = i\int\left(-e^{c(a+bx)}-\frac{8e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^3}-\frac{12e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}-\frac{6e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx$$
$$= -\left(i\int e^{c(a+bx)}\,dx\right)-6i\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx-8i\int\frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^3}\,dx-12$$
$$= -\frac{ie^{c(a+bx)}}{bc}+\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};\right.}{bc}$$

Mathematica [A]
time = 2.23, size = 210, normalized size = 1.12

$$\frac{1}{2}e^{c(a+bx)}\left(-\frac{2\cot(d)}{bc}-\frac{\csc^2(d+ex)}{e}+\frac{2(b^2c^2-2e^2)\,e^{2id}\left(ibce^{2iex}\,{}_2F_1\left(1,1-\frac{ibc}{2e};2-\frac{ibc}{2e};e^{2i(d+ex)}\right)+(-ibc+2e)\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)\right)}{bc(bc+2ie)e^2\left(-1+e^{2id}\right)}+\frac{bc\csc(d)\csc(d+ex)\sin(ex)}{e^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((-2*Cot[d])/(b*c) - Csc[d + e*x]^2/e + (2*(b^2*c^2 - 2*e^
2)*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeometric2F1[1, 1 - ((I/2)*b*c)/e,
 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + ((-I)*b*c + 2*e)*Hypergeometric2
F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))]))/(b*c*(b*c
 + (2*I)*e)*e^2*(-1 + E^((2*I)*d))) + (b*c*Csc[d]*Csc[d + e*x]*Sin[e*x])/e^
2))/2
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\left(\cot^3(ex+d)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -2*(18*(b^4*c^4*e + 52*b^2*c^2*e^3 + 576*e^5)*cos(4*x*e + 4*d)^2*e^(b*c*x +
 a*c) - 54*(b^4*c^4*e + 28*b^2*c^2*e^3 - 288*e^5)*cos(2*x*e + 2*d)^2*e^(b*c
*x + a*c) + 18*(b^4*c^4*e + 52*b^2*c^2*e^3 + 576*e^5)*e^(b*c*x + a*c)*sin(4
*x*e + 4*d)^2 - 54*(b^4*c^4*e + 28*b^2*c^2*e^3 - 288*e^5)*e^(b*c*x + a*c)*s
in(2*x*e + 2*d)^2 - 18*(3*b^4*c^4*e - 212*b^2*c^2*e^3 + 640*e^5)*cos(2*x*e
+ 2*d)*e^(b*c*x + a*c) + 3*(b^5*c^5 - 268*b^3*c^3*e^2 + 1216*b*c*e^4)*e^(b*
c*x + a*c)*sin(2*x*e + 2*d) - 3*(2*(b^4*c^4*e + 52*b^2*c^2*e^3 + 576*e^5)*c
os(4*x*e + 4*d)*e^(b*c*x + a*c) + 6*(b^4*c^4*e + 28*b^2*c^2*e^3 - 288*e^5)*
cos(2*x*e + 2*d)*e^(b*c*x + a*c) + (b^5*c^5 + 52*b^3*c^3*e^2 + 576*b*c*e^4)
*e^(b*c*x + a*c)*sin(4*x*e + 4*d) - 36*(b^3*c^3*e^2 + 36*b*c*e^4)*e^(b*c*x
+ a*c)*sin(2*x*e + 2*d) + 8*(b^4*c^4*e - 46*b^2*c^2*e^3 + 88*e^5)*e^(b*c*x
+ a*c))*cos(6*x*e + 6*d) + 3*(12*(b^4*c^4*e + 16*b^2*c^2*e^3 - 720*e^5)*cos
(2*x*e + 2*d)*e^(b*c*x + a*c) + 3*(b^5*c^5 + 16*b^3*c^3*e^2 - 720*b*c*e^4)*
e^(b*c*x + a*c)*sin(2*x*e + 2*d) + 2*(13*b^4*c^4*e - 500*b^2*c^2*e^3 + 1632
*e^5)*e^(b*c*x + a*c))*cos(4*x*e + 4*d) + 24*(b^4*c^4*e - 46*b^2*c^2*e^3 +
88*e^5)*e^(b*c*x + a*c) - 24*((b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5
 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(6*x*e + 6*d)^2*e^(a*c) + 9*(b^8*c^8*e +
54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(4*x*e +
4*d)^2*e^(a*c) + 9*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*
c^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)^2*e^(a*c) + (b^8*c^8*e + 54*b^6*c^6*e^
3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c)*sin(6*x*e + 6*d)^
2 + 9*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 460
8*e^9)*e^(a*c)*sin(4*x*e + 4*d)^2 - 18*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^
4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c)*sin(4*x*e + 4*d)*sin(2*x*e
+ 2*d) + 9*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7
- 4608*e^9)*e^(a*c)*sin(2*x*e + 2*d)^2 - 6*(b^8*c^8*e + 54*b^6*c^6*e^3 + 67
2*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)*e^(a*c) - 2*(3
*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9
)*cos(4*x*e + 4*d)*e^(a*c) - 3*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^
5 + 736*b^2*c^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)*e^(a*c) + (b^8*c^8*e + 54*
b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c))*cos(6*
x*e + 6*d) - 6*(3*(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c
^2*e^7 - 4608*e^9)*cos(2*x*e + 2*d)*e^(a*c) - (b^8*c^8*e + 54*b^6*c^6*e^3 +
 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c))*cos(4*x*e + 4*d) +
(b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)
*e^(a*c) - 6*((b^8*c^8*e + 54*b^6*c^6*e^3 + 672*b^4*c^4*e^5 + 736*b^2*c^2*e
^7 - 4608*e^9)*e^(a*c)*sin(4*x*e + 4*d) - (b^8*c^8*e + 54*b^6*c^6*e^3 + 672
*b^4*c^4*e^5 + 736*b^2*c^2*e^7 - 4608*e^9)*e^(a*c)*sin(2*x*e + 2*d))*sin(6*
x*e + 6*d))*integrate(((b^3*c^3 - 44*b*c*e^2)*cos(8*x*e + 8*d)*e^(b*c*x) -
4*(b^3*c^3 - 44*b*c*e^2)*cos(6*x*e + 6*d)*e^(b*c*x) + 6*(b^3*c^3 - 44*b*c*e
^2)*cos(4*x*e + 4*d)*e^(b*c*x) - 4*(b^3*c^3 - 44*b*c*e^2)*cos(2*x*e + 2*d)*
```

```
e^(b*c*x) + 12*(b^2*c^2*e - 4*e^3)*e^(b*c*x)*sin(8*x*e + 8*d) - 48*(b^2*c^2
*e - 4*e^3)*e^(b*c*x)*sin(6*x*e + 6*d) + 72*(b^2*c^2*e - 4*e^3)*e^(b*c*x)*s
in(4*x*e + 4*d) - 48*(b^2*c^2*e - 4*e^3)*e^(b*c*x)*sin(2*x*e + 2*d) + (b^3*
c^3 - 44*b*c*e^2)*e^(b*c*x))/(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 +
(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(8*x*e + 8*d)^2
+ 16*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(6*x*e + 6*
d)^2 + 36*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(4*x*e
 + 4*d)^2 + 16*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(
2*x*e + 2*d)^2 + (b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*si
n(8*x*e + 8*d)^2 + 16*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^
6)*sin(6*x*e + 6*d)^2 + 36*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 23
04*e^6)*sin(4*x*e + 4*d)^2 - 48*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4
 + 2304*e^6)*sin(4*x*e + 4*d)*sin(2*x*e + 2*d) + 16*(b^6*c^6 + 56*b^4*c^4*e
^2 + 784*b^2*c^2*e^4 + 2304*e^6)*sin(2*x*e + 2*d)^2 + 2*(b^6*c^6 + 56*b^4*c
^4*e^2 + 784*b^2*c^2*e^4 - 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 +
2304*e^6)*cos(6*x*e + 6*d) + 6*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4
+ 2304*e^6)*cos(4*x*e + 4*d) - 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^
4 + 2304*e^6)*cos(2*x*e + 2*d) + 2304*e^6)*cos(8*x*e + 8*d) - 8*(b^6*c^6 +
56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 6*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^
2*e^4 + 2304*e^6)*cos(4*x*e + 4*d) - 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*
c^2*e^4 + 2304*e^6)*cos(2*x*e + 2*d) + 2304*e^6)*cos(6*x*e + 6*d) + 12*(b^6
*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 - 4*(b^6*c^6 + 56*b^4*c^4*e^2 + 784
*b^2*c^2*e^4 + 2304*e^6)*cos(2*x*e + 2*d) + 2304*e^6)*cos(4*x*e + 4*d) - 8*
(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*cos(2*x*e + 2*d) -
4*(2*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*e^4 + 2304*e^6)*sin(6*x*e + 6*
d) - 3*(b^6*c^6 + 56*b^4*c^4*e^2 + 784*b^2*c^2*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(cot(x*e + d)^3*e^(b*c*x + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\cot^{3}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)**3,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(cot(e*x + d)^3*e^((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(d+e\,x)^3\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d + e*x)^3*exp(c*(a + b*x)),x)
```

```
[Out] int(cot(d + e*x)^3*exp(c*(a + b*x)), x)
```

3.25 $\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx$

Optimal. Leaf size=76

$$\frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{b\log(F)}$$

```
[Out] I*F^(b*x+a)/b/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F)/
d],I*exp(I*(d*x+c)))/b/ln(F)
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4552, 4527, 2225, 2283}

$$\frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{b\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

```
[Out] (I*F^(a + b*x))/(b*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4552

```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}
\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)dx &= -\int F^{a+bx}\tan\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)dx\\
&= -\left(i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx\right)\\
&= i\int F^{a+bx}\,dx-2i\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx\\
&= \frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{b\log(F)}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 133, normalized size = 1.75

$$\frac{F^{a+bx}\left(be^{i(c+dx)}\,{}_2F_1\left(1,1-\frac{ib\log(F)}{d};2-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)\log(F)+{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)(d-ib\log(F))\right)}{b\log(F)(id+b\log(F))}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

```
[Out] (F^(a + b*x)*(b*E^(I*(c + d*x))*Hypergeometric2F1[1, 1 - (I*b*Log[F])/d, 2
- (I*b*Log[F])/d, I*E^(I*(c + d*x))]*Log[F] + Hypergeometric2F1[1, ((-I)*b*
Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))]*(d - I*b*Log[F])))/(b*Log
[F]*(I*d + b*Log[F]))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int F^{bx+a}\cot\left(\frac{\pi}{4}+\frac{dx}{2}+\frac{c}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*cot(1/4*Pi+1/2*d*x+1/2*c),x)
```

```
[Out] int(F^(b*x+a)*cot(1/4*Pi+1/2*d*x+1/2*c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="maxima")
[Out] integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="fricas")
[Out] integral(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x)
[Out] Integral(F**(a + b*x)*cot(c/2 + d*x/2 + pi/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="giac")
[Out] integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int F^{a+b\,x}\,\mathrm{cot}\left(\frac{\Pi}{4}+\frac{c}{2}+\frac{d\,x}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a + b*x)*cot(Pi/4 + c/2 + (d*x)/2),x)
[Out] int(F^(a + b*x)*cot(Pi/4 + c/2 + (d*x)/2), x)
```

3.26 $\int F^{c(a+bx)} \sec^n(d+ex)\,dx$

Optimal. Leaf size=100

$$\frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\,{}_2F_1\left(n,\frac{en-ibc\log(F)}{2e};\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)\sec^n(d+ex)}{ien+bc\log(F)}$$

```
[Out] (1+exp(2*I*(e*x+d)))^n*F^(b*c*x+a*c)*hypergeom([n, 1/2*(e*n-I*b*c*ln(F))/e]
,[1+1/2*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*sec(e*x+d)^n/(b*c*ln(F)+I*e
*n)
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4539, 2291}

$$\frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\sec^n(d+ex)\,{}_2F_1\left(n,\frac{en-ibc\log(F)}{2e};\frac{1}{2}\left(n-\frac{ibc\log(F)}{e}+2\right);-e^{2i(d+ex)}\right)}{bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

```
[Out] ((1 + E^((2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n - I*
b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*Sec[
d + e*x]^n)/(I*e*n + b*c*Log[F])
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4539

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[(1 + E^(2*I*(d + e*x)))^n*(Sec[d + e*x]^n/E^(I*n*(d + e*x))), I
nt[SimplifyIntegrand[F^(c*(a + b*x))*(E^(I*n*(d + e*x))/(1 + E^(2*I*(d + e*
x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx = \left(e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^n \sec^n(d+ex)\right)\int e^{idn+ienx}\left(1+e^{2i(d+ex)}\right)^{-n}F^{ac+bcx}$$

$$= \frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\,{}_2F_1\left(n,\frac{en-ibc\log(F)}{2e};\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)\text{se}}{ien+bc\log(F)}$$

Mathematica [A]
time = 0.10, size = 102, normalized size = 1.02

$$-\frac{i\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,{}_2F_1\left(n,\frac{en-ibc\log(F)}{2e};\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)\sec^n(d+ex)}{en-ibc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
[Out] ((-I)*(1 + E^((2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n
 - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]
*Sec[d + e*x]^n)/(e*n - I*b*c*Log[F])
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(\sec^n(ex+d))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*sec(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(x*e + d)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\sec^{n}(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\,{\left(\frac{1}{\cos\left(d+e\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(1/cos(d + e*x))^n,x)
[Out] int(F^(c*(a + b*x))*(1/cos(d + e*x))^n, x)
```

3.27 $\int F^{c(a+bx)} \csc^n(d+ex)\,dx$

Optimal. Leaf size=102

$$-\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,{}_2F_1\left(n,\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right);e^{-2i(d+ex)}\right)}{ien-bc\log(F)}$$

```
[Out] -(1-1/exp(2*I*(e*x+d)))^n*F^(b*c*x+a*c)*csc(e*x+d)^n*hypergeom([n, 1/2*(I*b
*c*ln(F)+e*n)/e],[1+1/2*n+1/2*I*b*c*ln(F)/e],exp(-2*I*(e*x+d)))/(I*e*n-b*c*
ln(F))
```

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4540, 2291}

$$-\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,{}_2F_1\left(n,\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(n+\frac{ibc\log(F)}{e}+2\right);e^{-2i(d+ex)}\right)}{-bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

```
[Out] -(((1 - E^((-2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(I*e*n - b*c*Log[F]))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4540

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(1 - E^(-2*I*(d + e*x)))^n*(Csc[d + e*x]^n/E^((-I)*n*(d + e*x))
), Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(I*n*(d + e*x))*(1 - E^(-2*I
*(d + e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[
n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx = \left(e^{in(d+ex)}\left(1-e^{-2i(d+ex)}\right)^n\csc^n(d+ex)\right)\int e^{-idn-ienx}\left(1-e^{-2i(d+ex)}\right)^{-n}F^{ac+bcx}$$

$$= -\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,{}_2F_1\left(n,\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right)\right.}{ien-bc\log(F)}$$

Mathematica [A]
time = 0.11, size = 102, normalized size = 1.00

$$\frac{i\left(1-e^{-2i(d+ex)}\right)^n F^{c(a+bx)}\csc^n(d+ex)\,{}_2F_1\left(n,\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right);e^{-2i(d+ex)}\right)}{en+ibc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

```
[Out] (I*(1 - E^((-2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(e*n + I*b*c*Log[F])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\csc^n(ex+d)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(x*e + d)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\csc^n\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\,{\left(\frac{1}{\sin\left(d+e\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(1/sin(d + e*x))^n,x)
```

```
[Out] int(F^(c*(a + b*x))*(1/sin(d + e*x))^n, x)
```

3.28 $\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$

Optimal. Leaf size=139

$$-\frac{e^{-id}F^{ac}(fx)^m\text{Gamma}(1+m,x(ie-bc\log(F)))(x(ie-bc\log(F)))^{-m}}{2(e+ibc\log(F))}-\frac{e^{id}F^{ac}(fx)^m\text{Gamma}(1+m,-x(ie}{2(e-i}$$

```
[Out] -1/2*F^(a*c)*(f*x)^m*GAMMA(1+m,x*(I*e-b*c*ln(F)))/exp(I*d)/((x*(I*e-b*c*ln(
F)))^m)/(e+I*b*c*ln(F))-1/2*exp(I*d)*F^(a*c)*(f*x)^m*GAMMA(1+m,-x*(b*c*ln(F
)+I*e))/(e-I*b*c*ln(F))/((-x*(b*c*ln(F)+I*e))^m)
```

Rubi [F]
time = 0.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x], x]
```

Rubi steps

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx=\int F^{ac+bcx}(fx)^m \sin(d+ex)\,dx$$

Mathematica [A]
time = 0.62, size = 143, normalized size = 1.03

$$\frac{1}{2}F^{ac}(fx)^m(x(-ie-bc\log(F)))^{-m}\left(-ix\text{Gamma}(1+m,iex-bcx\log(F))(ix(e+ibc\log(F)))^{-1-m}(-iex-bcx\log(F))^m(\cos(d)-i\sin(d))-\frac{\text{Gamma}(1+m,-iex-bcx\log(F))(\cos(d)+i\sin(d))}{e-ibc\log(F)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
[Out] (F^(a*c)*(f*x)^m*((-I)*x*Gamma[1 + m, I*e*x - b*c*x*Log[F]]*(I*x*(e + I*b*c
*Log[F]))^(-1 - m)*((-I)*e*x - b*c*x*Log[F])^m*(Cos[d] - I*Sin[d]) - (Gamma
[1 + m, (-I)*e*x - b*c*x*Log[F]]*(Cos[d] + I*Sin[d]))/(e - I*b*c*Log[F])))/
(2*(x*((-I)*e - b*c*Log[F]))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}(fx)^m \sin(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)*sin(x*e + d), x)
```

Fricas [A]
time = 1.04, size = 135, normalized size = 0.97

$$\frac{(i\,bc\log\left(F\right)-e)e^{\left(ac\log(F)-m\log\left(-\frac{bc\log(F)-i\,e}{f}\right)-i\,d\right)}\Gamma(m+1,-bcx\log\left(F\right)+i\,xe)+(-i\,bc\log\left(F\right)-e)e^{\left(ac\log(F)-m\log\left(-\frac{bc\log(F)+i\,e}{f}\right)+i\,d\right)}\Gamma(m+1,-bcx\log\left(F\right)-i\,xe)}{2\left(b^2c^2\log\left(F\right)^2+e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*((I*b*c*log(F) - e)*e^(a*c*log(F) - m*log(-(b*c*log(F) - I*e)/f) - I*d)
*gamma(m + 1, -b*c*x*log(F) + I*x*e) + (-I*b*c*log(F) - e)*e^(a*c*log(F) -
m*log(-(b*c*log(F) + I*e)/f) + I*d)*gamma(m + 1, -b*c*x*log(F) - I*x*e))/(b
^2*c^2*log(F)^2 + e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(fx)^m\sin\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m*sin(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*(f*x)**m*sin(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)\,{\left(f\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m,x)
[Out] int(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m, x)
```

3.29 $\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(F^{ac+bcx}(fx)^m \csc(d+ex),x\right)$$

```
[Out] CannotIntegrate(F^(b*c*x+a*c)*(f*x)^m*csc(e*x+d),x)
```

Rubi [A]
time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x], x]
```

Rubi steps

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int F^{ac+bcx}(fx)^m \csc(d+ex)\,dx$$

Mathematica [A]
time = 9.17, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
[Out] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x], x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}(fx)^m}{\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="maxima")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(x*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="fricas")
[Out] integral((f*x)^m*F^(b*c*x + a*c)/sin(x*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}(fx)^m}{\sin(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d),x)
[Out] Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="giac")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F^{c\,(a+b\,x)}\,{(f\,x)}^m}{\sin\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x),x)
[Out] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x), x)
```

3.30 $\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(F^{ac+bcx}(fx)^m \csc^2(d+ex),x\right)$$

```
[Out] CannotIntegrate(F^(b*c*x+a*c)*(f*x)^m*csc(e*x+d)^2,x)
```

Rubi [A]
time = 0.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

```
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x]^2, x]
```

Rubi steps

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int F^{ac+bcx}(fx)^m \csc^2(d+ex)\,dx$$

Mathematica [A]
time = 10.41, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

```
[Out] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}(fx)^m}{\sin(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(x*e + d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="fricas")
[Out] integral(-(f*x)^m*F^(b*c*x + a*c)/(cos(x*e + d)^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}(fx)^m}{\sin^2(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d)**2,x)
[Out] Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="giac")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F^{c\,(a+b\,x)}\,{(f\,x)}^m}{{\sin\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x)^2,x)
[Out] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x)^2, x)
```

3.31 $\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$

Optimal. Leaf size=24

$$F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)$$

```
[Out] F^(b*c*x+a*c)*(f*x)^(-1+m)*sin(e*x+d)
```

Rubi [A]
time = 2.85, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {12, 6873, 6874, 4556, 4555}

$$(fx)^{m-1}\sin(d+ex)F^{ac+bcx}$$

Antiderivative was successfully verified.

```
[In] Int[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*c*x*Lo
g[F])*Sin[d + e*x]),x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^(-1 + m)*Sin[d + e*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 4556

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^
(m_), x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Cos[d +
 e*x], x] + (Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d +
e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a + b
*x))*Cos[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m,
 -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = f\int F^{c(a+bx)}(fx)^{-2+m}(ex\,c$$
$$= f\int F^{ac+bcx}(fx)^{-2+m}(ex\,c$$
$$= f\int \Bigg(\frac{eF^{ac+bcx}(fx)^{-1+m}\,c}{f}$$
$$= e\int F^{ac+bcx}(fx)^{-1+m}\cos(d$$
$$= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm}$$
$$= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm}$$
$$= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm}$$
$$= F^{ac+bcx}(fx)^{-1+m}\sin(d+e$$

Mathematica [A]
time = 0.97, size = 26, normalized size = 1.08

$$fF^{ac+bcx}x(fx)^{-2+m}\sin(d+ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*
c*x*Log[F])*Sin[d + e*x]),x]
```

```
[Out] f*F^(a*c + b*c*x)*x*(f*x)^(-2 + m)*Sin[d + e*x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 213, normalized size = 8.88

method	result
risch	$-\frac{iF^{c(bx+a)}xf\left(\frac{f^m x^m e^{iex} e^{id} e^{-\frac{i\pi \operatorname{csgn}(ifx)^3 m}{2}} e^{\frac{i\pi \operatorname{csgn}(ifx)^2 \operatorname{csgn}(if) m}{2}} e^{\frac{i\pi \operatorname{csgn}(ifx)^2 \operatorname{csgn}(ix) m}{2}} e^{-\frac{i\pi \operatorname{csgn}(ifx) \operatorname{csgn}(if) \operatorname{csgn}(ix) m}{2}}}{f^2 x^2} - \frac{f^m x^m}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))*sin(e*x
+d)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*F^(c*(b*x+a))*x*f*(f^m*x^m/f^2/x^2*exp(I*e*x)*exp(I*d)*exp(-1/2*I*Pi
*csgn(I*f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn
(I*f*x)^2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m)-f^m
*x^m/f^2/x^2*exp(-I*e*x)*exp(-I*d)*exp(-1/2*I*Pi*csgn(I*f*x)^3*m)*exp(1/2*I
*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*x)*m)*exp(
-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m))
```

Maxima [A]
time = 0.52, size = 32, normalized size = 1.33

$$\frac{F^{ac} f^{m-1} e^{(bcx\log(F)+m\log(x))}\sin(ex+d)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^(m - 1)*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)/x
```

Fricas [A]
time = 2.30, size = 27, normalized size = 1.12

$$(fx)^{m-2} F^{bcx+ac} fx \sin(xe+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] (f*x)^(m - 2)*F^(b*c*x + a*c)*f*x*sin(x*e + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F**(c*(b*x+a))*(f*x)**(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))
*sin(e*x+d)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6436 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6346 vs. $2(24) = 48$.
time = 0.67, size = 6346, normalized size = 264.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="giac")
```

```
[Out] (x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*
abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x -
2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4
*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c -
 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*lo
g(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1
/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*
sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x -
 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^
2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
- 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi
*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs
(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn
(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*p
i*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*
pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) -
```

```
 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) +
m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*
pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4
*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1
) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
 + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sg
n(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*
pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) -
x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*a
bs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2
*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*
tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x
) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi
*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) +
 m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn
(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*p
i*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 +
 x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*
abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x -
2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4
*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F))
+ m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*...
```

Mupad [B]
time = 2.90, size = 27, normalized size = 1.12

$$\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)\,{\left(f\,x\right)}^m}{f\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*f*(f*x)^(m - 2)*(sin(d + e*x)*(m + b*c*x*log(F) - 1) +
e*x*cos(d + e*x)),x)
```

```
[Out] (F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m)/(f*x)
```

3.32 $\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$

Optimal. Leaf size=23

$$fF^{c(a+bx)}x(fx)^m\sin(d+ex)$$

```
[Out] f*F^(c*(b*x+a))*x*(f*x)^m*sin(e*x+d)
```

Rubi [F]
time = 1.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$

Verification is not applicable to the result.

```
[In] Int[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[F])*Si
n[d + e*x]),x]
```

```
[Out] e*Defer[Int][F^(a*c + b*c*x)*(f*x)^(1 + m)*Cos[d + e*x], x] + f*(1 + m)*Def
er[Int][F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x], x] + b*c*Log[F]*Defer[Int][F^
(a*c + b*c*x)*(f*x)^(1 + m)*Sin[d + e*x], x]
```

Rubi steps

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = f\int F^{c(a+bx)}(fx)^m(ex\cos(d+$$
$$= f\int F^{ac+bcx}(fx)^m(ex\cos(d+e$$
$$= f\int\left(\frac{eF^{ac+bcx}(fx)^{1+m}\cos(d+}{f}\right.$$
$$= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex$$
$$= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex$$
$$= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex$$

Mathematica [A]
time = 0.58, size = 23, normalized size = 1.00

$$fF^{c(a+bx)}x(fx)^m\sin(d+ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[
F])*Sin[d + e*x]),x]
```

```
[Out] f*F^(c*(a + b*x))*x*(f*x)^m*Sin[d + e*x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.27, size = 201, normalized size = 8.74

method	result
risch	$-\frac{i F^{c(bx+a)} x f \left(f^m x^m e^{iex} e^{id} e^{-\frac{i\pi \mathrm{csgn}(ifx)^3 m}{2}} e^{\frac{i\pi \mathrm{csgn}(ifx)^2 \mathrm{csgn}(if) m}{2}} e^{\frac{i\pi \mathrm{csgn}(ifx)^2 \mathrm{csgn}(ix) m}{2}} e^{-\frac{i\pi\, \mathrm{csgn}(ifx) \mathrm{csgn}(if) \mathrm{csgn}(ix) m}{2}} - f^m x^m e \right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e*x+d)),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*F^(c*(b*x+a))*x*f*(f^m*x^m*exp(I*e*x)*exp(I*d)*exp(-1/2*I*Pi*csgn(I*
f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^
2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m)-f^m*x^m*exp
(-I*e*x)*exp(-I*d)*exp(-1/2*I*Pi*csgn(I*f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^
2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I
*f*x)*csgn(I*f)*csgn(I*x)*m))
```

Maxima [A]
time = 0.51, size = 31, normalized size = 1.35

$$F^{ac} f^{m+1} x e^{(bcx \log(F) + m \log(x))} \sin(xe + d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^(m + 1)*x*e^(b*c*x*log(F) + m*log(x))*sin(x*e + d)
```

Fricas [A]
time = 2.49, size = 25, normalized size = 1.09

$$(fx)^m F^{bcx+ac} fx \sin(xe + d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="fricas")
```

```
[Out] (f*x)^m*F^(b*c*x + a*c)*f*x*sin(x*e + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$f\left(\int F^{ac}F^{bcx}(fx)^m \sin(d+ex)\,dx + \int F^{ac}F^{bcx}m(fx)^m \sin(d+ex)\,dx + \int F^{ac}F^{bcx}ex(fx)^m \cos(d+ex)\,dx + \int F^{ac}F^{bcx}bcx(fx)^m \log(F)\sin(d+ex)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e
*x+d)),x)
```

```
[Out] f*(Integral(F**(a*c)*F**(b*c*x)*(f*x)**m*sin(d + e*x), x) + Integral(F**(a*
c)*F**(b*c*x)*m*(f*x)**m*sin(d + e*x), x) + Integral(F**(a*c)*F**(b*c*x)*e*
x*(f*x)**m*cos(d + e*x), x) + Integral(F**(a*c)*F**(b*c*x)*b*c*x*(f*x)**m*l
og(F)*sin(d + e*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4746 vs. 2(23) = 46.
time = 0.59, size = 4746, normalized size = 206.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="giac")
```

```
[Out] (x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c
*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*p
i*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F
) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(
f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi
*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c
)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c
*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*
sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log
(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + p
i*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x)
 - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*p
i*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c
*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*l
og(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m +
1/2*e*x)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
```

```
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2
*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*
pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(
x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*
sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan
(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*
log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/
4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m +
 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f)
 - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x
)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*l
og(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x +
 pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(
x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*p
i*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + x*abs(F)^(
a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) -
 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f)
+ 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*
b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi
*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*
d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi
*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1
/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*a*c*sgn
(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x
*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F
)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F
) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(
f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*
pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4
*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2
*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*b*c*x*
sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m
*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1
/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*a
bs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*
```

```
a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(ab...
```

Mupad [B]
time = 2.86, size = 23, normalized size = 1.00

$$F^{c\,(a+b\,x)}\,f\,x\,\sin\left(d+e\,x\right)\,{\left(f\,x\right)}^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*f*(f*x)^m*(sin(d + e*x)*(m + b*c*x*log(F) + 1) + e*x*co
s(d + e*x)),x)
```

```
[Out] F^(c*(a + b*x))*f*x*sin(d + e*x)*(f*x)^m
```

3.33 $\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$

Optimal. Leaf size=22

$$F^{ac+bcx}(fx)^m\sin(d+ex)$$

```
[Out] F^(b*c*x+a*c)*(f*x)^m*sin(e*x+d)
```

Rubi [A]

time = 1.86, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {16, 6873, 6874, 4555}

$$(fx)^m\sin(d+ex)F^{ac+bcx}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*Sin[d +
 e*x]))/x,x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = f\int F^{c(a+bx)}(fx)^{-1+m}(ex\cos(d+e$$
$$= f\int F^{ac+bcx}(fx)^{-1+m}(ex\cos(d+ex$$
$$= f\int\left(\frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{f}+\right.$$
$$= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+$$
$$= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+$$
$$= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+$$
$$= F^{ac+bcx}(fx)^m\sin(d+ex)$$

Mathematica [**A**]
time = 0.62, size = 22, normalized size = 1.00

$$F^{ac+bcx}(fx)^m\sin(d+ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*S
in[d + e*x]))/x,x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.26, size = 199, normalized size = 9.05

method	result
risch	$-\frac{iF^{c(bx+a)}\left(f^m x^m e^{iex}e^{id}e^{-\frac{i\pi\,\mathrm{csgn}(ifx)^3 m}{2}}e^{\frac{i\pi\,\mathrm{csgn}(ifx)^2\mathrm{csgn}(if)m}{2}}e^{\frac{i\pi\,\mathrm{csgn}(ifx)^2\mathrm{csgn}(ix)m}{2}}e^{-\frac{i\pi\,\mathrm{csgn}(ifx)\mathrm{csgn}(if)\mathrm{csgn}(ix)m}{2}}-f^m x^m e^{-}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d))/x,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/2*I*F^(c*(b*x+a))*(f^m*x^m*exp(I*e*x)*exp(I*d)*exp(-1/2*I*Pi*csgn(I*f*x)
^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*cs
gn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m)-f^m*x^m*exp(-I*
e*x)*exp(-I*d)*exp(-1/2*I*Pi*csgn(I*f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*cs
```

```
gn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x
)*csgn(I*f)*csgn(I*x)*m))
```

Maxima [**A**]
time = 0.53, size = 28, normalized size = 1.27

$$F^{ac}f^m e^{(bcx\log(F)+m\log(x))}\sin(xe+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^m*e^(b*c*x*log(F) + m*log(x))*sin(x*e + d)
```

Fricas [**A**]
time = 3.32, size = 23, normalized size = 1.05

$$(fx)^m F^{bcx+ac}\sin(xe+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="fricas")
```

```
[Out] (f*x)^m*F^(b*c*x + a*c)*sin(x*e + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{c(a+bx)}(fx)^m(bcx\log(F)\sin(d+ex)+ex\cos(d+ex)+m\sin(d+ex))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d
))/x,x)
```

```
[Out] Integral(F**(c*(a + b*x))*(f*x)**m*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d +
 e*x) + m*sin(d + e*x))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) + m)*sin(e*x + d))*(f*x)^m*F^((
b*x + a)*c)/x, x)
```

Mupad [B]
time = 2.80, size = 21, normalized size = 0.95

$$F^{c(a+bx)} \sin(d+ex) (fx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*(f*x)^m*(sin(d + e*x)*(m + b*c*x*log(F)) + e*x*cos(d +
 e*x)))/x,x)
```

```
[Out] F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m
```

3.34 $\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx$

Optimal. Leaf size=17

$$F^{c(a+bx)}x\sin(d+ex)$$

```
[Out] F^(c*(b*x+a))*x*sin(e*x+d)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(17) = 34$.
time = 0.54, antiderivative size = 327, normalized size of antiderivative = 19.24, number of steps used = 14, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {6873, 6874, 4518, 4554, 4517, 4553}

$$\frac{b^2c^2x\log^2(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{e^2x\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{bce^2\log(F)\sin(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}-\frac{e\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{b^2c^2e\log^2(F)\cos(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}+\frac{e^3\cos(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}-\frac{b^3c^3\log^3(F)\sin(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*x]),x]
```

```
[Out] (e^3*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (b^2*c^2*e*
F^(a*c + b*c*x)*Cos[d + e*x]*Log[F]^2)/(e^2 + b^2*c^2*Log[F]^2)^2 - (e*F^(a
*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*e^2*F^(a*c + b*c*
x)*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 - (b^3*c^3*F^(a*c + b*c*
x)*Log[F]^3*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (e^2*F^(a*c + b*c*x)
*x*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[F]*Sin
[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b^2*c^2*F^(a*c + b*c*x)*x*Log[F]^2*S
in[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
```

```
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = \int F^{ac+bcx}(ex\cos(d+ex)+(1+bcx\log(F)$$
$$= \int \left(eF^{ac+bcx}x\cos(d+ex)+F^{ac+bcx}(1+bc\right.$$
$$= e\int F^{ac+bcx}x\cos(d+ex)\,dx+\int F^{ac+bcx}(1$$
$$= \frac{bceF^{ac+bcx}x\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{e^2F^{ac+bc}}{e^2+b}$$
$$= \frac{bceF^{ac+bcx}x\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{e^2F^{ac+bc}}{e^2+b}$$
$$= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{b^2c^2eF^{ac+bcx}\cos(d}{(e^2+b^2c^2\mathrm{lo}}$$
$$= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{b^2c^2eF^{ac+bcx}\cos(d}{(e^2+b^2c^2\mathrm{lo}}$$
$$= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}+\frac{b^2c^2eF^{ac+bcx}\cos(d}{(e^2+b^2c^2\mathrm{lo}}$$

Mathematica [A]
time = 0.25, size = 17, normalized size = 1.00

$$F^{c(a+bx)}x\sin(d+ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*
x]),x]
```

```
[Out] F^(c*(a + b*x))*x*Sin[d + e*x]
```

Maple [A]
time = 0.15, size = 18, normalized size = 1.06

method	result	size
risch	$F^{c(bx+a)}x\sin(ex+d)$	18
norman	$\frac{2x\,e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x,method=_RET
URNVERBOSE)
```

```
[Out] F^(c*(b*x+a))*x*sin(e*x+d)
```

Maximа [B] Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(18) = 36$.
time = 0.38, size = 1358, normalized size = 79.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="maxima")
```

```
[Out] 1/2*((2*F^(a*c)*b*c*cos(d)*e*log(F) - ((F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c
)*e^3)*cos(d) + (F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*sin(d))
*x + (F^(a*c)*b^2*c^2*log(F)^2 - F^(a*c)*e^2)*sin(d))*F^(b*c*x)*cos(x*e + 2
*d) + (2*F^(a*c)*b*c*cos(d)*e*log(F) - ((F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*
c)*e^3)*cos(d) - (F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*sin(d)
)*x - (F^(a*c)*b^2*c^2*log(F)^2 - F^(a*c)*e^2)*sin(d))*F^(b*c*x)*cos(x*e) +
 (2*F^(a*c)*b*c*e*log(F)*sin(d) + ((F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*
e^2*log(F))*cos(d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(d))*x -
 (F^(a*c)*b^2*c^2*log(F)^2 - F^(a*c)*e^2)*cos(d))*F^(b*c*x)*sin(x*e + 2*d)
- (2*F^(a*c)*b*c*e*log(F)*sin(d) - ((F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c
*e^2*log(F))*cos(d) + (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(d))*x
+ (F^(a*c)*b^2*c^2*log(F)^2 - F^(a*c)*e^2)*cos(d))*F^(b*c*x)*sin(x*e))*b*c*
log(F)/((b^4*c^4*log(F)^4 + 2*b^2*c^2*e^2*log(F)^2 + e^4)*cos(d)^2 + (b^4*c
^4*log(F)^4 + 2*b^2*c^2*e^2*log(F)^2 + e^4)*sin(d)^2) + 1/2*((2*F^(a*c)*b*c
```

```
*e*log(F)*sin(d) + ((F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*cos
(d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(d))*x - (F^(a*c)*b^2*c
^2*log(F)^2 - F^(a*c)*e^2)*cos(d))*F^(b*c*x)*cos(x*e + 2*d) - (2*F^(a*c)*b*
c*e*log(F)*sin(d) - ((F^(a*c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*co
s(d) + (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(d))*x + (F^(a*c)*b^2*
c^2*log(F)^2 - F^(a*c)*e^2)*cos(d))*F^(b*c*x)*cos(x*e) - (2*F^(a*c)*b*c*cos
(d)*e*log(F) - ((F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(d) + (F^(a*c
)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*sin(d))*x + (F^(a*c)*b^2*c^2*l
og(F)^2 - F^(a*c)*e^2)*sin(d))*F^(b*c*x)*sin(x*e + 2*d) - (2*F^(a*c)*b*c*co
s(d)*e*log(F) - ((F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(d) - (F^(a*
c)*b^3*c^3*log(F)^3 + F^(a*c)*b*c*e^2*log(F))*sin(d))*x - (F^(a*c)*b^2*c^2*
log(F)^2 - F^(a*c)*e^2)*sin(d))*F^(b*c*x)*sin(x*e))*e/((b^4*c^4*log(F)^4 +
2*b^2*c^2*e^2*log(F)^2 + e^4)*cos(d)^2 + (b^4*c^4*log(F)^4 + 2*b^2*c^2*e^2*
log(F)^2 + e^4)*sin(d)^2) - 1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d
)*e)*F^(b*c*x)*cos(x*e + 2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)
*e)*F^(b*c*x)*cos(x*e) - (F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(
b*c*x)*sin(x*e + 2*d) - (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b
*c*x)*sin(x*e))/((b^2*c^2*log(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^
2)*sin(d)^2)
```

Fricas [A]
time = 2.40, size = 19, normalized size = 1.12

$$F^{bcx+ac}x\sin\left(xe+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*x*sin(x*e + d)
```

Sympy [A]
time = 3.57, size = 19, normalized size = 1.12

$$F^{ac}F^{bcx}x\sin\left(d+ex\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x)
```

```
[Out] F**(a*c)*F**(b*c*x)*x*sin(d + e*x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.45, size = 1941, normalized size = 114.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="giac")
```

```
[Out] -1/4*((pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) - 2*I*b^2*c^2*x*log
(F)*log(abs(F)) - I*pi*b*c*e*x*sgn(F) + I*pi*b*c*e*x + 2*b*c*e*x*log(F) - 2
*b*c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x + 2*I*b*c*log(F)
- 2*I*b*c*log(abs(F)) + 4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/
2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*
pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) +
 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e + 4*I*b*c*e*log(a
bs(F)) - 2*e^2) - (pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) + 2*I*b
^2*c^2*x*log(F)*log(abs(F)) - I*pi*b*c*e*x*sgn(F) + I*pi*b*c*e*x + 2*b*c*e*
x*log(F) + 2*b*c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x - 2*I
*b*c*log(F) + 2*I*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c
*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F)
 - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(ab
s(F)) + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 4*I*b*c*
e*log(abs(F)) - 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*I*((-
I*pi*b^2*c^2*x*log(F)*sgn(F) + I*pi*b^2*c^2*x*log(F) - 2*b^2*c^2*x*log(F)*l
og(abs(F)) - pi*b*c*e*x*sgn(F) + pi*b*c*e*x - 2*I*b*c*e*x*log(F) + 2*I*b*c*
e*x*log(abs(F)) - I*pi*b*c*sgn(F) + I*pi*b*c - 2*e^2*x + 2*b*c*log(F) - 2*b
*c*log(abs(F)) - 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^
2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^
2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e + 4*I*b*c*e*log(abs(F)
) - 2*e^2) - (I*pi*b^2*c^2*x*log(F)*sgn(F) - I*pi*b^2*c^2*x*log(F) - 2*b^2*
c^2*x*log(F)*log(abs(F)) + pi*b*c*e*x*sgn(F) - pi*b*c*e*x + 2*I*b*c*e*x*log
(F) + 2*I*b*c*e*x*log(abs(F)) + I*pi*b*c*sgn(F) - I*pi*b*c + 2*e^2*x + 2*b*
c*log(F) - 2*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F) - 2*
I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F))
 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 4*I*b*c*e*log
(abs(F)) - 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*((pi*b^2*c
^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) - 2*I*b^2*c^2*x*log(F)*log(abs(F))
 + I*pi*b*c*e*x*sgn(F) - I*pi*b*c*e*x - 2*b*c*e*x*log(F) + 2*b*c*e*x*log(ab
s(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x + 2*I*b*c*log(F) - 2*I*b*c*log(a
bs(F)) - 4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(
F) - 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(
abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(
abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 4*I*b*c*e*log(abs(F)) - 2*e^2)
 - (pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) + 2*I*b^2*c^2*x*log(F)
*log(abs(F)) + I*pi*b*c*e*x*sgn(F) - I*pi*b*c*e*x - 2*b*c*e*x*log(F) - 2*b*
c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x - 2*I*b*c*log(F) + 2
*I*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c
```

```
^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c
^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e + 4*I*b*c*e*log(abs(F)) -
 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*I*((I*pi*b^2*c^2*x*l
og(F)*sgn(F) - I*pi*b^2*c^2*x*log(F) + 2*b^2*c^2*x*log(F)*log(abs(F)) - pi*
b*c*e*x*sgn(F) + pi*b*c*e*x - 2*I*b*c*e*x*log(F) + 2*I*b*c*e*x*log(abs(F))
+ I*pi*b*c*sgn(F) - I*pi*b*c + 2*e^2*x - 2*b*c*log(F) + 2*b*c*log(abs(F)) -
 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1
/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F)
)*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F)
)^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 4*I*b*c*e*log(abs(F)) - 2*e^2) - (-I
*pi*b^2*c^2*x*log(F)*sgn(F) + I*pi*b^2*c^2*x*log(F) + 2*b^2*c^2*x*log(F)*lo
g(abs(F)) + pi*b*c*e*x*sgn(F) - pi*b*c*e*x + 2*I*b*c*e*x*log(F) + 2*I*b*c*e
*x*log(abs(F)) - I*pi*b*c*sgn(F) + I*pi*b*c - 2*e^2*x - 2*b*c*log(F) + 2*b*
c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*lo
g(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*lo
g(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e + 4*I*b*c*e*log(abs(F)) - 2*e^
2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 2.87, size = 17, normalized size = 1.00

$$F^{c\,(a+b\,x)}\,x\,\sin\left(d+e\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) + 1) + e*x*cos(d + e*x)),x)
[Out] F^(c*(a + b*x))*x*sin(d + e*x)
```

3.35 $\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx$

Optimal. Leaf size=16

$$F^{c(a+bx)}\sin(d+ex)$$

```
[Out] F^(c*(b*x+a))*sin(e*x+d)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2326}

$$\sin(d+ex)F^{c(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
```

```
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{c(a+bx)}\sin(d+ex)$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$F^{c(a+bx)}\sin(d+ex)$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
```

```
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Maple [A]
time = 0.11, size = 17, normalized size = 1.06

method	result	size
risch	$F^{c(bx+a)}\sin(ex+d)$	17
norman	$\frac{2\,e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{1+\tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x,method=_RETURNVERBO
SE)
```

```
[Out] F^(c*(b*x+a))*sin(e*x+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(17) = 34$.
time = 0.30, size = 399, normalized size = 24.94

$$-\frac{\left(\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)+F^{ac}\cos\left(d\right)e\right)F^{bcx}\cos\left(xe+2\,d\right)-\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)-F^{ac}\cos\left(d\right)e\right)F^{bcx}\cos\left(xe\right)-\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)-F^{ac}e\sin\left(d\right)\right)F^{bcx}\sin\left(xe+2\,d\right)-\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)+F^{ac}e\sin\left(d\right)\right)F^{bcx}\sin\left(xe\right)\right)bc\log\left(F\right)}{2\left(\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)\cos\left(d\right)^{2}+\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)\sin\left(d\right)^{2}\right)}+\frac{\left(\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)-F^{ac}e\sin\left(d\right)\right)F^{bcx}\cos\left(xe+2\,d\right)+\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)+F^{ac}e\sin\left(d\right)\right)F^{bcx}\cos\left(xe\right)+\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)+F^{ac}\cos\left(d\right)e\right)F^{bcx}\sin\left(xe+2\,d\right)-\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)-F^{ac}\cos\left(d\right)e\right)F^{bcx}\sin\left(xe\right)\right)e}{2\left(\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)\cos\left(d\right)^{2}+\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)\sin\left(d\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*cos(x*e + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*cos(x*e) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(x*e + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(x*e))*b*c*log(F)/
((b^2*c^2*log(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(d)^2) + 1
/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(x*e + 2*d)
 + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(x*e) + (F^(
a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(x*e + 2*d) - (F^(a
*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(x*e))*e/((b^2*c^2*l
og(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(d)^2)
```

Fricas [A]
time = 1.87, size = 18, normalized size = 1.12

$$F^{bcx+ac}\sin\left(xe+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
fricas")
```

```
[Out] F^(b*c*x + a*c)*sin(x*e + d)
```

Sympy [A]
time = 0.60, size = 17, normalized size = 1.06

$$F^{ac}F^{bcx}\sin\left(d+ex\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x)
```

```
[Out] F**(a*c)*F**(b*c*x)*sin(d + e*x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 639, normalized size = 39.94

$$-i\left(\frac{(bc\log\left(F\right)-ie)e^{\left(\frac{1}{2}i\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\pi bcx+\frac{1}{2}i\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\pi ac+iex+id\right)}}{2i\pi bc\mathrm{sgn}\left(F\right)-2i\pi bc+4bc\log\left(|F|\right)+4ie}-\frac{(bc\log\left(F\right)-ie)e^{\left(-\frac{1}{2}i\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\pi bcx-\frac{1}{2}i\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\pi ac-iex-id\right)}}{-2i\pi bc\mathrm{sgn}\left(F\right)+2i\pi bc+4bc\log\left(|F|\right)-4ie}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}-\left(\frac{(-ibc\log\left(F\right)-e)e^{\left(\frac{1}{2}i\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\pi bcx+\frac{1}{2}i\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\pi ac+iex+id\right)}}{2i\pi bc\mathrm{sgn}\left(F\right)-2i\pi bc+4bc\log\left(|F|\right)+4ie}+\frac{(-ibc\log\left(F\right)-e)e^{\left(-\frac{1}{2}i\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\pi bcx-\frac{1}{2}i\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\pi ac-iex-id\right)}}{-2i\pi bc\mathrm{sgn}\left(F\right)+2i\pi bc+4bc\log\left(|F|\right)-4ie}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}+i\left(\frac{(bc\log\left(F\right)+ie)e^{\left(\frac{1}{2}i\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\pi bcx+\frac{1}{2}i\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\pi ac-iex-id\right)}}{2i\pi bc\mathrm{sgn}\left(F\right)-2i\pi bc+4bc\log\left(|F|\right)-4ie}-\frac{(bc\log\left(F\right)+ie)e^{\left(-\frac{1}{2}i\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\pi bcx-\frac{1}{2}i\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\pi ac+iex+id\right)}}{-2i\pi bc\mathrm{sgn}\left(F\right)+2i\pi bc+4bc\log\left(|F|\right)+4ie}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}-\left(\frac{(ibc\log\left(F\right)-e)e^{\left(\frac{1}{2}i\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\pi bcx+\frac{1}{2}i\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\pi ac-iex-id\right)}}{2i\pi bc\mathrm{sgn}\left(F\right)-2i\pi bc+4bc\log\left(|F|\right)-4ie}+\frac{(ibc\log\left(F\right)-e)e^{\left(-\frac{1}{2}i\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\pi bcx-\frac{1}{2}i\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\pi ac+iex+id\right)}}{-2i\pi bc\mathrm{sgn}\left(F\right)+2i\pi bc+4bc\log\left(|F|\right)+4ie}\right)e^{\left(bcx\log\left(|F|\right)+ac\log\left(|F|\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
giac")
```

```
[Out] -I*((b*c*log(F) - I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi
*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c +
 4*b*c*log(abs(F)) + 4*I*e) - (b*c*log(F) - I*e)*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*
pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F
)) + a*c*log(abs(F))) - ((-I*b*c*log(F) - e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2
*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) + (-I*b*c*log(F) - e)*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I
*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*((b*c*log(F) + I*e)*e^(1/2
*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c -
I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e) -
 (b*c*log(F) + I*e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c +
4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - ((I*b
*c*log(F) - e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*l
og(abs(F)) - 4*I*e) + (I*b*c*log(F) - e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*
pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*s
gn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F)))
```

Mupad [B]
time = 2.35, size = 16, normalized size = 1.00

$$F^{c\,\left(a+b\,x\right)}\,\sin\left(d+e\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(e*cos(d + e*x) + b*c*sin(d + e*x)*log(F)),x)
[Out] F^(c*(a + b*x))*sin(d + e*x)
```

3.36 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$

Optimal. Leaf size=20

$$\frac{F^{ac+bcx}\sin(d+ex)}{x}$$

[Out] F^(b*c*x+a*c)*sin(e*x+d)/x

Rubi [A]
time = 1.26, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {6873, 6874, 4555}

$$\frac{\sin(d+ex)F^{ac+bcx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d + e*x]))
/x^2,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \int \frac{F^{ac+bcx}(ex\cos(d+ex)+(-1+bcx\,\mathrm{lo}}{x^2}$$

$$= \int \left(\frac{eF^{ac+bcx}\cos(d+ex)}{x} + \frac{F^{ac+bcx}(-1}{}\right.$$

$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx + \int \frac{F^{ac+bcx}(}{}$$

$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx + \int \left(-\frac{F^{ac+}}{}\right.$$

$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx + (bc\log(F))$$

$$= \frac{F^{ac+bcx}\sin(d+ex)}{x}$$

Mathematica [A]
time = 0.44, size = 19, normalized size = 0.95

$$\frac{F^{c(a+bx)}\sin(d+ex)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d +
e*x]))/x^2,x]
```

```
[Out] (F^(c*(a + b*x))*Sin[d + e*x])/x
```

Maple [A]
time = 0.17, size = 20, normalized size = 1.00

method	result	size
risch	$\frac{F^{c(bx+a)}\sin(ex+d)}{x}$	20
norman	$\frac{2\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)x}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x^2,x,method
=_RETURNVERBOSE)
```

```
[Out] 1/x*F^(c*(b*x+a))*sin(e*x+d)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.74, size = 613, normalized size = 30.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="maxima")
```

```
[Out] -1/4*(F^(a*c)*(I*Ei((b*c*log(F) + I*e)*x) - I*Ei((b*c*log(F) - I*e)*x) - I*
conjugate(Ei((b*c*log(F) + I*e)*x)) + I*conjugate(Ei((b*c*log(F) - I*e)*x))
)*cos(d) - F^(a*c)*(Ei((b*c*log(F) + I*e)*x) + Ei((b*c*log(F) - I*e)*x) + c
onjugate(Ei((b*c*log(F) + I*e)*x)) + conjugate(Ei((b*c*log(F) - I*e)*x)))*s
in(d))*b*c*log(F) - 1/4*(F^(a*c)*b*c*(I*conjugate(gamma(-1, -(b*c*log(F) +
I*e)*x)) - I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) - I*gamma(-1, -(b*
c*log(F) + I*e)*x) + I*gamma(-1, -(b*c*log(F) - I*e)*x))*log(F) + (conjugat
e(gamma(-1, -(b*c*log(F) + I*e)*x))*e + conjugate(gamma(-1, -(b*c*log(F) -
I*e)*x))*e + e*gamma(-1, -(b*c*log(F) + I*e)*x) + e*gamma(-1, -(b*c*log(F)
- I*e)*x))*F^(a*c))*cos(d) + 1/4*(F^(a*c)*(Ei((b*c*log(F) + I*e)*x) + Ei((b
*c*log(F) - I*e)*x) + conjugate(Ei((b*c*log(F) + I*e)*x)) + conjugate(Ei((b
*c*log(F) - I*e)*x)))*cos(d) - F^(a*c)*(-I*Ei((b*c*log(F) + I*e)*x) + I*Ei(
(b*c*log(F) - I*e)*x) + I*conjugate(Ei((b*c*log(F) + I*e)*x)) - I*conjugate
(Ei((b*c*log(F) - I*e)*x)))*sin(d))*e - 1/4*(F^(a*c)*b*c*(conjugate(gamma(-
1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) +
gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e)*x))*log(F)
 + (-I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x))*e + I*conjugate(gamma(-1
, -(b*c*log(F) - I*e)*x))*e + I*e*gamma(-1, -(b*c*log(F) + I*e)*x) - I*e*ga
mma(-1, -(b*c*log(F) - I*e)*x))*F^(a*c))*sin(d)
```

Fricas [A]
time = 2.48, size = 21, normalized size = 1.05

$$\frac{F^{bcx+ac}\sin\left(xe+d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*sin(x*e + d)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{F^{c(a+bx)}\left(bcx\log\left(F\right)\sin\left(d+ex\right)+ex\cos\left(d+ex\right)-\sin\left(d+ex\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x**2,
x)
```

```
[Out] Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) - s
in(d + e*x))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 1)*sin(e*x + d))*F^((b*x + a)
*c)/x^2, x)
```

Mupad [B]
time = 2.74, size = 19, normalized size = 0.95

$$\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) - 1) + e*x*cos(d + e*x)))/
x^2,x)
```

```
[Out] (F^(c*(a + b*x))*sin(d + e*x))/x
```

3.37 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx$

Optimal. Leaf size=20

$$\frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

[Out] F^(b*c*x+a*c)*sin(e*x+d)/x^2

Rubi [A]
time = 1.39, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6873, 6874, 4556, 4555}

$$\frac{\sin(d+ex)F^{ac+bcx}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d + e*x]))
/x^3,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 4556

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^
(m_), x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Cos[d +
 e*x], x] + (Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d +
e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a + b
*x))*Cos[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m,
 -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \int \frac{F^{ac+bcx}(ex\cos(d+ex)+(-2+bcx\log \cdots}{x^3}$$

$$= \int \left(\frac{eF^{ac+bcx}\cos(d+ex)}{x^2}+\frac{F^{ac+bcx}(-2 \cdots}{\cdots}\right.$$

$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x^2}\,dx+\int \frac{F^{ac+bcx}(\cdots}{\cdots}$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}-e^2\int \frac{F^{ac+bcx}\sin \cdots}{x \cdots}$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}-2\int \frac{F^{ac+bcx}\sin \cdots}{x^3 \cdots}$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}+\frac{F^{ac+bcx}\sin(d+ \cdots}{x^2}$$

$$= \frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

Mathematica [A]
time = 0.43, size = 19, normalized size = 0.95

$$\frac{F^{c(a+bx)}\sin(d+ex)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d +
e*x]))/x^3,x]
```

```
[Out] (F^(c*(a + b*x))*Sin[d + e*x])/x^2
```

Maple [A]
time = 0.17, size = 20, normalized size = 1.00

method	result	size
risch	$\frac{\sin(ex+d)F^{c(bx+a)}}{x^2}$	20
norman	$\frac{2\,\mathrm{e}^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)x^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x^3,x,method
=_RETURNVERBOSE)
```

```
[Out] sin(e*x+d)*F^(c*(b*x+a))/x^2
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.97, size = 1189, normalized size = 59.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b*c*(I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) - I*conjug
ate(gamma(-1, -(b*c*log(F) - I*e)*x)) - I*gamma(-1, -(b*c*log(F) + I*e)*x)
+ I*gamma(-1, -(b*c*log(F) - I*e)*x))*log(F) + (conjugate(gamma(-1, -(b*c*l
og(F) + I*e)*x))*e + conjugate(gamma(-1, -(b*c*log(F) - I*e)*x))*e + e*gamm
a(-1, -(b*c*log(F) + I*e)*x) + e*gamma(-1, -(b*c*log(F) - I*e)*x))*F^(a*c))
*cos(d) + (F^(a*c)*b*c*(conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + conju
gate(gamma(-1, -(b*c*log(F) - I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) +
 gamma(-1, -(b*c*log(F) - I*e)*x))*log(F) + (-I*conjugate(gamma(-1, -(b*c*l
og(F) + I*e)*x))*e + I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x))*e + I*e*
gamma(-1, -(b*c*log(F) + I*e)*x) - I*e*gamma(-1, -(b*c*log(F) - I*e)*x))*F^
(a*c))*sin(d))*b*c*log(F) + 1/2*(F^(a*c)*b^2*c^2*(I*conjugate(gamma(-2, -(b
*c*log(F) + I*e)*x)) - I*conjugate(gamma(-2, -(b*c*log(F) - I*e)*x)) - I*ga
mma(-2, -(b*c*log(F) + I*e)*x) + I*gamma(-2, -(b*c*log(F) - I*e)*x))*log(F)
^2 + 2*(conjugate(gamma(-2, -(b*c*log(F) + I*e)*x))*e + conjugate(gamma(-2,
 -(b*c*log(F) - I*e)*x))*e + e*gamma(-2, -(b*c*log(F) + I*e)*x) + e*gamma(-
2, -(b*c*log(F) - I*e)*x))*F^(a*c)*b*c*log(F) + (-I*conjugate(gamma(-2, -(b
*c*log(F) + I*e)*x))*e^2 + I*conjugate(gamma(-2, -(b*c*log(F) - I*e)*x))*e^
2 + I*e^2*gamma(-2, -(b*c*log(F) + I*e)*x) - I*e^2*gamma(-2, -(b*c*log(F) -
 I*e)*x))*F^(a*c))*cos(d) + 1/4*((F^(a*c)*b*c*(conjugate(gamma(-1, -(b*c*lo
g(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + gamma(-1, -
(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e)*x))*log(F) + (-I*conj
ugate(gamma(-1, -(b*c*log(F) + I*e)*x))*e + I*conjugate(gamma(-1, -(b*c*log
(F) - I*e)*x))*e + I*e*gamma(-1, -(b*c*log(F) + I*e)*x) - I*e*gamma(-1, -(b
*c*log(F) - I*e)*x))*F^(a*c))*cos(d) + (F^(a*c)*b*c*(-I*conjugate(gamma(-1,
 -(b*c*log(F) + I*e)*x)) + I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) +
I*gamma(-1, -(b*c*log(F) + I*e)*x) - I*gamma(-1, -(b*c*log(F) - I*e)*x))*lo
g(F) - (conjugate(gamma(-1, -(b*c*log(F) + I*e)*x))*e + conjugate(gamma(-1,
 -(b*c*log(F) - I*e)*x))*e + e*gamma(-1, -(b*c*log(F) + I*e)*x) + e*gamma(-
```

```
1, -(b*c*log(F) - I*e)*x))*F^(a*c))*sin(d))*e + 1/2*(F^(a*c)*b^2*c^2*(conju
gate(gamma(-2, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-2, -(b*c*log(F) -
 I*e)*x)) + gamma(-2, -(b*c*log(F) + I*e)*x) + gamma(-2, -(b*c*log(F) - I*e
)*x))*log(F)^2 - 2*(I*conjugate(gamma(-2, -(b*c*log(F) + I*e)*x))*e - I*con
jugate(gamma(-2, -(b*c*log(F) - I*e)*x))*e - I*e*gamma(-2, -(b*c*log(F) + I
*e)*x) + I*e*gamma(-2, -(b*c*log(F) - I*e)*x))*F^(a*c)*b*c*log(F) - (conjug
ate(gamma(-2, -(b*c*log(F) + I*e)*x))*e^2 + conjugate(gamma(-2, -(b*c*log(F
) - I*e)*x))*e^2 + e^2*gamma(-2, -(b*c*log(F) + I*e)*x) + e^2*gamma(-2, -(b
*c*log(F) - I*e)*x))*F^(a*c))*sin(d)
```

Fricas [A]
time = 2.62, size = 21, normalized size = 1.05

$$\frac{F^{bcx+ac}\sin{(xe+d)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*sin(x*e + d)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{F^{c(a+bx)}(bcx\log{(F)}\sin{(d+ex)}+ex\cos{(d+ex)}-2\sin{(d+ex)})}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x**3,
x)
```

```
[Out] Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) - 2
*sin(d + e*x))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 2)*sin(e*x + d))*F^((b*x + a)
*c)/x^3, x)
```

Mupad [B]
time = 2.75, size = 19, normalized size = 0.95

$$\frac{F^{c(a+bx)} \sin(d+ex)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) - 2) + e*x*cos(d + e*x)))/
x^3,x)
```

```
[Out] (F^(c*(a + b*x))*sin(d + e*x))/x^2
```

3.38 $\int e^{a+bx} \cos(c+dx) \sin(c+dx) \, dx$

Optimal. Leaf size=63

$$-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}+\frac{be^{a+bx}\sin(2c+2dx)}{2\left(b^2+4d^2\right)}$$

```
[Out] -d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*b*exp(b*x+a)*sin(2*d*x+2*c)/(b
^2+4*d^2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4557, 12, 4517}

$$\frac{be^{a+bx}\sin(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] -((d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2)) + (b*E^(a + b*x)*Sin[2*c
+ 2*d*x])/(2*(b^2 + 4*d^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx=\int \frac{1}{2}e^{a+bx}\sin(2c+2dx)\,dx$$
$$=\frac{1}{2}\int e^{a+bx}\sin(2c+2dx)\,dx$$
$$=-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}+\frac{be^{a+bx}\sin(2c+2dx)}{2\left(b^2+4d^2\right)}$$

Mathematica [A]
time = 0.14, size = 44, normalized size = 0.70

$$\frac{e^{a+bx}(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{2\left(b^2+4d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
[Out] (E^(a + b*x)*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(2*(b^2 + 4*d^2)
)
```

Maple [A]
time = 0.15, size = 60, normalized size = 0.95

method	result	size
risch	$-\frac{i e^{bx+a} e^{2idx} e^{2ic}}{4(2id+b)}+\frac{i e^{bx+a} e^{-2idx} e^{-2ic}}{-8id+4b}$	58
default	$-\frac{d\,e^{bx+a}\cos(2dx+2c)}{b^2+4d^2}+\frac{b\,e^{bx+a}\sin(2dx+2c)}{2b^2+8d^2}$	60
norman	$\frac{-\frac{d\,e^{bx+a}}{b^2+4d^2}+\frac{2b\,e^{bx+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2+4d^2}-\frac{2b\,e^{bx+a}\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2+4d^2}+\frac{6d\,e^{bx+a}\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2+4d^2}-\frac{d\,e^{bx+a}\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2+4d^2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	160

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] -d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*b*exp(b*x+a)*sin(2*d*x+2*c)/(b
^2+4*d^2)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.70

$$-\frac{(2\,d\cos\left(2\,dx+2\,c\right)-b\sin\left(2\,dx+2\,c\right))e^{(bx+a)}}{2\left(b^2+4\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*d*cos(2*d*x + 2*c) - b*sin(2*d*x + 2*c))*e^(b*x + a)/(b^2 + 4*d^2)
```

Fricas [A]
time = 5.04, size = 56, normalized size = 0.89

$$\frac{b\cos(dx+c)\,e^{(bx+a)}\sin(dx+c)-\left(2\,d\cos(dx+c)^2-d\right)e^{(bx+a)}}{b^2+4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cos(d*x + c)*e^(b*x + a)*sin(d*x + c) - (2*d*cos(d*x + c)^2 - d)*e^(b*x
+ a))/(b^2 + 4*d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 325, normalized size = 5.16

$$\begin{cases} xe^{a}\sin(c)\cos(c) & \text{for } b=0\wedge d=0 \\ \frac{ixe^{a}e^{-2idx}\sin^2(c+dx)}{4}+\frac{xe^{a}e^{-2idx}\sin(c+dx)\cos(c+dx)}{2}-\frac{ixe^{a}e^{-2idx}\cos^2(c+dx)}{4}+\frac{ie^{a}e^{-2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b=-2id \\ -\frac{ixe^{a}e^{2idx}\sin^2(c+dx)}{4}+\frac{xe^{a}e^{2idx}\sin(c+dx)\cos(c+dx)}{2}+\frac{ixe^{a}e^{2idx}\cos^2(c+dx)}{4}-\frac{ie^{a}e^{2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b=2id \\ \frac{be^{a}e^{bx}\sin(c+dx)\cos(c+dx)}{b^2+4d^2}+\frac{de^{a}e^{bx}\sin^2(c+dx)}{b^2+4d^2}-\frac{de^{a}e^{bx}\cos^2(c+dx)}{b^2+4d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp(-2
*I*d*x)*sin(c + d*x)**2/4 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x
)/2 - I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**2/4 + I*exp(a)*exp(-2*I*d*x)*s
in(c + d*x)*cos(c + d*x)/(4*d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*s
in(c + d*x)**2/4 + x*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c + d*x)/2 + I*x*
exp(a)*exp(2*I*d*x)*cos(c + d*x)**2/4 - I*exp(a)*exp(2*I*d*x)*sin(c + d*x)*
cos(c + d*x)/(4*d), Eq(b, 2*I*d)), (b*exp(a)*exp(b*x)*sin(c + d*x)*cos(c +
d*x)/(b**2 + 4*d**2) + d*exp(a)*exp(b*x)*sin(c + d*x)**2/(b**2 + 4*d**2) -
d*exp(a)*exp(b*x)*cos(c + d*x)**2/(b**2 + 4*d**2), True))
```

Giac [A]
time = 0.40, size = 55, normalized size = 0.87

$$-\frac{1}{2}\left(\frac{2\,d\cos(2\,dx+2\,c)}{b^2+4\,d^2}-\frac{b\sin(2\,dx+2\,c)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*c)/(b^2 + 4*d^2)
)*e^(b*x + a)
```

Mupad [B]
time = 0.50, size = 46, normalized size = 0.73

$$-\frac{e^{a+b\,x}\,\left(2\,d\,\cos\left(2\,c+2\,d\,x\right)-b\,\sin\left(2\,c+2\,d\,x\right)\right)}{2\,\left(b^2+4\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x),x)
```

```
[Out] -(exp(a + b*x)*(2*d*cos(2*c + 2*d*x) - b*sin(2*c + 2*d*x)))/(2*(b^2 + 4*d^2
))
```

3.39 $\int e^{a+bx} \cos(c+dx) \sin^2(c+dx) \, dx$

Optimal. Leaf size=119

$$\frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)} - \frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)} + \frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)} - \frac{3de^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}$$

```
[Out] 1/4*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/4*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2+
9*d^2)+1/4*d*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*sin(3*d*x+3*c
)/(b^2+9*d^2)
```

Rubi [A]

time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4557, 4518}

$$\frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)} - \frac{3de^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)} + \frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)} - \frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(4*(b^2 + 9*d^2)) + (d*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) - (3*
d*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = \int \left(\frac{1}{4}e^{a+bx}\cos(c+dx) - \frac{1}{4}e^{a+bx}\cos(3c+3dx)\right)dx$$
$$= \frac{1}{4}\int e^{a+bx}\cos(c+dx)\,dx - \frac{1}{4}\int e^{a+bx}\cos(3c+3dx)\,dx$$
$$= \frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)} - \frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)} + \frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)} - \frac{3de}{}$$

Mathematica [A]
time = 0.70, size = 74, normalized size = 0.62

$$\frac{1}{4}e^{a+bx}\left(\frac{b\cos(c+dx)+d\sin(c+dx)}{b^2+d^2} - \frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*((b*Cos[c + d*x] + d*Sin[c + d*x])/(b^2 + d^2) - (b*Cos[3*(c +
 d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

Maple [A]
time = 0.26, size = 108, normalized size = 0.91

method	result
default	$\frac{b\,e^{bx+a}\cos(dx+c)}{4b^2+4d^2} - \frac{b\,e^{bx+a}\cos(3dx+3c)}{4(b^2+9d^2)} + \frac{d\,e^{bx+a}\sin(dx+c)}{4b^2+4d^2} - \frac{3d\,e^{bx+a}\sin(3dx+3c)}{4(b^2+9d^2)}$
risch	$-\frac{e^{bx+a}e^{3idx}e^{3ic}}{8(3id+b)} + \frac{e^{bx+a}e^{idx}e^{ic}}{8id+8b} + \frac{e^{bx+a}e^{-idx}e^{-ic}}{-8id+8b} - \frac{e^{bx+a}e^{-3idx}e^{-3ic}}{8(-3id+b)}$
norman	$\frac{\frac{2b\,d^2e^{bx+a}}{b^4+10b^2d^2+9d^4} - \frac{2b\,d^2e^{bx+a}\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+10b^2d^2+9d^4} + \frac{2b\left(2b^2+3d^2\right)e^{bx+a}\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+10b^2d^2+9d^4} - \frac{2b\left(2b^2+3d^2\right)e^{bx+a}\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+10b^2d^2+9d^4} - \frac{4b^2d\,e^{bx+a}\tan\left(\frac{dx}{2}\right.}{b^4+10b^2d^2+9d^4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/4*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2+
9*d^2)+1/4*d*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*sin(3*d*x+3*c
)/(b^2+9*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(107) = 214.
time = 0.30, size = 538, normalized size = 4.52

$$-\frac{(b^3\cos(3c)e^a + bd^2\cos(3c)e^a + 3b^2de^a\sin(3c) + 3d^3e^a\sin(3c))\cos(3dx)e^{(bx)} + (b^3\cos(3c)e^a + bd^2\cos(3c)e^a - 3b^2de^a\sin(3c) - 3d^3e^a\sin(3c))\cos(3dx+6c)e^{(bx)} - (b^3\cos(3c)e^a + 9bd^2\cos(3c)e^a - b^2de^a\sin(3c) - 9d^3e^a\sin(3c))\cos(dx+4c)e^{(bx)} - (b^3\cos(3c)e^a + 9bd^2\cos(3c)e^a + b^2de^a\sin(3c) + 9d^3e^a\sin(3c))\cos(dx-2c)e^{(bx)} + (3b^2d\cos(3c)e^a + 3d^3\cos(3c)e^a - b^3e^a\sin(3c) - bd^2e^a\sin(3c))e^{(bx)}\sin(3dx) + (3b^2d\cos(3c)e^a + 3d^3\cos(3c)e^a + b^3e^a\sin(3c) + bd^2e^a\sin(3c))e^{(bx)}\sin(3dx+6c) - (b^2d\cos(3c)e^a + 9d^3\cos(3c)e^a + b^3e^a\sin(3c) + 9bd^2e^a\sin(3c))e^{(bx)}\sin(dx+4c) - (b^2d\cos(3c)e^a + 9d^3\cos(3c)e^a - b^3e^a\sin(3c) - 9bd^2e^a\sin(3c))e^{(bx)}\sin(dx-2c)}{8\left(b^4\cos(3c)^2 + b^4\sin(3c)^2 + 9\left(\cos(3c)^2+\sin(3c)^2\right)d^4 + 10\left(b^2\cos(3c)^2 + b^2\sin(3c)^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3
*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a
- 3*b^2*d*e^a*sin(3*c) - 3*d^3*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) - (b^
3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(
3*c))*cos(d*x + 4*c)*e^(b*x) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b
^2*d*e^a*sin(3*c) + 9*d^3*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) + (3*b^2*d*c
os(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2*e^a*sin(3*c))*e
^(b*x)*sin(3*d*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a + b^3*e^a*si
n(3*c) + b*d^2*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^2*d*cos(3*c)*e^a
 + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(3*c))*e^(b*x)*si
n(d*x + 4*c) - (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c)
- 9*b*d^2*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2 + b^4*sin(3
*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)
^2)*d^2)
```

Fricas [A]
time = 2.94, size = 109, normalized size = 0.92

$$\frac{\left(b^2d+3\,d^3-3\,(b^2d+d^3)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(\left(b^3+bd^2\right)\cos\left(dx+c\right)^3-\left(b^3+3\,bd^2\right)\cos\left(dx+c\right)\right)e^{(bx+a)}}{b^4+10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*d + 3*d^3 - 3*(b^2*d + d^3)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c)
- ((b^3 + b*d^2)*cos(d*x + c)^3 - (b^3 + 3*b*d^2)*cos(d*x + c))*e^(b*x + a)
)/(b^4 + 10*b^2*d^2 + 9*d^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.20, size = 1040, normalized size = 8.74

$$\begin{cases} xe^a\sin^2(c)\cos(c) & \text{for}\ b=0\wedge d=0\\ \frac{ixe^ae^{-3idx}\sin^3(c+dx)}{8}+\frac{3xe^ae^{-3idx}\sin^2(c+dx)\cos(c+dx)}{8}-\frac{3ixe^ae^{-3idx}\sin(c+dx)\cos^2(c+dx)}{8}-\frac{xe^ae^{-3idx}\cos^3(c+dx)}{8}-\frac{e^ae^{-3idx}\sin^3(c+dx)}{24d}+\frac{ie^ae^{-3idx}\sin^2(c+dx)\cos(c+dx)}{4d}+\frac{ie^ae^{-3idx}\cos^3(c+dx)}{24d} & \text{for}\ b=-3id\\ \frac{ixe^ae^{-idx}\sin^3(c+dx)}{8}+\frac{xe^ae^{-idx}\sin^2(c+dx)\cos(c+dx)}{8}+\frac{ixe^ae^{-idx}\sin(c+dx)\cos^2(c+dx)}{8}+\frac{xe^ae^{-idx}\cos^3(c+dx)}{8}+\frac{3e^ae^{-idx}\sin^3(c+dx)}{8d}-\frac{ie^ae^{-idx}\sin^2(c+dx)\cos(c+dx)}{4d}-\frac{ie^ae^{-idx}\cos^3(c+dx)}{8d} & \text{for}\ b=-id\\ -\frac{ixe^ae^{idx}\sin^3(c+dx)}{8}+\frac{xe^ae^{idx}\sin^2(c+dx)\cos(c+dx)}{8}-\frac{ixe^ae^{idx}\sin(c+dx)\cos^2(c+dx)}{8}+\frac{xe^ae^{idx}\cos^3(c+dx)}{8}+\frac{3e^ae^{idx}\sin^3(c+dx)}{8d}+\frac{ie^ae^{idx}\sin^2(c+dx)\cos(c+dx)}{4d}+\frac{ie^ae^{idx}\cos^3(c+dx)}{8d} & \text{for}\ b=id\\ -\frac{ixe^ae^{3idx}\sin^3(c+dx)}{8}+\frac{3xe^ae^{3idx}\sin^2(c+dx)\cos(c+dx)}{8}+\frac{3ixe^ae^{3idx}\sin(c+dx)\cos^2(c+dx)}{8}-\frac{xe^ae^{3idx}\cos^3(c+dx)}{8}-\frac{e^ae^{3idx}\sin^3(c+dx)}{24d}-\frac{ie^ae^{3idx}\sin^2(c+dx)\cos(c+dx)}{4d}-\frac{ie^ae^{3idx}\cos^3(c+dx)}{24d} & \text{for}\ b=3id\\ \frac{b^3e^ae^{bx}\sin^2(c+dx)\cos(c+dx)}{b^4+10b^2d^2+9d^4}+\frac{b^2de^ae^{bx}\sin^3(c+dx)}{b^4+10b^2d^2+9d^4}-\frac{2b^2de^ae^{bx}\sin(c+dx)\cos^2(c+dx)}{b^4+10b^2d^2+9d^4}+\frac{3bd^2e^ae^{bx}\sin^2(c+dx)\cos(c+dx)}{b^4+10b^2d^2+9d^4}+\frac{2bd^2e^ae^{bx}\cos^3(c+dx)}{b^4+10b^2d^2+9d^4}+\frac{3d^3e^ae^{bx}\sin^3(c+dx)}{b^4+10b^2d^2+9d^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp
(-3*I*d*x)*sin(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos
(c + d*x)/8 - 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - x
```

```
*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/8 - exp(a)*exp(-3*I*d*x)*sin(c + d*x)
**3/(24*d) + I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) + I*
exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b, -3*I*d)), (I*x*exp(a)*ex
p(-I*d*x)*sin(c + d*x)**3/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c +
d*x)/8 + I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + x*exp(a)*e
xp(-I*d*x)*cos(c + d*x)**3/8 + 3*exp(a)*exp(-I*d*x)*sin(c + d*x)**3/(8*d) -
 I*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) - I*exp(a)*exp(-I*
d*x)*cos(c + d*x)**3/(8*d), Eq(b, -I*d)), (-I*x*exp(a)*exp(I*d*x)*sin(c + d
*x)**3/8 + x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 - I*x*exp(a)*
exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + x*exp(a)*exp(I*d*x)*cos(c + d*x
)**3/8 + 3*exp(a)*exp(I*d*x)*sin(c + d*x)**3/(8*d) + I*exp(a)*exp(I*d*x)*si
n(c + d*x)**2*cos(c + d*x)/(4*d) + I*exp(a)*exp(I*d*x)*cos(c + d*x)**3/(8*d
), Eq(b, I*d)), (-I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/8 + 3*x*exp(a)*ex
p(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*I*x*exp(a)*exp(3*I*d*x)*sin(c
 + d*x)*cos(c + d*x)**2/8 - x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/8 - exp(a
)*exp(3*I*d*x)*sin(c + d*x)**3/(24*d) - I*exp(a)*exp(3*I*d*x)*sin(c + d*x)*
*2*cos(c + d*x)/(4*d) - I*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b,
 3*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 + 10*b**
2*d**2 + 9*d**4) + b**2*d*exp(a)*exp(b*x)*sin(c + d*x)**3/(b**4 + 10*b**2*d
**2 + 9*d**4) - 2*b**2*d*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4
 + 10*b**2*d**2 + 9*d**4) + 3*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c
+ d*x)/(b**4 + 10*b**2*d**2 + 9*d**4) + 2*b*d**2*exp(a)*exp(b*x)*cos(c + d*
x)**3/(b**4 + 10*b**2*d**2 + 9*d**4) + 3*d**3*exp(a)*exp(b*x)*sin(c + d*x)*
*3/(b**4 + 10*b**2*d**2 + 9*d**4), True))
```

Giac [A]
time = 0.40, size = 98, normalized size = 0.82

$$-\frac{1}{4}\left(\frac{b\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}+\frac{3\,d\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)}+\frac{1}{4}\left(\frac{b\cos\left(dx+c\right)}{b^2+d^2}+\frac{d\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x + 3*c)/(b^2 + 9*d^2)
)*e^(b*x + a) + 1/4*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(d*x + c)/(b^2 + d^2
))*e^(b*x + a)
```

Mupad [B]
time = 3.01, size = 166, normalized size = 1.39

$$\frac{e^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1{}\mathrm{i}\right)}{8\,\left(b-d\,1{}\mathrm{i}\right)}-\frac{e^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(-3\,d+b\,1{}\mathrm{i}\right)}+\frac{e^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(-d+b\,1{}\mathrm{i}\right)}-\frac{e^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1{}\mathrm{i}\right)}{8\,\left(b-d\,3{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(8*(b - d*1i))
 - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)/
(8*(b*1i - 3*d)) + (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*
1i)*1i)/(8*(b*1i - d)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*
c) - sin(3*c)*1i))/(8*(b - d*3i))
```

3.40 $\int e^{a+bx} \cos(c+dx) \sin^3(c+dx) \, dx$

Optimal. Leaf size=129

$$-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}+\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}+\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}$$

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4557, 4517}

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}+\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

```
[Out] -1/2*(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) + (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) - (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx) - \frac{1}{8}e^{a+bx}\sin(4c+4dx)\right)dx$$
$$= -\left(\frac{1}{8}\int e^{a+bx}\sin(4c+4dx)\,dx\right) + \frac{1}{4}\int e^{a+bx}\sin(2c+2dx)\,dx$$
$$= -\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)} + \frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)} + \frac{be^{a+bx}\sin(2c+2dx}{4(b^2+4d^2)}$$

Mathematica [A]
time = 0.99, size = 82, normalized size = 0.64

$$\frac{1}{8}e^{a+bx}\left(\frac{2(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{b^2+4d^2}+\frac{4d\cos(4(c+dx))-b\sin(4(c+dx))}{b^2+16d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^2
) + (4*d*Cos[4*(c + d*x)] - b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

Maple [A]
time = 0.28, size = 118, normalized size = 0.91

method	result
risch	$\frac{i e^{bx+a} e^{4idx} e^{4ic}}{64id+16b} - \frac{i e^{bx+a} e^{2idx} e^{2ic}}{8(2id+b)} + \frac{i e^{bx+a} e^{-2idx} e^{-2ic}}{-16id+8b} - \frac{i e^{bx+a} e^{-4idx} e^{-4ic}}{16(-4id+b)}$
default	$-\frac{d\,e^{bx+a}\cos(2dx+2c)}{2(b^2+4d^2)} + \frac{d\,e^{bx+a}\cos(4dx+4c)}{2b^2+32d^2} + \frac{b\,e^{bx+a}\sin(2dx+2c)}{4b^2+16d^2} - \frac{b\,e^{bx+a}\sin(4dx+4c)}{8(b^2+16d^2)}$
norman	$\frac{-\frac{6d^3 e^{bx+a}}{b^4+20b^2d^2+64d^4} - \frac{6d^3 e^{bx+a}\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+20b^2d^2+64d^4} + \frac{12b\,d^2 e^{bx+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^4+20b^2d^2+64d^4} - \frac{12b\,d^2 e^{bx+a}\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+20b^2d^2+64d^4} + \frac{4b\left(2b^2+11d^2\right)e^{bx+a}\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{b^4+20b^2d^2+64d^4}}{\big(1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.
time = 0.29, size = 550, normalized size = 4.26

$$\frac{(4b^2d\cos(4c)e^a + 16d^3\cos(4c)e^a - b^3e^a\sin(4c) - 4bd^2e^a\sin(4c))\cos(4dx)e^{(bx)} + (4b^2d\cos(4c)e^a + 16d^3\cos(4c)e^a + b^3e^a\sin(4c) + 4bd^2e^a\sin(4c))\cos(4dx+8c)e^{(bx)} - 2(2b^2d\cos(4c)e^a + 32d^3\cos(4c)e^a + b^3e^a\sin(4c) + 16bd^2e^a\sin(4c))\cos(2dx+6c)e^{(bx)} - 2(2b^2d\cos(4c)e^a + 32d^3\cos(4c)e^a - b^3e^a\sin(4c) - 16bd^2e^a\sin(4c))\cos(2dx-2c)e^{(bx)} - (b^3\cos(4c)e^a + 4bd^2\cos(4c)e^a + 4b^2de^a\sin(4c) + 16d^3e^a\sin(4c))e^{(bx)}\sin(4dx) - (b^3\cos(4c)e^a + 4bd^2\cos(4c)e^a - 4b^2de^a\sin(4c) - 16d^3e^a\sin(4c))e^{(bx)}\sin(4dx+8c) + 2(b^3\cos(4c)e^a + 16bd^2\cos(4c)e^a - 2b^2de^a\sin(4c) - 32d^3e^a\sin(4c))e^{(bx)}\sin(2dx+6c) + 2(b^3\cos(4c)e^a + 16bd^2\cos(4c)e^a + 2b^2de^a\sin(4c) + 32d^3e^a\sin(4c))e^{(bx)}\sin(2dx-2c)}{16(b^4\cos(4c)^2 + b^4\sin(4c)^2 + 64(\cos(4c)^2 + \sin(4c)^2)d^4 + 20(b^2\cos(4c)^2 + b^2\sin(4c)^2)d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b*
d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4
*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x)
 - 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 16*b*
d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) - 2*(2*b^2*d*cos(4*c)*e^a + 32*d
^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x - 2*c
)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*sin(4*c)
 + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*co
s(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x
+ 8*c) + 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^a*sin(4*c)
 - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) + 2*(b^3*cos(4*c)*e^a + 16
*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c))*e^(b*x)*s
in(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*c)^2 + sin(4*
c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

Fricas [A]
time = 2.00, size = 135, normalized size = 1.05

$$-\frac{\left(\left(b^{3}+4\,bd^{2}\right)\cos\left(dx+c\right)^{3}-\left(b^{3}+10\,bd^{2}\right)\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(4\left(b^{2}d+4\,d^{3}\right)\cos\left(dx+c\right)^{4}+b^{2}d+10\,d^{3}-\left(5\,b^{2}d+32\,d^{3}\right)\cos\left(dx+c\right)^{2}\right)e^{(bx+a)}}{b^{4}+20\,b^{2}d^{2}+64\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(((b^3 + 4*b*d^2)*cos(d*x + c)^3 - (b^3 + 10*b*d^2)*cos(d*x + c))*e^(b*x +
 a)*sin(d*x + c) - (4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + b^2*d + 10*d^3 - (5*
b^2*d + 32*d^3)*cos(d*x + c)^2)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.84, size = 1357, normalized size = 10.52

$$\begin{cases} xe^{a}\sin^{3}(c)\cos(c) & \text{for}\ b=0\wedge d=0\\ \frac{ixe^{a}e^{-4idx}\sin^{4}(c+dx)}{16}+\frac{xe^{a}e^{-4idx}\sin^{3}(c+dx)\cos(c+dx)}{4}-\frac{3ixe^{a}e^{-4idx}\sin^{2}(c+dx)\cos^{2}(c+dx)}{8}-\frac{xe^{a}e^{-4idx}\sin(c+dx)\cos^{3}(c+dx)}{4}+\frac{ixe^{a}e^{-4idx}\cos^{4}(c+dx)}{16}-\frac{e^{a}e^{-4idx}\sin^{4}(c+dx)}{24d}+\frac{11ie^{a}e^{-4idx}\sin^{3}(c+dx)\cos(c+dx)}{48d}+\frac{5ie^{a}e^{-4idx}\sin(c+dx)\cos^{3}(c+dx)}{48d}+\frac{e^{a}e^{-4idx}\cos^{4}(c+dx)}{24d} & \text{for}\ b=-4id\\ \frac{ixe^{a}e^{-2idx}\sin^{4}(c+dx)}{8}+\frac{xe^{a}e^{-2idx}\sin^{3}(c+dx)\cos(c+dx)}{4}+\frac{xe^{a}e^{-2idx}\sin(c+dx)\cos^{3}(c+dx)}{4}-\frac{ixe^{a}e^{-2idx}\cos^{4}(c+dx)}{8}+\frac{7e^{a}e^{-2idx}\sin^{4}(c+dx)}{48d}-\frac{ie^{a}e^{-2idx}\sin^{3}(c+dx)\cos(c+dx)}{6d}-\frac{e^{a}e^{-2idx}\sin^{2}(c+dx)\cos^{2}(c+dx)}{4d}-\frac{e^{a}e^{-2idx}\cos^{4}(c+dx)}{16d} & \text{for}\ b=-2id\\ -\frac{ixe^{a}e^{2idx}\sin^{4}(c+dx)}{8}+\frac{xe^{a}e^{2idx}\sin^{3}(c+dx)\cos(c+dx)}{4}+\frac{xe^{a}e^{2idx}\sin(c+dx)\cos^{3}(c+dx)}{4}+\frac{ixe^{a}e^{2idx}\cos^{4}(c+dx)}{8}+\frac{7e^{a}e^{2idx}\sin^{4}(c+dx)}{48d}+\frac{ie^{a}e^{2idx}\sin^{3}(c+dx)\cos(c+dx)}{6d}-\frac{e^{a}e^{2idx}\sin^{2}(c+dx)\cos^{2}(c+dx)}{4d}-\frac{e^{a}e^{2idx}\cos^{4}(c+dx)}{16d} & \text{for}\ b=2id\\ -\frac{ixe^{a}e^{4idx}\sin^{4}(c+dx)}{16}+\frac{xe^{a}e^{4idx}\sin^{3}(c+dx)\cos(c+dx)}{4}+\frac{3ixe^{a}e^{4idx}\sin^{2}(c+dx)\cos^{2}(c+dx)}{8}-\frac{xe^{a}e^{4idx}\sin(c+dx)\cos^{3}(c+dx)}{4}-\frac{ixe^{a}e^{4idx}\cos^{4}(c+dx)}{16}-\frac{e^{a}e^{4idx}\sin^{4}(c+dx)}{24d}-\frac{11ie^{a}e^{4idx}\sin^{3}(c+dx)\cos(c+dx)}{48d}-\frac{5ie^{a}e^{4idx}\sin(c+dx)\cos^{3}(c+dx)}{48d}+\frac{e^{a}e^{4idx}\cos^{4}(c+dx)}{24d} & \text{for}\ b=4id\\ \frac{b^{3}e^{a}e^{bx}\sin^{3}(c+dx)\cos(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}+\frac{b^{2}de^{a}e^{bx}\sin^{4}(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}-\frac{3b^{2}de^{a}e^{bx}\sin^{2}(c+dx)\cos^{2}(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}+\frac{10bd^{2}e^{a}e^{bx}\sin^{3}(c+dx)\cos(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}+\frac{6bd^{2}e^{a}e^{bx}\sin(c+dx)\cos^{3}(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}+\frac{10d^{3}e^{a}e^{bx}\sin^{4}(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}-\frac{12d^{3}e^{a}e^{bx}\sin^{2}(c+dx)\cos^{2}(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}}-\frac{6d^{3}e^{a}e^{bx}\cos^{4}(c+dx)}{b^{4}+20b^{2}d^{2}+64d^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp
(-4*I*d*x)*sin(c + d*x)**4/16 + x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(
c + d*x)/4 - 3*I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8 -
 x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(-4*
I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*d) + 1
```

```
1*I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) + 5*I*exp(a)*e
xp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(-4*I*d*x)*cos
(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)
**4/8 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*ex
p(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-2*I*d*x)*cos(c
 + d*x)**4/8 + 7*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**4/(48*d) - I*exp(a)*exp
(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(6*d) - exp(a)*exp(-2*I*d*x)*sin(c
+ d*x)**2*cos(c + d*x)**2/(4*d) - exp(a)*exp(-2*I*d*x)*cos(c + d*x)**4/(16*
d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/8 + x*exp(a)*
exp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*exp(2*I*d*x)*sin(c +
 d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/8 + 7*exp
(a)*exp(2*I*d*x)*sin(c + d*x)**4/(48*d) + I*exp(a)*exp(2*I*d*x)*sin(c + d*x
)**3*cos(c + d*x)/(6*d) - exp(a)*exp(2*I*d*x)*sin(c + d*x)**2*cos(c + d*x)*
*2/(4*d) - exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/(16*d), Eq(b, 2*I*d)), (-I*x
*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 + x*exp(a)*exp(4*I*d*x)*sin(c + d*x
)**3*cos(c + d*x)/4 + 3*I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c + d*x
)**2/8 - x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*
exp(4*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/(24*d
) - 11*I*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 5*I*exp(
a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(4*I*d*x)*c
os(c + d*x)**4/(24*d), Eq(b, 4*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)**3
*cos(c + d*x)/(b**4 + 20*b**2*d**2 + 64*d**4) + b**2*d*exp(a)*exp(b*x)*sin(
c + d*x)**4/(b**4 + 20*b**2*d**2 + 64*d**4) - 3*b**2*d*exp(a)*exp(b*x)*sin(
c + d*x)**2*cos(c + d*x)**2/(b**4 + 20*b**2*d**2 + 64*d**4) + 10*b*d**2*exp
(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)/(b**4 + 20*b**2*d**2 + 64*d**4) +
 6*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**3/(b**4 + 20*b**2*d**2
 + 64*d**4) + 10*d**3*exp(a)*exp(b*x)*sin(c + d*x)**4/(b**4 + 20*b**2*d**2
+ 64*d**4) - 12*d**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**2/(b**4
+ 20*b**2*d**2 + 64*d**4) - 6*d**3*exp(a)*exp(b*x)*cos(c + d*x)**4/(b**4 +
20*b**2*d**2 + 64*d**4), True))
```

Giac [A]
time = 0.40, size = 111, normalized size = 0.86

$$\frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2}-\frac{b\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)}-\frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^2
))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*
c)/(b^2 + 4*d^2))*e^(b*x + a)
```

Mupad [B]
time = 3.03, size = 178, normalized size = 1.38

$$-\frac{e^{a+bx}\,\left(\cos\left(2\,d\,x\right)-\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)-\sin\left(2\,c\right)\,1{}\mathrm{i}\right)}{8\,\left(2\,d+b\,1{}\mathrm{i}\right)}+\frac{e^{a+bx}\,\left(\cos\left(4\,d\,x\right)-\sin\left(4\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(4\,c\right)-\sin\left(4\,c\right)\,1{}\mathrm{i}\right)}{16\,\left(4\,d+b\,1{}\mathrm{i}\right)}-\frac{e^{a+bx}\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(b+d\,2{}\mathrm{i}\right)}+\frac{e^{a+bx}\,\left(\cos\left(4\,d\,x\right)+\sin\left(4\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(4\,c\right)+\sin\left(4\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,\left(b+d\,4{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] (exp(a + b*x)*(cos(4*d*x) - sin(4*d*x)*1i)*(cos(4*c) - sin(4*c)*1i))/(16*(b
*1i + 4*d)) - (exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(2*
c)*1i))/(8*(b*1i + 2*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(
2*c) + sin(2*c)*1i)*1i)/(8*(b + d*2i)) + (exp(a + b*x)*(cos(4*d*x) + sin(4*
d*x)*1i)*(cos(4*c) + sin(4*c)*1i)*1i)/(16*(b + d*4i))
```

3.41 $\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx$

Optimal. Leaf size=119

$$-\frac{de^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}+\frac{be^{a+bx}\sin(c+dx)}{4(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}$$

```
[Out] -1/4*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)+1/4*b*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)+1/4*b*exp(b*x+a)*sin(3*d*x+3*
c)/(b^2+9*d^2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4557, 4517}

$$\frac{be^{a+bx}\sin(c+dx)}{4(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}-\frac{de^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

```
[Out] -1/4*(d*E^(a + b*x)*Cos[c + d*x])/(b^2 + d^2) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(4*(b^2 + 9*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) +
(b*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx &= \int\left(\frac{1}{4}e^{a+bx}\sin(c+dx)+\frac{1}{4}e^{a+bx}\sin(3c+3dx)\right)dx\\ &= \frac{1}{4}\int e^{a+bx}\sin(c+dx)\,dx+\frac{1}{4}\int e^{a+bx}\sin(3c+3dx)\,dx\\ &= -\frac{de^{a+bx}\cos(c+dx)}{4\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4\left(b^2+9d^2\right)}+\frac{be^{a+bx}\sin(c+dx)}{4\left(b^2+d^2\right)}+\end{aligned}$$

Mathematica [A]
time = 0.71, size = 74, normalized size = 0.62

$$\frac{1}{4}e^{a+bx}\left(\frac{-d\cos(c+dx)+b\sin(c+dx)}{b^2+d^2}+\frac{-3d\cos(3(c+dx))+b\sin(3(c+dx))}{b^2+9d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

```
[Out] (E^(a + b*x)*((-(d*Cos[c + d*x]) + b*Sin[c + d*x])/(b^2 + d^2) + (-3*d*Cos[
3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

Maple [A]
time = 0.20, size = 108, normalized size = 0.91

method	result
default	$-\frac{d\,\mathrm{e}^{bx+a}\cos(dx+c)}{4(b^2+d^2)}-\frac{3d\,\mathrm{e}^{bx+a}\cos(3dx+3c)}{4(b^2+9d^2)}+\frac{b\,\mathrm{e}^{bx+a}\sin(dx+c)}{4b^2+4d^2}+\frac{b\,\mathrm{e}^{bx+a}\sin(3dx+3c)}{4b^2+36d^2}$
risch	$-\frac{i\mathrm{e}^{bx+a}\mathrm{e}^{3idx}\mathrm{e}^{3ic}}{8(3id+b)}-\frac{i\mathrm{e}^{bx+a}\mathrm{e}^{idx}\mathrm{e}^{ic}}{8(id+b)}+\frac{i\mathrm{e}^{bx+a}\mathrm{e}^{-idx}\mathrm{e}^{-ic}}{-8id+8b}+\frac{i\mathrm{e}^{bx+a}\mathrm{e}^{-3idx}\mathrm{e}^{-3ic}}{-24id+8b}$
norman	$\frac{\frac{d\left(b^2+3d^2\right)\mathrm{e}^{bx+a}\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+10b^2d^2+9d^4}+\frac{d\left(11b^2+9d^2\right)\mathrm{e}^{bx+a}\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+10b^2d^2+9d^4}-\frac{d\left(b^2+3d^2\right)\mathrm{e}^{bx+a}}{b^4+10b^2d^2+9d^4}-\frac{4b\left(b^2-d^2\right)\mathrm{e}^{bx+a}\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+10b^2d^2+9d^4}+\frac{2b\left(b^2+3d^2\right)\mathrm{e}^{b}}{b^4+10}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)+1/4*b*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)+1/4*b*exp(b*x+a)*sin(3*d*x+3*
c)/(b^2+9*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(107) = 214.
time = 0.29, size = 538, normalized size = 4.52

$$-\frac{\left(3b^2d\cos(3c)e^a+3d^3\cos(3c)e^a-b^3e^a\sin(3c)-bd^2e^a\sin(3c)\right)\cos(3dx)e^{(bx)}+\left(3b^2d\cos(3c)e^a+3d^3\cos(3c)e^a+b^3e^a\sin(3c)+bd^2e^a\sin(3c)\right)\cos(3dx+6c)e^{(bx)}+\left(b^2d\cos(3c)e^a+9d^3\cos(3c)e^a+b^3e^a\sin(3c)+9bd^2e^a\sin(3c)\right)\cos(dx+4c)e^{(bx)}+\left(b^2d\cos(3c)e^a+9d^3\cos(3c)e^a-b^3e^a\sin(3c)-9bd^2e^a\sin(3c)\right)\cos(dx-2c)e^{(bx)}-\left(b^3\cos(3c)e^a+bd^2\cos(3c)e^a+3b^2de^a\sin(3c)+3d^3e^a\sin(3c)\right)e^{(bx)}\sin(3dx)-\left(b^3\cos(3c)e^a+bd^2\cos(3c)e^a-3b^2de^a\sin(3c)-3d^3e^a\sin(3c)\right)e^{(bx)}\sin(3dx+6c)-\left(b^3\cos(3c)e^a+9bd^2\cos(3c)e^a-b^2de^a\sin(3c)-9d^3e^a\sin(3c)\right)e^{(bx)}\sin(dx+4c)-\left(b^3\cos(3c)e^a+9bd^2\cos(3c)e^a+b^2de^a\sin(3c)+9d^3e^a\sin(3c)\right)e^{(bx)}\sin(dx-2c)}{8\left(b^4\cos(3c)^2+b^4\sin(3c)^2+9\left(\cos(3c)^2+\sin(3c)^2\right)d^4+10\left(b^2\cos(3c)^2+b^2\sin(3c)^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*((3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2
*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*
e^a + b^3*e^a*sin(3*c) + b*d^2*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) + (b^
2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(
3*c))*cos(d*x + 4*c)*e^(b*x) + (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b
^3*e^a*sin(3*c) - 9*b*d^2*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) - (b^3*cos(3
*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3*e^a*sin(3*c))*e
^(b*x)*sin(3*d*x) - (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a - 3*b^2*d*e^a*si
n(3*c) - 3*d^3*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^3*cos(3*c)*e^a +
 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(3*c))*e^(b*x)*si
n(d*x + 4*c) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b^2*d*e^a*sin(3*c
) + 9*d^3*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2 + b^4*sin(3
*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)
^2)*d^2)
```

Fricas [A]
time = 0.94, size = 98, normalized size = 0.82

$$\frac{\left(2\,bd^2+\left(b^3+bd^2\right)\cos\left(dx+c\right)^2\right)e^{\left(bx+a\right)}\sin\left(dx+c\right)+\left(2\,b^2d\cos\left(dx+c\right)-3\,\left(b^2d+d^3\right)\cos\left(dx+c\right)^3\right)e^{\left(bx+a\right)}}{b^4+10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((2*b*d^2 + (b^3 + b*d^2)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (2*b^2
*d*cos(d*x + c) - 3*(b^2*d + d^3)*cos(d*x + c)^3)*e^(b*x + a))/(b^4 + 10*b^
2*d^2 + 9*d^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.21, size = 1030, normalized size = 8.66

$$\begin{cases} xe^{a}\sin\left(c\right)\cos^{2}\left(c\right) & \text{for}\: b=0\wedge d=0 \\ -\frac{xe^{a}e^{-3idx}\sin^{3}\left(c+dx\right)}{8}+\frac{3ixe^{a}e^{-3idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{3xe^{a}e^{-3idx}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{ixe^{a}e^{-3idx}\cos^{3}\left(c+dx\right)}{8}+\frac{ie^{a}e^{-3idx}\sin^{3}\left(c+dx\right)}{8d}+\frac{e^{a}e^{-3idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{e^{a}e^{-3idx}\cos^{3}\left(c+dx\right)}{24d} & \text{for}\: b=-3id \\ \frac{xe^{a}e^{-idx}\sin^{3}\left(c+dx\right)}{8}-\frac{ixe^{a}e^{-idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{xe^{a}e^{-idx}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{ixe^{a}e^{-idx}\cos^{3}\left(c+dx\right)}{8}+\frac{ie^{a}e^{-idx}\sin^{3}\left(c+dx\right)}{8d}+\frac{e^{a}e^{-idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{e^{a}e^{-idx}\cos^{3}\left(c+dx\right)}{8d} & \text{for}\: b=-id \\ \frac{xe^{a}e^{idx}\sin^{3}\left(c+dx\right)}{8}+\frac{ixe^{a}e^{idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{xe^{a}e^{idx}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{ixe^{a}e^{idx}\cos^{3}\left(c+dx\right)}{8}-\frac{ie^{a}e^{idx}\sin^{3}\left(c+dx\right)}{8d}+\frac{e^{a}e^{idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{e^{a}e^{idx}\cos^{3}\left(c+dx\right)}{8d} & \text{for}\: b=id \\ -\frac{xe^{a}e^{3idx}\sin^{3}\left(c+dx\right)}{8}-\frac{3ixe^{a}e^{3idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{3xe^{a}e^{3idx}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{ixe^{a}e^{3idx}\cos^{3}\left(c+dx\right)}{8}-\frac{ie^{a}e^{3idx}\sin^{3}\left(c+dx\right)}{8d}+\frac{e^{a}e^{3idx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{e^{a}e^{3idx}\cos^{3}\left(c+dx\right)}{24d} & \text{for}\: b=3id \\ \frac{b^{3}e^{a}e^{bx}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{b^{4}+10b^{2}d^{2}+9d^{4}}+\frac{2b^{2}de^{a}e^{bx}\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{b^{2}de^{a}e^{bx}\cos^{3}\left(c+dx\right)}{b^{4}+10b^{2}d^{2}+9d^{4}}+\frac{2bd^{2}e^{a}e^{bx}\sin^{3}\left(c+dx\right)}{b^{4}+10b^{2}d^{2}+9d^{4}}+\frac{3bd^{2}e^{a}e^{bx}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{3d^{3}e^{a}e^{bx}\cos^{3}\left(c+dx\right)}{b^{4}+10b^{2}d^{2}+9d^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(a)*exp(
-3*I*d*x)*sin(c + d*x)**3/8 + 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*co
s(c + d*x)/8 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - I*
```

```
x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/8 + I*exp(a)*exp(-3*I*d*x)*sin(c + d
*x)**3/(8*d) + exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) - ex
p(a)*exp(-3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b, -3*I*d)), (x*exp(a)*exp(-I
*d*x)*sin(c + d*x)**3/8 - I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*
x)/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - I*x*exp(a)*exp
(-I*d*x)*cos(c + d*x)**3/8 + I*exp(a)*exp(-I*d*x)*sin(c + d*x)**3/(8*d) + e
xp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) - exp(a)*exp(-I*d*x)*c
os(c + d*x)**3/(8*d), Eq(b, -I*d)), (x*exp(a)*exp(I*d*x)*sin(c + d*x)**3/8
+ I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + x*exp(a)*exp(I*d*x
)*sin(c + d*x)*cos(c + d*x)**2/8 + I*x*exp(a)*exp(I*d*x)*cos(c + d*x)**3/8
- I*exp(a)*exp(I*d*x)*sin(c + d*x)**3/(8*d) + exp(a)*exp(I*d*x)*sin(c + d*x
)**2*cos(c + d*x)/(4*d) - exp(a)*exp(I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, I*
d)), (-x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/8 - 3*I*x*exp(a)*exp(3*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos(c
 + d*x)**2/8 + I*x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/8 - I*exp(a)*exp(3*I
*d*x)*sin(c + d*x)**3/(8*d) + exp(a)*exp(3*I*d*x)*sin(c + d*x)**2*cos(c + d
*x)/(4*d) - exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b, 3*I*d)), (b**
3*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 + 10*b**2*d**2 + 9*d**
4) + 2*b**2*d*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 + 10*b**2*
d**2 + 9*d**4) - b**2*d*exp(a)*exp(b*x)*cos(c + d*x)**3/(b**4 + 10*b**2*d**
2 + 9*d**4) + 2*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**3/(b**4 + 10*b**2*d**2
 + 9*d**4) + 3*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 +
10*b**2*d**2 + 9*d**4) - 3*d**3*exp(a)*exp(b*x)*cos(c + d*x)**3/(b**4 + 10*
b**2*d**2 + 9*d**4), True))
```

Giac [A]
time = 0.43, size = 100, normalized size = 0.84

$$-\frac{1}{4}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)}-\frac{1}{4}\left(\frac{d\cos\left(dx+c\right)}{b^2+d^2}-\frac{b\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x + 3*c)/(b^2 + 9*d^2)
)*e^(b*x + a) - 1/4*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d*x + c)/(b^2 + d^2
))*e^(b*x + a)
```

Mupad [B]
time = 0.86, size = 167, normalized size = 1.40

$$-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1{}\mathrm{i}\right)}{8\,\left(d+b\,1{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(b+d\,1{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1{}\mathrm{i}\right)}{8\,\left(3\,d+b\,1{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(b+d\,3{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x),x)
```

```
[Out] - (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(8*(b*1i + d
)) - (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/(8*(b
+ d*1i)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*c) - sin(3*c)*
1i))/(8*(b*1i + 3*d)) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c
) + sin(3*c)*1i)*1i)/(8*(b + d*3i))
```

3.42 $\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=79

$$\frac{e^{a+bx}}{8b}-\frac{be^{a+bx}\cos(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\sin(4c+4dx)}{2\left(b^2+16d^2\right)}$$

```
[Out] 1/8*exp(b*x+a)/b-1/8*b*exp(b*x+a)*cos(4*d*x+4*c)/(b^2+16*d^2)-1/2*d*exp(b*x
+a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4557, 2225, 4518}

$$-\frac{de^{a+bx}\sin(4c+4dx)}{2\left(b^2+16d^2\right)}-\frac{be^{a+bx}\cos(4c+4dx)}{8\left(b^2+16d^2\right)}+\frac{e^{a+bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] E^(a + b*x)/(8*b) - (b*E^(a + b*x)*Cos[4*c + 4*d*x])/(8*(b^2 + 16*d^2)) - (
d*E^(a + b*x)*Sin[4*c + 4*d*x])/(2*(b^2 + 16*d^2))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = \int\left(\frac{1}{8}e^{a+bx}-\frac{1}{8}e^{a+bx}\cos(4c+4dx)\right)dx$$
$$=\frac{1}{8}\int e^{a+bx}\,dx-\frac{1}{8}\int e^{a+bx}\cos(4c+4dx)\,dx$$
$$=\frac{e^{a+bx}}{8b}-\frac{be^{a+bx}\cos(4c+4dx)}{8(b^2+16d^2)}-\frac{de^{a+bx}\sin(4c+4dx)}{2(b^2+16d^2)}$$

Mathematica [A]
time = 0.37, size = 57, normalized size = 0.72

$$\frac{e^{a+bx}(b^2+16d^2-b^2\cos(4(c+dx))-4bd\sin(4(c+dx)))}{8(b^3+16bd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*(b^2 + 16*d^2 - b^2*Cos[4*(c + d*x)] - 4*b*d*Sin[4*(c + d*x)])
)/(8*(b^3 + 16*b*d^2))
```

Maple [A]
time = 0.31, size = 71, normalized size = 0.90

method	result
risch	$\frac{e^{bx+a}}{8b}-\frac{e^{bx+a}e^{4idx}e^{4ic}}{16(4id+b)}-\frac{e^{bx+a}e^{-4idx}e^{-4ic}}{16(-4id+b)}$
default	$\frac{e^{bx+a}}{8b}-\frac{b\,e^{bx+a}\cos(4dx+4c)}{8(b^2+16d^2)}-\frac{d\,e^{bx+a}\sin(4dx+4c)}{2(b^2+16d^2)}$
norman	$\frac{-\frac{4d\,e^{bx+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2+16d^2}+\frac{28d\,e^{bx+a}\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2+16d^2}-\frac{28d\,e^{bx+a}\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2+16d^2}+\frac{4d\,e^{bx+a}\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2+16d^2}+\frac{2d^2e^{bx+a}}{\left(b^2+16d^2\right)b}+\frac{2d^2e^{bx+a}\left(\tan^8\left(\cdots\right.\right.}{\left(b^2+16d^2\cdots\right.}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*exp(b*x+a)/b-1/8*b*exp(b*x+a)*cos(4*d*x+4*c)/(b^2+16*d^2)-1/2*d*exp(b*x
+a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(70) = 140.
time = 0.31, size = 236, normalized size = 2.99

$$-\frac{(b^2\cos(4c)e^a+4bde^a\sin(4c))\cos(4dx)e^{(bx)}+(b^2\cos(4c)e^a-4bde^a\sin(4c))\cos(4dx+8c)e^{(bx)}+(4bd\cos(4c)e^a-b^2e^a\sin(4c))e^{(bx)}\sin(4dx)+(4bd\cos(4c)e^a+b^2e^a\sin(4c))e^{(bx)}\sin(4dx+8c)-2\left(b^2\cos(4c)^2e^a+b^2e^a\sin(4c)^2+16\left(\cos(4c)^2e^a+e^a\sin(4c)^2\right)d^2\right)e^{(bx)}}{16\left(b^3\cos(4c)^2+b^3\sin(4c)^2+16\left(b\cos(4c)^2+b\sin(4c)^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="maxima")

[Out] -1/16*((b^2*cos(4*c)*e^a + 4*b*d*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (b^2*cos(4*c)*e^a - 4*b*d*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x) + (4*b*d*cos(4*c)*e^a - b^2*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) + (4*b*d*cos(4*c)*e^a + b^2*e^a*sin(4*c))*e^(b*x)*sin(4*d*x + 8*c) - 2*(b^2*cos(4*c)^2*e^a + b^2*e^a*sin(4*c)^2 + 16*(cos(4*c)^2*e^a + e^a*sin(4*c)^2)*d^2)*e^(b*x))/(b^3*cos(4*c)^2 + b^3*sin(4*c)^2 + 16*(b*cos(4*c)^2 + b*sin(4*c)^2)*d^2)

Fricas [A]
time = 1.84, size = 90, normalized size = 1.14

$$-\frac{2\left(2\,bd\cos\left(dx+c\right)^{3}-bd\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(b^{2}\cos\left(dx+c\right)^{4}-b^{2}\cos\left(dx+c\right)^{2}-2\,d^{2}\right)e^{(bx+a)}}{b^{3}+16\,bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="fricas")

[Out] -(2*(2*b*d*cos(d*x + c)^3 - b*d*cos(d*x + c))*e^(b*x + a)*sin(d*x + c) + (b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*d^2)*e^(b*x + a))/(b^3 + 16*b*d^2)

Sympy [C] Result contains complex when optimal does not.
time = 6.98, size = 850, normalized size = 10.76

$$\begin{cases} xe^{a}\sin^{2}\left(c\right)\cos^{2}\left(c\right) & \text{for}\: b=0\wedge d=0 \\ \left(\frac{x\sin^{4}\left(c+dx\right)}{8}+\frac{x\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{x\cos^{4}\left(c+dx\right)}{8}+\frac{\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}\right)e^{a} & \text{for}\: b=0 \\ -\frac{xe^{a}e^{-4idx}\sin^{4}\left(c+dx\right)}{16}+\frac{ixe^{a}e^{-4idx}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{4}+\frac{3xe^{a}e^{-4idx}\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{ixe^{a}e^{-4idx}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{4}-\frac{xe^{a}e^{-4idx}\cos^{4}\left(c+dx\right)}{16}+\frac{ie^{a}e^{-4idx}\sin^{4}\left(c+dx\right)}{24d}+\frac{5e^{a}e^{-4idx}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{48d}-\frac{5e^{a}e^{-4idx}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{48d}+\frac{ie^{a}e^{-4idx}\cos^{4}\left(c+dx\right)}{24d} & \text{for}\: b=-4id \\ -\frac{xe^{a}e^{4idx}\sin^{4}\left(c+dx\right)}{16}-\frac{ixe^{a}e^{4idx}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{4}+\frac{3xe^{a}e^{4idx}\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{ixe^{a}e^{4idx}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{4}-\frac{xe^{a}e^{4idx}\cos^{4}\left(c+dx\right)}{16}-\frac{ie^{a}e^{4idx}\sin^{4}\left(c+dx\right)}{24d}+\frac{5e^{a}e^{4idx}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{48d}-\frac{5e^{a}e^{4idx}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{48d}-\frac{ie^{a}e^{4idx}\cos^{4}\left(c+dx\right)}{24d} & \text{for}\: b=4id \\ \frac{b^{2}e^{a}e^{bx}\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{b^{3}+16bd^{2}}+\frac{2bde^{a}e^{bx}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{b^{3}+16bd^{2}}-\frac{2bde^{a}e^{bx}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{b^{3}+16bd^{2}}+\frac{2d^{2}e^{a}e^{bx}\sin^{4}\left(c+dx\right)}{b^{3}+16bd^{2}}+\frac{4d^{2}e^{a}e^{bx}\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{b^{3}+16bd^{2}}+\frac{2d^{2}e^{a}e^{bx}\cos^{4}\left(c+dx\right)}{b^{3}+16bd^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**2,x)

[Out] Piecewise((x*exp(a)*sin(c)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**4/8 + x*sin(c + d*x)**2*cos(c + d*x)**2/4 + x*cos(c + d*x)**4/8 + sin(c + d*x)**3*cos(c + d*x)/(8*d) - sin(c + d*x)*cos(c + d*x)**3/(8*d))*exp(a), Eq(b, 0)), (-x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/16 + I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + 3*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8 - I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - x*exp(a)*exp(-4*I*d*x)*cos(c + d*x)**4/16 + I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*d) + 5*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 5*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + I*exp(a)*exp(-4*I*d*x)*cos(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (-x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 - I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + 3*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8 + I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - x*exp(a)*exp(4*I*

```
d*x)*cos(c + d*x)**4/16 - I*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/(24*d) + 5*
exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 5*exp(a)*exp(4*I*
d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) - I*exp(a)*exp(4*I*d*x)*cos(c + d*
x)**4/(24*d), Eq(b, 4*I*d)), (b**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c +
d*x)**2/(b**3 + 16*b*d**2) + 2*b*d*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c +
d*x)/(b**3 + 16*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**
3/(b**3 + 16*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sin(c + d*x)**4/(b**3 + 16*b*
d**2) + 4*d**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**2/(b**3 + 16*b
*d**2) + 2*d**2*exp(a)*exp(b*x)*cos(c + d*x)**4/(b**3 + 16*b*d**2), True))
```

Giac [**A**]
time = 0.40, size = 66, normalized size = 0.84

$$-\frac{1}{8}\left(\frac{b\cos\left(4\,dx+4\,c\right)}{b^{2}+16\,d^{2}}+\frac{4\,d\sin\left(4\,dx+4\,c\right)}{b^{2}+16\,d^{2}}\right)e^{(bx+a)}+\frac{e^{(bx+a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(b*cos(4*d*x + 4*c)/(b^2 + 16*d^2) + 4*d*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) + 1/8*e^(b*x + a)/b
```

Mupad [**B**]
time = 0.37, size = 58, normalized size = 0.73

$$\frac{\mathrm{e}^{a+b\,x}\,\left(b^2+16\,d^2-b^2\,\cos\left(4\,c+4\,d\,x\right)-4\,b\,d\,\sin\left(4\,c+4\,d\,x\right)\right)}{8\,b\,\left(b^2+16\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] (exp(a + b*x)*(b^2 + 16*d^2 - b^2*cos(4*c + 4*d*x) - 4*b*d*sin(4*c + 4*d*x)
))/(8*b*(b^2 + 16*d^2))
```

3.43 $\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=183

$$-\frac{de^{a+bx}\cos(c+dx)}{8\,(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16\,(b^2+9d^2)}+\frac{5de^{a+bx}\cos(5c+5dx)}{16\,(b^2+25d^2)}+\frac{be^{a+bx}\sin(c+dx)}{8\,(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3d}{16\,(b^2+9d^2)}$$

```
[Out] -1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*sin
(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*exp(b
*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Rubi [A]
time = 0.08, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4557, 4517}

$$\frac{be^{a+bx}\sin(c+dx)}{8\,(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{16\,(b^2+9d^2)}-\frac{be^{a+bx}\sin(5c+5dx)}{16\,(b^2+25d^2)}-\frac{de^{a+bx}\cos(c+dx)}{8\,(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16\,(b^2+9d^2)}+\frac{5de^{a+bx}\cos(5c+5dx)}{16\,(b^2+25d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] -1/8*(d*E^(a + b*x)*Cos[c + d*x])/(b^2 + d^2) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(16*(b^2 + 9*d^2)) + (5*d*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 +
25*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) + (b*E^(a + b*x)*Si
n[3*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(
b^2 + 25*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \int\left(\frac{1}{8}e^{a+bx}\sin(c+dx)+\frac{1}{16}e^{a+bx}\sin(3c+3dx)-\frac{1}{16}e^{a+bx}\sin(5c+\right.$$
$$=\frac{1}{16}\int e^{a+bx}\sin(3c+3dx)\,dx-\frac{1}{16}\int e^{a+bx}\sin(5c+5dx)\,dx+\frac{1}{8}\int e$$
$$=-\frac{de^{a+bx}\cos(c+dx)}{8(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)}+\frac{5de^{a+bx}\cos(5c+5c}{16(b^2+25d^2)}$$

Mathematica [A]
time = 0.95, size = 110, normalized size = 0.60

$$\frac{1}{16}e^{a+bx}\left(\frac{2(-d\cos(c+dx)+b\sin(c+dx))}{b^2+d^2}+\frac{-3d\cos(3(c+dx))+b\sin(3(c+dx))}{b^2+9d^2}+\frac{5d\cos(5(c+dx))-b\sin(5(c+dx))}{b^2+25d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
[Out] (E^(a + b*x)*((2*(-(d*Cos[c + d*x]) + b*Sin[c + d*x]))/(b^2 + d^2) + (-3*d*
Cos[3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2) + (5*d*Cos[5*(c + d*x)
] - b*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

Maple [A]
time = 0.21, size = 166, normalized size = 0.91

method	result
default	$-\frac{d\,e^{bx+a}\cos(dx+c)}{8(b^2+d^2)}-\frac{3d\,e^{bx+a}\cos(3dx+3c)}{16(b^2+9d^2)}+\frac{5d\,e^{bx+a}\cos(5dx+5c)}{16(b^2+25d^2)}+\frac{b\,e^{bx+a}\sin(dx+c)}{8b^2+8d^2}+\frac{b\,e^{bx+a}\sin(3dx+3c)}{16b^2+144d^2}-\frac{b\,e^{bx+}}{16}$
risch	$\frac{ie^{bx+a}e^{5idx}e^{5ic}}{160id+32b}-\frac{ie^{bx+a}e^{3idx}e^{3ic}}{32(3id+b)}-\frac{ie^{bx+a}e^{idx}e^{ic}}{16(id+b)}+\frac{ie^{bx+a}e^{-idx}e^{-ic}}{-16id+16b}+\frac{ie^{bx+a}e^{-3idx}e^{-3ic}}{-96id+32b}-\frac{ie^{bx+a}e^{-5idx}e^{-5ic}}{32(-5id+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*sin
(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*exp(b
*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(165) = 330$.
time = 0.33, size = 1148, normalized size = 6.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*cos(5*c)*e^a
 - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^4*e^a*sin(5*c))*cos(5
*d*x)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*co
s(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c) + 9*b*d^4*e^a*sin(5
*c))*cos(5*d*x + 10*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c
)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^3*d^2*e^a*sin(5*c) +
25*b*d^4*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 7
8*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 26*b^3*d^
2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) - 2*(b^4*d
*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)*e^a + b^5*e^a*si
n(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5*c))*cos(d*x + 6*c)*e
^(b*x) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)
*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 225*b*d^4*e^a*sin(5*c))
*cos(d*x - 4*c)*e^(b*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b
*d^4*cos(5*c)*e^a + 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5
*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)
*e^a + 9*b*d^4*cos(5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c
) - 45*d^5*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (b^5*cos(5*c)*e^a + 26
*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b
^2*d^3*e^a*sin(5*c) - 75*d^5*e^a*sin(5*c))*e^(b*x)*sin(3*d*x + 8*c) + (b^5*
cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^
a*sin(5*c) + 78*b^2*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*e^(b*x)*sin(3*d
*x - 2*c) + 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5
*c)*e^a - b^4*d*e^a*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*
c))*e^(b*x)*sin(d*x + 6*c) + 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a
+ 225*b*d^4*cos(5*c)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 2
25*d^5*e^a*sin(5*c))*e^(b*x)*sin(d*x - 4*c))/(b^6*cos(5*c)^2 + b^6*sin(5*c)
^2 + 225*(cos(5*c)^2 + sin(5*c)^2)*d^6 + 259*(b^2*cos(5*c)^2 + b^2*sin(5*c)
^2)*d^4 + 35*(b^4*cos(5*c)^2 + b^4*sin(5*c)^2)*d^2)
```

<u>Fricas</u> [A]
time = 1.70, size = 201, normalized size = 1.10

$$\frac{\left(2\,b^3d^2+26\,bd^4-\left(b^5+10\,b^3d^2+9\,bd^4\right)\cos\left(dx+c\right)^4+\left(b^5+14\,b^3d^2+13\,bd^4\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(5\left(b^4d+10\,b^2d^3+9\,d^5\right)\cos\left(dx+c\right)^5-\left(7\,b^4d+82\,b^2d^3+75\,d^5\right)\cos\left(dx+c\right)^3+2\left(b^4d+13\,b^2d^3\right)\cos\left(dx+c\right)\right)e^{(bx+a)}}{b^6+35\,b^4d^2+259\,b^2d^4+225\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] ((2*b^3*d^2 + 26*b*d^4 - (b^5 + 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^4 + (b^5
 + 14*b^3*d^2 + 13*b*d^4)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (5*(b^
4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^5 - (7*b^4*d + 82*b^2*d^3 + 75*d^5)*
cos(d*x + c)^3 + 2*(b^4*d + 13*b^2*d^3)*cos(d*x + c))*e^(b*x + a))/(b^6 + 3
5*b^4*d^2 + 259*b^2*d^4 + 225*d^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 30.19, size = 2751, normalized size = 15.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(a)*e
xp(-5*I*d*x)*sin(c + d*x)**5/32 + 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**
4*cos(c + d*x)/32 + 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**
2/16 - 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 - 5*x*
exp(a)*exp(-5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 + I*x*exp(a)*exp(-5*I*
d*x)*cos(c + d*x)**5/32 + I*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**5/(64*d) + 3
*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(64*d) - exp(a)*exp(-5*I
*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(6*d) + 25*I*exp(a)*exp(-5*I*d*x)*sin
(c + d*x)*cos(c + d*x)**4/(192*d) + 31*exp(a)*exp(-5*I*d*x)*cos(c + d*x)**5
/(960*d), Eq(b, -5*I*d)), (-x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**5/32 + 3*I
*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 + x*exp(a)*exp(-3*I
*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16 + I*x*exp(a)*exp(-3*I*d*x)*sin(c +
 d*x)**2*cos(c + d*x)**3/16 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c +
 d*x)**4/32 - I*x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/32 + 3*I*exp(a)*exp(
-3*I*d*x)*sin(c + d*x)**5/(64*d) + 7*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*c
os(c + d*x)/(64*d) - exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(
6*d) + 9*I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) + 7*exp
(a)*exp(-3*I*d*x)*cos(c + d*x)**5/(192*d), Eq(b, -3*I*d)), (x*exp(a)*exp(-I
*d*x)*sin(c + d*x)**5/16 - I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**4*cos(c + d
*x)/16 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/8 - I*x*exp(a
)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/8 + x*exp(a)*exp(-I*d*x)*sin(
c + d*x)*cos(c + d*x)**4/16 - I*x*exp(a)*exp(-I*d*x)*cos(c + d*x)**5/16 + I
*exp(a)*exp(-I*d*x)*sin(c + d*x)**5/(32*d) + 3*exp(a)*exp(-I*d*x)*sin(c + d
*x)**4*cos(c + d*x)/(32*d) - exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x
)**3/(6*d) + I*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(96*d) - 5*e
xp(a)*exp(-I*d*x)*cos(c + d*x)**5/(96*d), Eq(b, -I*d)), (x*exp(a)*exp(I*d*x
)*sin(c + d*x)**5/16 + I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(c + d*x)/1
6 + x*exp(a)*exp(I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/8 + I*x*exp(a)*exp(
I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/8 + x*exp(a)*exp(I*d*x)*sin(c + d*x)
*cos(c + d*x)**4/16 + I*x*exp(a)*exp(I*d*x)*cos(c + d*x)**5/16 - I*exp(a)*e
xp(I*d*x)*sin(c + d*x)**5/(32*d) + 3*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(
c + d*x)/(32*d) - exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(6*d) -
 I*exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(96*d) - 5*exp(a)*exp(I*d
*x)*cos(c + d*x)**5/(96*d), Eq(b, I*d)), (-x*exp(a)*exp(3*I*d*x)*sin(c + d*
x)**5/32 - 3*I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 + x*ex
p(a)*exp(3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16 - I*x*exp(a)*exp(3*I*d
*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 + 3*x*exp(a)*exp(3*I*d*x)*sin(c + d*
```

```
x)*cos(c + d*x)**4/32 + I*x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**5/32 - 3*I*ex
p(a)*exp(3*I*d*x)*sin(c + d*x)**5/(64*d) + 7*exp(a)*exp(3*I*d*x)*sin(c + d*
x)**4*cos(c + d*x)/(64*d) - exp(a)*exp(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x
)**3/(6*d) - 9*I*exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) +
7*exp(a)*exp(3*I*d*x)*cos(c + d*x)**5/(192*d), Eq(b, 3*I*d)), (-x*exp(a)*ex
p(5*I*d*x)*sin(c + d*x)**5/32 - 5*I*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**4*c
os(c + d*x)/32 + 5*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16
 + 5*I*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 - 5*x*exp(a
)*exp(5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 - I*x*exp(a)*exp(5*I*d*x)*co
s(c + d*x)**5/32 - I*exp(a)*exp(5*I*d*x)*sin(c + d*x)**5/(64*d) + 3*exp(a)*
exp(5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(64*d) - exp(a)*exp(5*I*d*x)*sin(
c + d*x)**2*cos(c + d*x)**3/(6*d) - 25*I*exp(a)*exp(5*I*d*x)*sin(c + d*x)*c
os(c + d*x)**4/(192*d) + 31*exp(a)*exp(5*I*d*x)*cos(c + d*x)**5/(960*d), Eq
(b, 5*I*d)), (b**5*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 +
35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 2*b**4*d*exp(a)*exp(b*x)*sin(c +
 d*x)**4*cos(c + d*x)/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) - 3*
b**4*d*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/(b**6 + 35*b**4*d**2
 + 259*b**2*d**4 + 225*d**6) + 2*b**3*d**2*exp(a)*exp(b*x)*sin(c + d*x)**5/
(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 18*b**3*d**2*exp(a)*exp(
b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 +
 225*d**6) + 6*b**3*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**4/(b**6
 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 26*b**2*d**3*exp(a)*exp(b*x)*
sin(c + d*x)**4*cos(c + d*x)/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**
6) - 30*b**2*d**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/(b**6 + 3
5*b**4*d**2 + 259*b**2*d**4 + 225*d**6) - 6*b**2*d**3*exp(a)*exp(b*x)*cos(c
 + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 26*b*d**4*exp
(a)*exp(b*x)*sin(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**
6) + 65*b*d**4*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 + 35*b
**4*d**2 + 259*b**2*d**4 + 225*d**6) + 30*b*d**4*exp(a)*exp(b*x)*sin(c + d*
x)*cos(c + d*x)**4/(b**6 + 35*b**4*d**2 + 259*b...
```

Giac [A]
time = 0.41, size = 155, normalized size = 0.85

$$\frac{1}{16}\left(\frac{5\,d\cos\left(5\,dx+5\,c\right)}{b^2+25\,d^2}-\frac{b\sin\left(5\,dx+5\,c\right)}{b^2+25\,d^2}\right)e^{(bx+a)}-\frac{1}{16}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)}-\frac{1}{8}\left(\frac{d\cos\left(dx+c\right)}{b^2+d^2}-\frac{b\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*d*cos(5*d*x + 5*c)/(b^2 + 25*d^2) - b*sin(5*d*x + 5*c)/(b^2 + 25*d^
2))*e^(b*x + a) - 1/16*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x +
3*c)/(b^2 + 9*d^2))*e^(b*x + a) - 1/8*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d
*x + c)/(b^2 + d^2))*e^(b*x + a)
```

Mupad [B]

time = 3.74, size = 255, normalized size = 1.39

$$-\frac{e^{a+bx}\left(\cos\left(dx\right)-\sin\left(dx\right)\,1\mathrm{i}\right)\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)}{16\,\left(d+b\,1\mathrm{i}\right)}-\frac{e^{a+bx}\left(\cos\left(dx\right)+\sin\left(dx\right)\,1\mathrm{i}\right)\left(\cos\left(c\right)+\sin\left(c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,\left(b+d\,1\mathrm{i}\right)}-\frac{e^{a+bx}\left(\cos\left(3\,dx\right)-\sin\left(3\,dx\right)\,1\mathrm{i}\right)\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1\mathrm{i}\right)}{32\,\left(3\,d+b\,1\mathrm{i}\right)}+\frac{e^{a+bx}\left(\cos\left(5\,dx\right)-\sin\left(5\,dx\right)\,1\mathrm{i}\right)\left(\cos\left(5\,c\right)-\sin\left(5\,c\right)\,1\mathrm{i}\right)}{32\,\left(5\,d+b\,1\mathrm{i}\right)}-\frac{e^{a+bx}\left(\cos\left(3\,dx\right)+\sin\left(3\,dx\right)\,1\mathrm{i}\right)\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,\left(b+d\,3\mathrm{i}\right)}+\frac{e^{a+bx}\left(\cos\left(5\,dx\right)+\sin\left(5\,dx\right)\,1\mathrm{i}\right)\left(\cos\left(5\,c\right)+\sin\left(5\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,\left(b+d\,5\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] (exp(a + b*x)*(cos(5*d*x) - sin(5*d*x)*1i)*(cos(5*c) - sin(5*c)*1i))/(32*(b
*1i + 5*d)) - (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1
i)/(16*(b + d*1i)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*c) -
 sin(3*c)*1i))/(32*(b*1i + 3*d)) - (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(
cos(c) - sin(c)*1i))/(16*(b*1i + d)) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*
x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)/(32*(b + d*3i)) + (exp(a + b*x)*(cos(5*
d*x) + sin(5*d*x)*1i)*(cos(5*c) + sin(5*c)*1i)*1i)/(32*(b + d*5i))
```

3.44 $\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$

Optimal. Leaf size=129

$$-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}-\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}+\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}+\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)-1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)+1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {4557, 4517}

$$\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}+\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}-\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

```
[Out] -1/2*(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) - (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) + (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx &= \int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx)+\frac{1}{8}e^{a+bx}\sin(4c+4dx)\right)dx\\ &= \frac{1}{8}\int e^{a+bx}\sin(4c+4dx)\,dx+\frac{1}{4}\int e^{a+bx}\sin(2c+2dx)\,dx\\ &= -\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}+\frac{be^{a+bx}\sin(2c+2dx}{4\left(b^2+4d^2\right)}\end{aligned}$$

Mathematica [A]
time = 0.70, size = 81, normalized size = 0.63

$$\frac{1}{8}e^{a+bx}\left(\frac{2(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{b^2+4d^2}+\frac{-4d\cos(4(c+dx))+b\sin(4(c+dx))}{b^2+16d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

```
[Out] (E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^2
) + (-4*d*Cos[4*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

Maple [A]
time = 0.26, size = 118, normalized size = 0.91

method	result
risch	$-\frac{i e^{bx+a} e^{4idx} e^{4ic}}{16(4id+b)}-\frac{i e^{bx+a} e^{2idx} e^{2ic}}{8(2id+b)}+\frac{i e^{bx+a} e^{-2idx} e^{-2ic}}{-16id+8b}+\frac{i e^{bx+a} e^{-4idx} e^{-4ic}}{-64id+16b}$
default	$-\frac{d\,e^{bx+a}\cos(2dx+2c)}{2(b^2+4d^2)}-\frac{d\,e^{bx+a}\cos(4dx+4c)}{2(b^2+16d^2)}+\frac{b\,e^{bx+a}\sin(2dx+2c)}{4b^2+16d^2}+\frac{b\,e^{bx+a}\sin(4dx+4c)}{8b^2+128d^2}$
norman	$\frac{-\frac{d\left(b^2+10d^2\right)e^{bx+a}}{b^4+20b^2d^2+64d^4}-\frac{6b\left(b^2+2d^2\right)e^{bx+a}\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+20b^2d^2+64d^4}+\frac{6b\left(b^2+2d^2\right)e^{bx+a}\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4+20b^2d^2+64d^4}+\frac{2b\left(b^2+10d^2\right)e^{bx+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^4+20b^2d^2+64d^4}-\frac{2b\left(b^2+10d\right.}{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)-1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)+1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.
time = 0.30, size = 550, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b
*d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(
4*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x
) + 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 16*b
*d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) + 2*(2*b^2*d*cos(4*c)*e^a + 32*
d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x - 2*
c)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*sin(4*c
) + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*c
os(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x
 + 8*c) - 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^a*sin(4*c
) - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) - 2*(b^3*cos(4*c)*e^a + 1
6*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c))*e^(b*x)*
sin(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*c)^2 + sin(4
*c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

Fricas [A]
time = 1.78, size = 114, normalized size = 0.88

$$\frac{\left(6\,bd^2\cos\left(dx+c\right)+\left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^3\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(3\,b^2d\cos\left(dx+c\right)^2-4\left(b^2d+4\,d^3\right)\cos\left(dx+c\right)^4+6\,d^3\right)e^{(bx+a)}}{b^4+20\,b^2d^2+64\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((6*b*d^2*cos(d*x + c) + (b^3 + 4*b*d^2)*cos(d*x + c)^3)*e^(b*x + a)*sin(d*
x + c) + (3*b^2*d*cos(d*x + c)^2 - 4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 6*d^3
)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.90, size = 1353, normalized size = 10.49

$$\begin{cases} xe^a\sin\left(c\right)\cos^3\left(c\right) & \text{for}\ b=0\wedge d=0\\ -\frac{ixe^ae^{-4idx}\sin^4\left(c+dx\right)}{16}-\frac{xe^ae^{-4idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4}+\frac{3ixe^ae^{-4idx}\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{8}+\frac{xe^ae^{-4idx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4}-\frac{ixe^ae^{-4idx}\cos^4\left(c+dx\right)}{16}-\frac{e^ae^{-4idx}\sin^4\left(c+dx\right)}{24d}+\frac{5ie^ae^{-4idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{48d}+\frac{11ie^ae^{-4idx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{48d}+\frac{e^ae^{-4idx}\cos^4\left(c+dx\right)}{24d} & \text{for}\ b=-4id\\ \frac{ixe^ae^{-2idx}\sin^4\left(c+dx\right)}{8}+\frac{xe^ae^{-2idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4}+\frac{xe^ae^{-2idx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4}-\frac{ixe^ae^{-2idx}\cos^4\left(c+dx\right)}{8}-\frac{e^ae^{-2idx}\sin^4\left(c+dx\right)}{48d}+\frac{ie^ae^{-2idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{6d}+\frac{e^ae^{-2idx}\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4d}-\frac{e^ae^{-2idx}\cos^4\left(c+dx\right)}{16d} & \text{for}\ b=-2id\\ -\frac{ixe^ae^{2idx}\sin^4\left(c+dx\right)}{8}+\frac{xe^ae^{2idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4}+\frac{xe^ae^{2idx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4}+\frac{ixe^ae^{2idx}\cos^4\left(c+dx\right)}{8}-\frac{e^ae^{2idx}\sin^4\left(c+dx\right)}{48d}-\frac{ie^ae^{2idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{6d}+\frac{e^ae^{2idx}\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4d}-\frac{e^ae^{2idx}\cos^4\left(c+dx\right)}{16d} & \text{for}\ b=2id\\ \frac{ixe^ae^{4idx}\sin^4\left(c+dx\right)}{16}-\frac{xe^ae^{4idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4}-\frac{3ixe^ae^{4idx}\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{8}+\frac{xe^ae^{4idx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4}+\frac{ixe^ae^{4idx}\cos^4\left(c+dx\right)}{16}-\frac{e^ae^{4idx}\sin^4\left(c+dx\right)}{24d}-\frac{5ie^ae^{4idx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{48d}-\frac{11ie^ae^{4idx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{48d}+\frac{e^ae^{4idx}\cos^4\left(c+dx\right)}{24d} & \text{for}\ b=4id\\ \frac{b^3e^ae^{bx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{b^4+20b^2d^2+64d^4}+\frac{3b^2de^ae^{bx}\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{b^4+20b^2d^2+64d^4}-\frac{b^2de^ae^{bx}\cos^4\left(c+dx\right)}{b^4+20b^2d^2+64d^4}+\frac{6bd^2e^ae^{bx}\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{b^4+20b^2d^2+64d^4}+\frac{10bd^2e^ae^{bx}\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{b^4+20b^2d^2+64d^4}+\frac{6d^3e^ae^{bx}\sin^4\left(c+dx\right)}{b^4+20b^2d^2+64d^4}+\frac{12d^3e^ae^{bx}\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{b^4+20b^2d^2+64d^4}-\frac{10d^3e^ae^{bx}\cos^4\left(c+dx\right)}{b^4+20b^2d^2+64d^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)*ex
p(-4*I*d*x)*sin(c + d*x)**4/16 - x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos
(c + d*x)/4 + 3*I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8
+ x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-4
*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*d) +
```

```
5*I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) + 11*I*exp(a)*
exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(-4*I*d*x)*co
s(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x
)**4/8 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*e
xp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-2*I*d*x)*cos(
c + d*x)**4/8 - exp(a)*exp(-2*I*d*x)*sin(c + d*x)**4/(48*d) + I*exp(a)*exp(
-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(6*d) + exp(a)*exp(-2*I*d*x)*sin(c +
 d*x)**2*cos(c + d*x)**2/(4*d) - exp(a)*exp(-2*I*d*x)*cos(c + d*x)**4/(16*d
), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/8 + x*exp(a)*e
xp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*exp(2*I*d*x)*sin(c +
d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/8 - exp(a)
*exp(2*I*d*x)*sin(c + d*x)**4/(48*d) - I*exp(a)*exp(2*I*d*x)*sin(c + d*x)**
3*cos(c + d*x)/(6*d) + exp(a)*exp(2*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/
(4*d) - exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/(16*d), Eq(b, 2*I*d)), (I*x*exp
(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 - x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3
*cos(c + d*x)/4 - 3*I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2
/8 + x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(
4*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/(24*d) -
5*I*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 11*I*exp(a)*e
xp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(4*I*d*x)*cos(c
 + d*x)**4/(24*d), Eq(b, 4*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)*cos(c
+ d*x)**3/(b**4 + 20*b**2*d**2 + 64*d**4) + 3*b**2*d*exp(a)*exp(b*x)*sin(c
+ d*x)**2*cos(c + d*x)**2/(b**4 + 20*b**2*d**2 + 64*d**4) - b**2*d*exp(a)*e
xp(b*x)*cos(c + d*x)**4/(b**4 + 20*b**2*d**2 + 64*d**4) + 6*b*d**2*exp(a)*e
xp(b*x)*sin(c + d*x)**3*cos(c + d*x)/(b**4 + 20*b**2*d**2 + 64*d**4) + 10*b
*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**3/(b**4 + 20*b**2*d**2 + 6
4*d**4) + 6*d**3*exp(a)*exp(b*x)*sin(c + d*x)**4/(b**4 + 20*b**2*d**2 + 64*
d**4) + 12*d**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**2/(b**4 + 20*
b**2*d**2 + 64*d**4) - 10*d**3*exp(a)*exp(b*x)*cos(c + d*x)**4/(b**4 + 20*b
**2*d**2 + 64*d**4), True))
```

Giac [A]

time = 0.40, size = 111, normalized size = 0.86

$$-\frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2}-\frac{b\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)}-\frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2
*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

Mupad [B]

time = 0.82, size = 179, normalized size = 1.39

$$-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)-\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)-\sin\left(2\,c\right)\,1{}\mathrm{i}\right)}{8\,\left(2\,d+b\,1{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(4\,d\,x\right)-\sin\left(4\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(4\,c\right)-\sin\left(4\,c\right)\,1{}\mathrm{i}\right)}{16\,\left(4\,d+b\,1{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(b+d\,2{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(4\,d\,x\right)+\sin\left(4\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(4\,c\right)+\sin\left(4\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,\left(b+d\,4{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x),x)
```

```
[Out] - (exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(2*c)*1i))/(8*(
b*1i + 2*d)) - (exp(a + b*x)*(cos(4*d*x) - sin(4*d*x)*1i)*(cos(4*c) - sin(4
*c)*1i))/(16*(b*1i + 4*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(co
s(2*c) + sin(2*c)*1i)*1i)/(8*(b + d*2i)) - (exp(a + b*x)*(cos(4*d*x) + sin(
4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i)*1i)/(16*(b + d*4i))
```

3.45 $\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=183

$$\frac{be^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{be^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{be^{a+bx}\cos(5c+5dx)}{16\left(b^2+25d^2\right)}+\frac{de^{a+bx}\sin(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\sin(3c+3dx)}{16\left(b^2+9d^2\right)}$$

```
[Out] 1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*sin(
d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp(b*
x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Rubi [A]
time = 0.08, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4557, 4518}

$$\frac{de^{a+bx}\sin(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\sin(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{5de^{a+bx}\sin(5c+5dx)}{16\left(b^2+25d^2\right)}+\frac{be^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{be^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{be^{a+bx}\cos(5c+5dx)}{16\left(b^2+25d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 + 25*d^
2)) + (d*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Sin[3
*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (5*d*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(b
^2 + 25*d^2))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \int \left(\frac{1}{8}e^{a+bx}\cos(c+dx) - \frac{1}{16}e^{a+bx}\cos(3c+3dx) - \frac{1}{16}e^{a+bx}\cos(5c+5$$
$$= -\left(\frac{1}{16}\int e^{a+bx}\cos(3c+3dx)\,dx\right) - \frac{1}{16}\int e^{a+bx}\cos(5c+5dx)\,dx + \frac{1}{8}$$
$$= \frac{be^{a+bx}\cos(c+dx)}{8(b^2+d^2)} - \frac{be^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)} +$$

Mathematica [A]
time = 0.80, size = 110, normalized size = 0.60

$$\frac{1}{16}e^{a+bx}\left(\frac{2(b\cos(c+dx)+d\sin(c+dx))}{b^2+d^2} - \frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2} - \frac{b\cos(5(c+dx))+5d\sin(5(c+dx))}{b^2+25d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*((2*(b*Cos[c + d*x] + d*Sin[c + d*x]))/(b^2 + d^2) - (b*Cos[3*
(c + d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2) - (b*Cos[5*(c + d*x)] + 5*
d*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

Maple [A]
time = 0.24, size = 166, normalized size = 0.91

method	result
risch	$-\frac{e^{bx+a}e^{5idx}e^{5ic}}{32(5id+b)} - \frac{e^{bx+a}e^{3idx}e^{3ic}}{32(3id+b)} + \frac{e^{bx+a}e^{idx}e^{ic}}{16id+16b} + \frac{e^{bx+a}e^{-idx}e^{-ic}}{-16id+16b} - \frac{e^{bx+a}e^{-3idx}e^{-3ic}}{32(-3id+b)} - \frac{e^{bx+a}e^{-5idx}e^{-5ic}}{32(-5id+b)}$
default	$\frac{b\,e^{bx+a}\cos(dx+c)}{8b^2+8d^2} - \frac{b\,e^{bx+a}\cos(3dx+3c)}{16(b^2+9d^2)} - \frac{b\,e^{bx+a}\cos(5dx+5c)}{16(b^2+25d^2)} + \frac{d\,e^{bx+a}\sin(dx+c)}{8b^2+8d^2} - \frac{3d\,e^{bx+a}\sin(3dx+3c)}{16(b^2+9d^2)} - \frac{5d\,e^{bx+a}\sin}{16(b^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*sin(
d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp(b*
x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. 2(165) = 330.
time = 0.33, size = 1144, normalized size = 6.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/32*((b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(5*c)*e^a +
 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5*e^a*sin(5*c))*cos(
5*d*x)*e^(b*x) + (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(
5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c) - 45*d^5*e^a*sin(
5*c))*cos(5*d*x + 10*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e
^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b^2*d^3*e^a*sin(5*c)
 - 75*d^5*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b
^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^a*sin(5*c) + 78*b^2
*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) - 2*(b^5*
cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c)*e^a - b^4*d*e^a
*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*c))*cos(d*x + 6*c)*
e^(b*x) - 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c
)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 225*d^5*e^a*sin(5*c)
)*cos(d*x - 4*c)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a
+ 45*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^
4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos
(5*c)*e^a + 45*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c
) + 9*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (3*b^4*d*cos(5*c)*e^a
 + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^
3*d^2*e^a*sin(5*c) + 25*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(3*d*x + 8*c) + (3*b
^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a
*sin(5*c) - 26*b^3*d^2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(3*
d*x - 2*c) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(
5*c)*e^a + b^5*e^a*sin(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5
*c))*e^(b*x)*sin(d*x + 6*c) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e
^a + 225*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 22
5*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(d*x - 4*c))/(b^6*cos(5*c)^2 + b^6*sin(5*c
)^2 + 225*(cos(5*c)^2 + sin(5*c)^2)*d^6 + 259*(b^2*cos(5*c)^2 + b^2*sin(5*c
)^2)*d^4 + 35*(b^4*cos(5*c)^2 + b^4*sin(5*c)^2)*d^2)
```

Fricas [A]
time = 1.44, size = 200, normalized size = 1.09

$$\frac{\left(6\,b^2d^3+30\,d^5-5\left(b^4d+10\,b^2d^3+9\,d^5\right)\cos\left(dx+c\right)^4+3\left(b^4d+6\,b^2d^3+5\,d^5\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(\left(b^5+10\,b^3d^2+9\,bd^4\right)\cos\left(dx+c\right)^5-\left(b^5+6\,b^3d^2+5\,bd^4\right)\cos\left(dx+c\right)^3-6\left(b^3d^2+5\,bd^4\right)\cos\left(dx+c\right)\right)e^{(bx+a)}}{b^6+35\,b^4d^2+259\,b^2d^4+225\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*d^3 + 30*d^5 - 5*(b^4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^4 + 3*(b
^4*d + 6*b^2*d^3 + 5*d^5)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) - ((b^5
+ 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^5 - (b^5 + 6*b^3*d^2 + 5*b*d^4)*cos(d*
x + c)^3 - 6*(b^3*d^2 + 5*b*d^4)*cos(d*x + c))*e^(b*x + a))/(b^6 + 35*b^4*d
^2 + 259*b^2*d^4 + 225*d^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 30.05, size = 2958, normalized size = 16.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)
*exp(-5*I*d*x)*sin(c + d*x)**5/32 - 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**
4*cos(c + d*x)/32 + 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)
**2/16 + 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 - 5*I*
x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp(-5*I*
d*x)*cos(c + d*x)**5/32 - 47*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**5/(960*d) +
 41*I*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(192*d) + exp(a)*ex
p(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - I*exp(a)*exp(-5*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)**3/(6*d) - 25*exp(a)*exp(-5*I*d*x)*sin(c + d*x
)*cos(c + d*x)**4/(192*d) + 31*I*exp(a)*exp(-5*I*d*x)*cos(c + d*x)**5/(960*
d), Eq(b, -5*I*d)), (I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**5/32 + 3*x*exp(
a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 - I*x*exp(a)*exp(-3*I*d*x)
*sin(c + d*x)**3*cos(c + d*x)**2/16 + x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**
2*cos(c + d*x)**3/16 - 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)
**4/32 - x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/32 - 23*exp(a)*exp(-3*I*d*x
)*sin(c + d*x)**5/(192*d) + 25*I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c
 + d*x)/(64*d) + exp(a)*exp(-3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(3*d)
 + I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(6*d) + 9*exp(a)*
exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) - 7*I*exp(a)*exp(-3*I*d*x
)*cos(c + d*x)**5/(192*d), Eq(b, -3*I*d)), (I*x*exp(a)*exp(-I*d*x)*sin(c +
d*x)**5/16 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)**4*cos(c + d*x)/16 + I*x*exp
(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/8 + x*exp(a)*exp(-I*d*x)*si
n(c + d*x)**2*cos(c + d*x)**3/8 + I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c
 + d*x)**4/16 + x*exp(a)*exp(-I*d*x)*cos(c + d*x)**5/16 + 13*exp(a)*exp(-I*
d*x)*sin(c + d*x)**5/(96*d) - 7*I*exp(a)*exp(-I*d*x)*sin(c + d*x)**4*cos(c
+ d*x)/(96*d) + exp(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) -
I*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(6*d) - exp(a)*exp(-I*
d*x)*sin(c + d*x)*cos(c + d*x)**4/(96*d) - 5*I*exp(a)*exp(-I*d*x)*cos(c + d
*x)**5/(96*d), Eq(b, -I*d)), (-I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**5/16 + x
*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(c + d*x)/16 - I*x*exp(a)*exp(I*d*x)*
sin(c + d*x)**3*cos(c + d*x)**2/8 + x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos
(c + d*x)**3/8 - I*x*exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**4/16 + x*
exp(a)*exp(I*d*x)*cos(c + d*x)**5/16 + 13*exp(a)*exp(I*d*x)*sin(c + d*x)**5
/(96*d) + 7*I*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(96*d) + exp(a
)*exp(I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + I*exp(a)*exp(I*d*x)*si
n(c + d*x)**2*cos(c + d*x)**3/(6*d) - exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c
+ d*x)**4/(96*d) + 5*I*exp(a)*exp(I*d*x)*cos(c + d*x)**5/(96*d), Eq(b, I*d)
```

```
), (-I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**5/32 + 3*x*exp(a)*exp(3*I*d*x)*s
in(c + d*x)**4*cos(c + d*x)/32 + I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3*co
s(c + d*x)**2/16 + x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16
 + 3*I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp
(3*I*d*x)*cos(c + d*x)**5/32 - 23*exp(a)*exp(3*I*d*x)*sin(c + d*x)**5/(192*
d) - 25*I*exp(a)*exp(3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(64*d) + exp(a)*
exp(3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - I*exp(a)*exp(3*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)**3/(6*d) + 9*exp(a)*exp(3*I*d*x)*sin(c + d*x)*
cos(c + d*x)**4/(64*d) + 7*I*exp(a)*exp(3*I*d*x)*cos(c + d*x)**5/(192*d), E
q(b, 3*I*d)), (I*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**5/32 - 5*x*exp(a)*exp(
5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 - 5*I*x*exp(a)*exp(5*I*d*x)*sin(c
+ d*x)**3*cos(c + d*x)**2/16 + 5*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**2*cos(
c + d*x)**3/16 + 5*I*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32
- x*exp(a)*exp(5*I*d*x)*cos(c + d*x)**5/32 - 47*exp(a)*exp(5*I*d*x)*sin(c +
 d*x)**5/(960*d) - 41*I*exp(a)*exp(5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(1
92*d) + exp(a)*exp(5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + I*exp(a
)*exp(5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(6*d) - 25*exp(a)*exp(5*I*d*
x)*sin(c + d*x)*cos(c + d*x)**4/(192*d) - 31*I*exp(a)*exp(5*I*d*x)*cos(c +
d*x)**5/(960*d), Eq(b, 5*I*d)), (b**5*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c
 + d*x)**3/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 3*b**4*d*exp(
a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 + 35*b**4*d**2 + 259*b**2
*d**4 + 225*d**6) - 2*b**4*d*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**4/(
b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 6*b**3*d**2*exp(a)*exp(b*
x)*sin(c + d*x)**4*cos(c + d*x)/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*
d**6) + 18*b**3*d**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/(b**6
+ 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 2*b**3*d**2*exp(a)*exp(b*x)*co
s(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 6*b**2*d**
3*exp(a)*exp(b*x)*sin(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 22
5*d**6) + 30*b**2*d**3*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**
6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) - ...
```

Giac [A]

time = 0.42, size = 152, normalized size = 0.83

$$-\frac{1}{16}\left(\frac{b\cos\left(5\,dx+5\,c\right)}{b^2+25\,d^2}+\frac{5\,d\sin\left(5\,dx+5\,c\right)}{b^2+25\,d^2}\right)e^{(bx+a)}-\frac{1}{16}\left(\frac{b\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}+\frac{3\,d\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)}+\frac{1}{8}\left(\frac{b\cos\left(dx+c\right)}{b^2+d^2}+\frac{d\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/16*(b*cos(5*d*x + 5*c)/(b^2 + 25*d^2) + 5*d*sin(5*d*x + 5*c)/(b^2 + 25*d
^2))*e^(b*x + a) - 1/16*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x +
 3*c)/(b^2 + 9*d^2))*e^(b*x + a) + 1/8*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(
d*x + c)/(b^2 + d^2))*e^(b*x + a)
```

Mupad [B]

time = 3.71, size = 255, normalized size = 1.39

$$\frac{e^{a+bx}\,(\cos(dx)-\sin(dx)\,1i)\,(\cos(c)-\sin(c)\,1i)}{16\,(b-d\,1i)}-\frac{e^{a+bx}\,(\cos(3\,d\,x)+\sin(3\,d\,x)\,1i)\,(\cos(3\,c)+\sin(3\,c)\,1i)\,1i}{32\,(-3\,d+b\,1i)}-\frac{e^{a+bx}\,(\cos(5\,d\,x)+\sin(5\,d\,x)\,1i)\,(\cos(5\,c)+\sin(5\,c)\,1i)\,1i}{32\,(-5\,d+b\,1i)}+\frac{e^{a+bx}\,(\cos(dx)+\sin(dx)\,1i)\,(\cos(c)+\sin(c)\,1i)\,1i}{16\,(-d+b\,1i)}-\frac{e^{a+bx}\,(\cos(3\,d\,x)-\sin(3\,d\,x)\,1i)\,(\cos(3\,c)-\sin(3\,c)\,1i)}{32\,(b-d\,3i)}-\frac{e^{a+bx}\,(\cos(5\,d\,x)-\sin(5\,d\,x)\,1i)\,(\cos(5\,c)-\sin(5\,c)\,1i)}{32\,(b-d\,5i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(16*(b - d*1i)
) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)
/(32*(b*1i - 3*d)) - (exp(a + b*x)*(cos(5*d*x) + sin(5*d*x)*1i)*(cos(5*c) +
 sin(5*c)*1i)*1i)/(32*(b*1i - 5*d)) + (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i
)*(cos(c) + sin(c)*1i)*1i)/(16*(b*1i - d)) - (exp(a + b*x)*(cos(3*d*x) - si
n(3*d*x)*1i)*(cos(3*c) - sin(3*c)*1i))/(32*(b - d*3i)) - (exp(a + b*x)*(cos
(5*d*x) - sin(5*d*x)*1i)*(cos(5*c) - sin(5*c)*1i))/(32*(b - d*5i))
```

3.46 $\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx) \, dx$

Optimal. Leaf size=129

$$-\frac{3de^{a+bx}\cos(2c+2dx)}{16(b^2+4d^2)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16(b^2+36d^2)}+\frac{3be^{a+bx}\sin(2c+2dx)}{32(b^2+4d^2)}-\frac{be^{a+bx}\sin(6c+6dx)}{32(b^2+36d^2)}$$

```
[Out] -3/16*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+3/16*d*exp(b*x+a)*cos(6*d*x+6
*c)/(b^2+36*d^2)+3/32*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/32*b*exp(b*
x+a)*sin(6*d*x+6*c)/(b^2+36*d^2)
```

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4557, 4517}

$$\frac{3be^{a+bx}\sin(2c+2dx)}{32(b^2+4d^2)}-\frac{be^{a+bx}\sin(6c+6dx)}{32(b^2+36d^2)}-\frac{3de^{a+bx}\cos(2c+2dx)}{16(b^2+4d^2)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16(b^2+36d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (-3*d*E^(a + b*x)*Cos[2*c + 2*d*x])/(16*(b^2 + 4*d^2)) + (3*d*E^(a + b*x)*C
os[6*c + 6*d*x])/(16*(b^2 + 36*d^2)) + (3*b*E^(a + b*x)*Sin[2*c + 2*d*x])/(
32*(b^2 + 4*d^2)) - (b*E^(a + b*x)*Sin[6*c + 6*d*x])/(32*(b^2 + 36*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \int \left(\frac{3}{32}e^{a+bx}\sin(2c+2dx)-\frac{1}{32}e^{a+bx}\sin(6c+6dx)\right)dx$$
$$= -\left(\frac{1}{32}\int e^{a+bx}\sin(6c+6dx)\,dx\right)+\frac{3}{32}\int e^{a+bx}\sin(2c+2dx)\,dx$$
$$= -\frac{3de^{a+bx}\cos(2c+2dx)}{16\,(b^2+4d^2)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16\,(b^2+36d^2)}+\frac{3be^{a+bx}\sin(2c+}{32\,(b^2+4d^2)}$$

Mathematica [A]
time = 0.96, size = 111, normalized size = 0.86

$$\frac{e^{a+bx}(-6d(b^2+36d^2)\cos(2(c+dx))+6d(b^2+4d^2)\cos(6(c+dx))-2b(-b^2-52d^2+(b^2+4d^2)\cos(4(c+dx)))\sin(2(c+dx)))}{32\,(b^4+40b^2d^2+144d^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(-6*d*(b^2 + 36*d^2)*Cos[2*(c + d*x)] + 6*d*(b^2 + 4*d^2)*Cos[
6*(c + d*x)] - 2*b*(-b^2 - 52*d^2 + (b^2 + 4*d^2)*Cos[4*(c + d*x)])*Sin[2*(
c + d*x)]))/(32*(b^4 + 40*b^2*d^2 + 144*d^4))
```

Maple [A]
time = 0.27, size = 118, normalized size = 0.91

method	result	size
risch	$\frac{i\mathrm{e}^{bx+a}\mathrm{e}^{6idx}\mathrm{e}^{6ic}}{384id+64b}-\frac{3i\mathrm{e}^{bx+a}\mathrm{e}^{2idx}\mathrm{e}^{2ic}}{64(2id+b)}+\frac{3i\mathrm{e}^{bx+a}\mathrm{e}^{-2idx}\mathrm{e}^{-2ic}}{64(-2id+b)}-\frac{i\mathrm{e}^{bx+a}\mathrm{e}^{-6idx}\mathrm{e}^{-6ic}}{64(-6id+b)}$	114
default	$-\frac{3d\,\mathrm{e}^{bx+a}\cos(2dx+2c)}{16(b^2+4d^2)}+\frac{3d\,\mathrm{e}^{bx+a}\cos(6dx+6c)}{16(b^2+36d^2)}+\frac{3b\,\mathrm{e}^{bx+a}\sin(2dx+2c)}{32(b^2+4d^2)}-\frac{b\,\mathrm{e}^{bx+a}\sin(6dx+6c)}{32(b^2+36d^2)}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+3/16*d*exp(b*x+a)*cos(6*d*x+6
*c)/(b^2+36*d^2)+3/32*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/32*b*exp(b*
x+a)*sin(6*d*x+6*c)/(b^2+36*d^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.
time = 0.30, size = 550, normalized size = 4.26

$$\frac{(6b^2d\cos(6c)e^a+24d^3\cos(6c)e^a-b^3e^a\sin(6c)-4bd^2e^a\sin(6c))\cos(6dx)e^{(bx)}+(6b^2d\cos(6c)e^a+24d^3\cos(6c)e^a+b^3e^a\sin(6c)+4bd^2e^a\sin(6c))\cos(6dx+12c)e^{(bx)}-3(2b^2d\cos(6c)e^a+72d^3\cos(6c)e^a+b^3e^a\sin(6c)+36bd^2e^a\sin(6c))\cos(2dx+8c)e^{(bx)}-3(2b^2d\cos(6c)e^a+72d^3\cos(6c)e^a-b^3e^a\sin(6c)-36bd^2e^a\sin(6c))\cos(2dx-4c)e^{(bx)}-(b^3\cos(6c)e^a+4bd^2\cos(6c)e^a+6b^2de^a\sin(6c)+24d^3e^a\sin(6c))e^{(bx)}\sin(6dx)-(b^3\cos(6c)e^a+4bd^2\cos(6c)e^a-6b^2de^a\sin(6c)-24d^3e^a\sin(6c))e^{(bx)}\sin(6dx+12c)+3(b^3\cos(6c)e^a+36bd^2\cos(6c)e^a-2b^2de^a\sin(6c)-72d^3e^a\sin(6c))e^{(bx)}\sin(2dx+8c)+3(b^3\cos(6c)e^a+36bd^2\cos(6c)e^a+2b^2de^a\sin(6c)+72d^3e^a\sin(6c))e^{(bx)}\sin(2dx-4c)}{64\,(b^4\cos(6c)^2+b^4\sin(6c)^2+144\,(\cos(6c)^2+\sin(6c)^2)d^4+40\,(b^2\cos(6c)^2+b^2\sin(6c)^2)d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*((6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 4*b*
d^2*e^a*sin(6*c))*cos(6*d*x)*e^(b*x) + (6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6
*c)*e^a + b^3*e^a*sin(6*c) + 4*b*d^2*e^a*sin(6*c))*cos(6*d*x + 12*c)*e^(b*x
) - 3*(2*b^2*d*cos(6*c)*e^a + 72*d^3*cos(6*c)*e^a + b^3*e^a*sin(6*c) + 36*b
*d^2*e^a*sin(6*c))*cos(2*d*x + 8*c)*e^(b*x) - 3*(2*b^2*d*cos(6*c)*e^a + 72*
d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 36*b*d^2*e^a*sin(6*c))*cos(2*d*x - 4*
c)*e^(b*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*cos(6*c)*e^a + 6*b^2*d*e^a*sin(6*c
) + 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*c
os(6*c)*e^a - 6*b^2*d*e^a*sin(6*c) - 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x
 + 12*c) + 3*(b^3*cos(6*c)*e^a + 36*b*d^2*cos(6*c)*e^a - 2*b^2*d*e^a*sin(6*
c) - 72*d^3*e^a*sin(6*c))*e^(b*x)*sin(2*d*x + 8*c) + 3*(b^3*cos(6*c)*e^a +
36*b*d^2*cos(6*c)*e^a + 2*b^2*d*e^a*sin(6*c) + 72*d^3*e^a*sin(6*c))*e^(b*x)
*sin(2*d*x - 4*c))/(b^4*cos(6*c)^2 + b^4*sin(6*c)^2 + 144*(cos(6*c)^2 + sin
(6*c)^2)*d^4 + 40*(b^2*cos(6*c)^2 + b^2*sin(6*c)^2)*d^2)
```

Fricas [A]
time = 1.33, size = 156, normalized size = 1.21

$$-\frac{\left(\left(b^{3}+4\,bd^{2}\right)\cos\left(dx+c\right)^{5}-6\,bd^{2}\cos\left(dx+c\right)-\left(b^{3}+4\,bd^{2}\right)\cos\left(dx+c\right)^{3}\right)e^{\left(bx+a\right)}\sin\left(dx+c\right)-3\left(2\left(b^{2}d+4\,d^{3}\right)\cos\left(dx+c\right)^{6}+b^{2}d\cos\left(dx+c\right)^{2}-3\left(b^{2}d+4\,d^{3}\right)\cos\left(dx+c\right)^{4}+2\,d^{3}\right)e^{\left(bx+a\right)}}{b^{4}+40\,b^{2}d^{2}+144\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(((b^3 + 4*b*d^2)*cos(d*x + c)^5 - 6*b*d^2*cos(d*x + c) - (b^3 + 4*b*d^2)*
cos(d*x + c)^3)*e^(b*x + a)*sin(d*x + c) - 3*(2*(b^2*d + 4*d^3)*cos(d*x + c
)^6 + b^2*d*cos(d*x + c)^2 - 3*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 2*d^3)*e^(b
*x + a))/(b^4 + 40*b^2*d^2 + 144*d^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 66.44, size = 1991, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)
*exp(-6*I*d*x)*sin(c + d*x)**6/64 - 3*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**
5*cos(c + d*x)/32 + 15*I*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**4*cos(c + d*x
)**2/64 + 5*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/16 - 15*
I*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 - 3*x*exp(a)*ex
p(-6*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32 + I*x*exp(a)*exp(-6*I*d*x)*cos(
c + d*x)**6/64 - exp(a)*exp(-6*I*d*x)*sin(c + d*x)**6/(160*d) + 7*I*exp(a)*
exp(-6*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/(320*d) + 11*I*exp(a)*exp(-6*I*d
```

```
*x)*sin(c + d*x)**3*cos(c + d*x)**3/(96*d) + 7*I*exp(a)*exp(-6*I*d*x)*sin(c
 + d*x)*cos(c + d*x)**5/(320*d) + exp(a)*exp(-6*I*d*x)*cos(c + d*x)**6/(160
*d), Eq(b, -6*I*d)), (3*I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**6/64 + 3*x*e
xp(a)*exp(-2*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/32 + 3*I*x*exp(a)*exp(-2*I
*d*x)*sin(c + d*x)**4*cos(c + d*x)**2/64 + 3*x*exp(a)*exp(-2*I*d*x)*sin(c +
 d*x)**3*cos(c + d*x)**3/16 - 3*I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**2*co
s(c + d*x)**4/64 + 3*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32
 - 3*I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**6/64 - 3*exp(a)*exp(-2*I*d*x)*s
in(c + d*x)**6/(32*d) + 15*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**5*cos(c + d
*x)/(64*d) + 13*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/(32*
d) + 15*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/(64*d) + 3*exp(
a)*exp(-2*I*d*x)*cos(c + d*x)**6/(32*d), Eq(b, -2*I*d)), (-3*I*x*exp(a)*exp
(2*I*d*x)*sin(c + d*x)**6/64 + 3*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**5*cos(
c + d*x)/32 - 3*I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4*cos(c + d*x)**2/64
+ 3*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/16 + 3*I*x*exp(a)
*exp(2*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 + 3*x*exp(a)*exp(2*I*d*x)*
sin(c + d*x)*cos(c + d*x)**5/32 + 3*I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**6
/64 - 3*exp(a)*exp(2*I*d*x)*sin(c + d*x)**6/(32*d) - 15*I*exp(a)*exp(2*I*d*
x)*sin(c + d*x)**5*cos(c + d*x)/(64*d) - 13*I*exp(a)*exp(2*I*d*x)*sin(c + d
*x)**3*cos(c + d*x)**3/(32*d) - 15*I*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c
 + d*x)**5/(64*d) + 3*exp(a)*exp(2*I*d*x)*cos(c + d*x)**6/(32*d), Eq(b, 2*I
*d)), (I*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)**6/64 - 3*x*exp(a)*exp(6*I*d*x)
*sin(c + d*x)**5*cos(c + d*x)/32 - 15*I*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)*
*4*cos(c + d*x)**2/64 + 5*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)**3*cos(c + d*x
)**3/16 + 15*I*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 - 3
*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32 - I*x*exp(a)*exp(6*I
*d*x)*cos(c + d*x)**6/64 - exp(a)*exp(6*I*d*x)*sin(c + d*x)**6/(160*d) - 7*
I*exp(a)*exp(6*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/(320*d) - 11*I*exp(a)*ex
p(6*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/(96*d) - 7*I*exp(a)*exp(6*I*d*x)
*sin(c + d*x)*cos(c + d*x)**5/(320*d) + exp(a)*exp(6*I*d*x)*cos(c + d*x)**6
/(160*d), Eq(b, 6*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)
**3/(b**4 + 40*b**2*d**2 + 144*d**4) + 3*b**2*d*exp(a)*exp(b*x)*sin(c + d*x
)**4*cos(c + d*x)**2/(b**4 + 40*b**2*d**2 + 144*d**4) - 3*b**2*d*exp(a)*exp
(b*x)*sin(c + d*x)**2*cos(c + d*x)**4/(b**4 + 40*b**2*d**2 + 144*d**4) + 6*
b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**5*cos(c + d*x)/(b**4 + 40*b**2*d**2 +
144*d**4) + 16*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**3/(b**4
 + 40*b**2*d**2 + 144*d**4) + 6*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c +
 d*x)**5/(b**4 + 40*b**2*d**2 + 144*d**4) + 6*d**3*exp(a)*exp(b*x)*sin(c +
d*x)**6/(b**4 + 40*b**2*d**2 + 144*d**4) + 18*d**3*exp(a)*exp(b*x)*sin(c +
d*x)**4*cos(c + d*x)**2/(b**4 + 40*b**2*d**2 + 144*d**4) - 18*d**3*exp(a)*e
xp(b*x)*sin(c + d*x)**2*cos(c + d*x)**4/(b**4 + 40*b**2*d**2 + 144*d**4) -
6*d**3*exp(a)*exp(b*x)*cos(c + d*x)**6/(b**4 + 40*b**2*d**2 + 144*d**4), Tr
ue))
```

Giac [A]

time = 0.42, size = 111, normalized size = 0.86

$$\frac{1}{32}\left(\frac{6\,d\cos\left(6\,dx+6\,c\right)}{b^2+36\,d^2}-\frac{b\sin\left(6\,dx+6\,c\right)}{b^2+36\,d^2}\right)e^{(bx+a)}-\frac{3}{32}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*(6*d*cos(6*d*x + 6*c)/(b^2 + 36*d^2) - b*sin(6*d*x + 6*c)/(b^2 + 36*d^
2))*e^(b*x + a) - 3/32*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x +
2*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

Mupad [B]
time = 1.01, size = 178, normalized size = 1.38

$$-\frac{3\,\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)-\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)-\sin\left(2\,c\right)\,1{}\mathrm{i}\right)}{64\,\left(2\,d+b\,1{}\mathrm{i}\right)}+\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(6\,d\,x\right)-\sin\left(6\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(6\,c\right)-\sin\left(6\,c\right)\,1{}\mathrm{i}\right)}{64\,\left(6\,d+b\,1{}\mathrm{i}\right)}-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{64\,\left(b+d\,2{}\mathrm{i}\right)}+\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(6\,d\,x\right)+\sin\left(6\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(6\,c\right)+\sin\left(6\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{64\,\left(b+d\,6{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] (exp(a + b*x)*(cos(6*d*x) - sin(6*d*x)*1i)*(cos(6*c) - sin(6*c)*1i))/(64*(b
*1i + 6*d)) - (3*exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(
2*c)*1i))/(64*(b*1i + 2*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(c
os(2*c) + sin(2*c)*1i)*3i)/(64*(b + d*2i)) + (exp(a + b*x)*(cos(6*d*x) + si
n(6*d*x)*1i)*(cos(6*c) + sin(6*c)*1i)*1i)/(64*(b + d*6i))
```

3.47 $\int e^x x \sin(x)\,dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^x\cos(x)-\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)$$

```
[Out] 1/2*exp(x)*cos(x)-1/2*exp(x)*x*cos(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4517, 4553, 4518}

$$\frac{1}{2}e^x x\sin(x)+\frac{1}{2}e^x\cos(x)-\frac{1}{2}e^x x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Sin[x],x]
```

```
[Out] (E^x*Cos[x])/2 - (E^x*x*Cos[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int e^x x \sin(x)\,dx = -\frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x) - \int \left(-\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx$$
$$= -\frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x) + \frac{1}{2}\int e^x\cos(x)\,dx - \frac{1}{2}\int e^x\sin(x)\,dx$$
$$= \frac{1}{2}e^x\cos(x) - \frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x)$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.63

$$\frac{1}{2}e^x(\cos(x) - x\cos(x) + x\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Sin[x],x]
[Out] (E^x*(Cos[x] - x*Cos[x] + x*Sin[x]))/2
```

Maple [A]
time = 0.06, size = 19, normalized size = 0.63

method	result	size
default	$\left(-\frac{x}{2}+\frac{1}{2}\right)\mathrm{e}^{x}\cos\left(x\right)+\frac{\mathrm{e}^{x} x \sin(x)}{2}$	19
risch	$\left(-\frac{1}{8}-\frac{i}{8}\right)\left(-1+i+2x\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{8}+\frac{i}{8}\right)\left(-1-i+2x\right)\mathrm{e}^{(1-i)x}$	36
norman	$\frac{\mathrm{e}^{x} x\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x} x}{2}-\frac{\mathrm{e}^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{\mathrm{e}^{x} x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{\mathrm{e}^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*sin(x),x,method=_RETURNVERBOSE)
[Out] (-1/2*x+1/2)*exp(x)*cos(x)+1/2*exp(x)*x*sin(x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.57

$$-\frac{1}{2}\left(x-1\right)\cos\left(x\right)e^{x} + \frac{1}{2}\,xe^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="maxima")
[Out] -1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

Fricas [A]
time = 1.50, size = 17, normalized size = 0.57

$$-\frac{1}{2}\,(x-1)\cos\left(x\right)e^{x}+\frac{1}{2}\,xe^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="fricas")
[Out] -1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

Sympy [A]
time = 0.16, size = 27, normalized size = 0.90

$$\frac{xe^{x}\sin\left(x\right)}{2}-\frac{xe^{x}\cos\left(x\right)}{2}+\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x)
[Out] x*exp(x)*sin(x)/2 - x*exp(x)*cos(x)/2 + exp(x)*cos(x)/2
```

Giac [A]
time = 0.39, size = 16, normalized size = 0.53

$$-\frac{1}{2}\,((x-1)\cos\left(x\right)-x\sin\left(x\right))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="giac")
[Out] -1/2*((x - 1)*cos(x) - x*sin(x))*e^x
```

Mupad [B]
time = 0.08, size = 16, normalized size = 0.53

$$\frac{\mathrm{e}^{x}\left(\cos\left(x\right)-x\,\cos\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*sin(x),x)
[Out] (exp(x)*(cos(x) - x*cos(x) + x*sin(x)))/2
```

3.48 $\int e^x x^2 \sin(x)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{2}e^x\cos(x)+e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x^2\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+exp(x)*x*cos(x)-1/2*exp(x)*x^2*cos(x)-1/2*exp(x)*sin(x)+
1/2*exp(x)*x^2*sin(x)
```

Rubi [A]
time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{1}{2}e^x x^2\sin(x)-\frac{1}{2}e^x x^2\cos(x)-\frac{1}{2}e^x\sin(x)+e^x x\cos(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E
^x*x^2*Sin[x])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \sin(x)\,dx &= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\int x\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)\,dx\\
&= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\int\left(-\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)\right)\,dx\\
&= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)+\int e^x x\cos(x)\,dx-\int e^x x\sin(x)\,dx\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)+\int\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)\,dx-\int\left(\frac{1}{2}\right.\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\left(\frac{1}{2}\int e^x\cos(x)\,dx\right)\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\left(\frac{1}{4}e^x\cos(x)+\frac{1}{4}e^x\sin(x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 0.50

$$\frac{1}{2}e^x\left(-(-1+x)^2\cos(x)+\left(-1+x^2\right)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Sin[x],x]
```

```
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Maple [A]
time = 0.06, size = 27, normalized size = 0.54

method	result	size

default	$\left(-\frac{1}{2}x^2+x-\frac{1}{2}\right)\mathrm{e}^x\cos\left(x\right)+\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^x\sin\left(x\right)$	27
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^2+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^2-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^x x+\mathrm{e}^x x^2\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^x x^2}{2}-\mathrm{e}^x\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\mathrm{e}^x x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{\mathrm{e}^x x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\mathrm{e}^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*x^2+x-1/2)*exp(x)*cos(x)+(1/2*x^2-1/2)*exp(x)*sin(x)
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.52

$$-\frac{1}{2}\left(x^2-2\,x+1\right)\cos\left(x\right)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Fricas [A]
time = 2.00, size = 26, normalized size = 0.52

$$-\frac{1}{2}\left(x^2-2\,x+1\right)\cos\left(x\right)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Sympy [A]
time = 0.31, size = 48, normalized size = 0.96

$$\frac{x^2e^x\sin\left(x\right)}{2}-\frac{x^2e^x\cos\left(x\right)}{2}+xe^x\cos\left(x\right)-\frac{e^x\sin\left(x\right)}{2}-\frac{e^x\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Giac [A]
time = 0.42, size = 25, normalized size = 0.50

$$-\frac{1}{2}\left(\left(x^2-2\,x+1\right)\cos\left(x\right)-\left(x^2-1\right)\sin\left(x\right)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

Mupad [B]
time = 2.38, size = 21, normalized size = 0.42

$$\frac{\mathrm{e}^x\,\left(x-1\right)\,\left(\cos\left(x\right)+\sin\left(x\right)-x\,\cos\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x)*sin(x),x)
```

```
[Out] (exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

3.49 $\int e^x x \cos(x) \, dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)$$

```
[Out] 1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4518, 4554, 4517}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int e^x x\cos(x)\,dx &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)\,dx\\ &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\ &= \frac{1}{2}e^x x\cos(x)-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 0.60

$$\frac{1}{2}e^x(x\cos(x)+(-1+x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.67

method	result	size
default	$\frac{e^x x\cos(x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)(-1+i+2x)\,e^{(1+i)x}+\left(\frac{1}{8}+\frac{i}{8}\right)(-1-i+2x)\,e^{(1-i)x}$	36
norman	$\frac{e^x x\tan\left(\frac{x}{2}\right)+\frac{e^x x}{2}-e^x\tan\left(\frac{x}{2}\right)-\frac{e^x x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.57

$$\frac{1}{2}\,x\cos(x)\,e^x+\frac{1}{2}\,(x-1)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A]
time = 1.46, size = 17, normalized size = 0.57

$$\frac{1}{2}\,x\cos\left(x\right)e^{x}+\frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A]
time = 0.16, size = 27, normalized size = 0.90

$$\frac{xe^{x}\sin\left(x\right)}{2}+\frac{xe^{x}\cos\left(x\right)}{2}-\frac{e^{x}\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Giac [A]
time = 0.40, size = 15, normalized size = 0.50

$$\frac{1}{2}\,(x\cos\left(x\right)+(x-1)\sin\left(x\right))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

Mupad [B]
time = 2.35, size = 17, normalized size = 0.57

$$\frac{\mathrm{e}^{x}\,\left(x\,\cos\left(x\right)-\sin\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*cos(x),x)
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.50 $\int e^x x^2 \cos(x)\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^xx^2\cos(x)+\frac{1}{2}e^x\sin(x)-e^xx\sin(x)+\frac{1}{2}e^xx^2\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*x^2*cos(x)+1/2*exp(x)*sin(x)-exp(x)*x*sin(x)+
1/2*exp(x)*x^2*sin(x)
```

Rubi [A]
time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4518, 4554, 14, 4517, 4553}

$$\frac{1}{2}e^xx^2\sin(x)+\frac{1}{2}e^xx^2\cos(x)-e^xx\sin(x)+\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2*Cos[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*x^2*Cos[x])/2 + (E^x*Sin[x])/2 - E^x*x*Sin[x] + (E
^x*x^2*Sin[x])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \cos(x)\, dx &= \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\int x\left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)\, dx \\
&= \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\int \left(\frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x)\right)\, dx \\
&= \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - \int e^x x \cos(x)\, dx - \int e^x x \sin(x)\, dx \\
&= \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x) + \int \left(-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)\, dx + \int \Bigg(\\
&= \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x) + 2\left(\frac{1}{2}\int e^x \sin(x)\, dx\right) \\
&= \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x) + 2\left(-\frac{1}{4}e^x \cos(x) + \frac{1}{4}e^x \sin(x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.45

$$\frac{1}{2}e^x(-1+x)((1+x)\cos(x)+(-1+x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Cos[x],x]
```

```
[Out] (E^x*(-1 + x)*((1 + x)*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A]
time = 0.06, size = 28, normalized size = 0.55

method	result	size

default	$\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^x\cos\left(x\right)-\left(-\frac{1}{2}x^2+x-\frac{1}{2}\right)\mathrm{e}^x\sin\left(x\right)$	28
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)\left(x^2+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(\frac{1}{4}+\frac{i}{4}\right)\left(x^2-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^x\tan\left(\frac{x}{2}\right)+\mathrm{e}^x x^2\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^x x^2}{2}+\frac{\mathrm{e}^x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-2\,\mathrm{e}^x x\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^x x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\mathrm{e}^x}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2*cos(x),x,method=_RETURNVERBOSE)
[Out] (1/2*x^2-1/2)*exp(x)*cos(x)-(-1/2*x^2+x-1/2)*exp(x)*sin(x)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.51

$$\frac{1}{2}\left(x^2-1\right)\cos\left(x\right)e^x+\frac{1}{2}\left(x^2-2\,x+1\right)e^x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

Fricas [A]
time = 1.52, size = 26, normalized size = 0.51

$$\frac{1}{2}\left(x^2-1\right)\cos\left(x\right)e^x+\frac{1}{2}\left(x^2-2\,x+1\right)e^x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

Sympy [A]
time = 0.32, size = 48, normalized size = 0.94

$$\frac{x^2e^x\sin\left(x\right)}{2}+\frac{x^2e^x\cos\left(x\right)}{2}-xe^x\sin\left(x\right)+\frac{e^x\sin\left(x\right)}{2}-\frac{e^x\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2*cos(x),x)
[Out] x**2*exp(x)*sin(x)/2 + x**2*exp(x)*cos(x)/2 - x*exp(x)*sin(x) + exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Giac [A]
time = 0.38, size = 24, normalized size = 0.47

$$\frac{1}{2}\left(\left(x^2-1\right)\cos\left(x\right)+\left(x^2-2\,x+1\right)\sin\left(x\right)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="giac")
[Out] 1/2*((x^2 - 1)*cos(x) + (x^2 - 2*x + 1)*sin(x))*e^x
```

Mupad [B]
time = 2.36, size = 22, normalized size = 0.43

$$\frac{\mathrm{e}^x\,\left(x-1\right)\,\left(\cos\left(x\right)-\sin\left(x\right)+x\,\cos\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x)*cos(x),x)
[Out] (exp(x)*(x - 1)*(cos(x) - sin(x) + x*cos(x) + x*sin(x)))/2
```

3.51 $\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx$

Optimal. Leaf size=27

$$-\frac{23}{25}e^{3x}\cos(4x)-\frac{14}{25}e^{3x}\sin(4x)$$

```
[Out] -23/25*exp(3*x)*cos(4*x)-14/25*exp(3*x)*sin(4*x)
```

Rubi [A]
time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6874, 4518, 4517}

$$-\frac{14}{25}e^{3x}\sin(4x)-\frac{23}{25}e^{3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

```
[Out] (-23*E^(3*x)*Cos[4*x])/25 - (14*E^(3*x)*Sin[4*x])/25
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx &= \int \left(-5e^{3x}\cos(4x)+2e^{3x}\sin(4x)\right)\,dx \\ &= 2\int e^{3x}\sin(4x)\,dx-5\int e^{3x}\cos(4x)\,dx \\ &= -\frac{23}{25}e^{3x}\cos(4x)-\frac{14}{25}e^{3x}\sin(4x)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 22, normalized size = 0.81

$$-\frac{1}{25}e^{3x}(23\cos(4x)+14\sin(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

```
[Out] -1/25*(E^(3*x)*(23*Cos[4*x] + 14*Sin[4*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(21) = 42$.
time = 0.10, size = 103, normalized size = 3.81

method	result
risch	$-\frac{23\,\mathrm{e}^{(3+4i)x}}{50}+\frac{7i\mathrm{e}^{(3+4i)x}}{25}-\frac{23\,\mathrm{e}^{(3-4i)x}}{50}-\frac{7i\mathrm{e}^{(3-4i)x}}{25}$
norman	$\frac{-\frac{28\,\mathrm{e}^{3x}\tan(2x)}{25}+\frac{23\,\mathrm{e}^{3x}\left(\tan^2(2x)\right)}{25}-\frac{23\,\mathrm{e}^{3x}}{25}}{1+\tan^2(2x)}$
default	$-\frac{8(3\cos(x)+4\sin(x))\mathrm{e}^{3x}\left(\cos^3(x)\right)}{5}+\frac{8(3\cos(x)+2\sin(x))\mathrm{e}^{3x}\cos(x)}{5}-\frac{3\,\mathrm{e}^{3x}}{5}-\frac{8\,\mathrm{e}^{3x}\cos(4x)}{25}+\frac{6\,\mathrm{e}^{3x}\sin(4x)}{25}-\frac{8\,\mathrm{e}^{3x}\cos(2x}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -8/5*(3*cos(x)+4*sin(x))*exp(3*x)*cos(x)^3+8/5*(3*cos(x)+2*sin(x))*exp(3*x)
*cos(x)-3/5*exp(x)^3-8/25*exp(3*x)*cos(4*x)+6/25*exp(3*x)*sin(4*x)-8/13*exp
(3*x)*cos(2*x)+12/13*exp(3*x)*sin(2*x)-4/13*exp(3*x)*(3*sin(2*x)-2*cos(2*x)
)
```

Maxima [A]
time = 0.27, size = 39, normalized size = 1.44

$$-\frac{2}{25}\,(4\,\cos\left(4\,x\right)-3\,\sin\left(4\,x\right))e^{(3\,x)}-\frac{1}{5}\,(3\,\cos\left(4\,x\right)+4\,\sin\left(4\,x\right))e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="maxima")
```

```
[Out] -2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^(
3*x)
```

Fricas [A]
time = 1.19, size = 21, normalized size = 0.78

$$-\frac{23}{25}\cos\left(4\,x\right)e^{\left(3\,x\right)}-\frac{14}{25}\,e^{\left(3\,x\right)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="fricas")
```

```
[Out] -23/25*cos(4*x)*e^(3*x) - 14/25*e^(3*x)*sin(4*x)
```

Sympy [A]
time = 0.10, size = 27, normalized size = 1.00

$$-\frac{14e^{3x}\sin\left(4x\right)}{25}-\frac{23e^{3x}\cos\left(4x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x)
```

```
[Out] -14*exp(3*x)*sin(4*x)/25 - 23*exp(3*x)*cos(4*x)/25
```

Giac [A]
time = 0.42, size = 39, normalized size = 1.44

$$-\frac{2}{25}\left(4\,\cos\left(4\,x\right)-3\,\sin\left(4\,x\right)\right)e^{\left(3\,x\right)}-\frac{1}{5}\left(3\,\cos\left(4\,x\right)+4\,\sin\left(4\,x\right)\right)e^{\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="giac")
```

```
[Out] -2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^(
3*x)
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.70

$$-\frac{\mathrm{e}^{3\,x}\left(23\,\cos\left(4\,x\right)+14\,\sin\left(4\,x\right)\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(3*x)*(5*cos(4*x) - 2*sin(4*x)),x)
```

```
[Out] -(exp(3*x)*(23*cos(4*x) + 14*sin(4*x)))/25
```

3.52 $\int (e^{-x}\sin(x)+e^{x}\sin(x))\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^{x}\sin(x)$$

```
[Out] -1/2*cos(x)/exp(x)-1/2*exp(x)*cos(x)-1/2*sin(x)/exp(x)+1/2*exp(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {4517}

$$-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^{x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/E^x + E^x*Sin[x],x]
[Out] -1/2*Cos[x]/E^x - (E^x*Cos[x])/2 - Sin[x]/(2*E^x) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int (e^{-x}\sin(x)+e^{x}\sin(x))\,dx &= \int e^{-x}\sin(x)\,dx+\int e^{x}\sin(x)\,dx\\ &= -\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^{x}\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 33, normalized size = 0.80

$$-\frac{1}{2}e^{x}\left(1+e^{-2x}\right)\cos(x)-\frac{1}{2}e^{x}\left(-1+e^{-2x}\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/E^x + E^x*Sin[x],x]
```

[Out] -1/2*(E^x*(1 + E^(-2*x))*Cos[x]) - (E^x*(-1 + E^(-2*x))*Sin[x])/2

Maple [A]
time = 0.08, size = 30, normalized size = 0.73

method	result	size
default	$-\frac{e^{x} \cos(x)}{2}+\frac{e^{x} \sin(x)}{2}-\frac{e^{-x} \cos(x)}{2}-\frac{e^{-x} \sin(x)}{2}$	30
norman	$\frac{\left(-\frac{1}{2}+e^{2x} \tan\left(\frac{x}{2}\right)-\frac{e^{2x}}{2}+\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{e^{2x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)\right) e^{-x}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	59
risch	$-\frac{e^{(-1+i)x}}{4}+\frac{i e^{(-1+i)x}}{4}-\frac{e^{(-1-i)x}}{4}-\frac{i e^{(-1-i)x}}{4}-\frac{e^{(1+i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{i e^{(1-i)x}}{4}$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/exp(x)+exp(x)*sin(x),x,method=_RETURNVERBOSE)

[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)-1/2*exp(-x)*cos(x)-1/2*exp(-x)*sin(x)

Maxima [A]
time = 0.27, size = 23, normalized size = 0.56

$$-\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{\left(-x\right)}-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="maxima")

[Out] -1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x

Fricas [A]
time = 1.47, size = 26, normalized size = 0.63

$$-\frac{1}{2}\left(\cos\left(x\right)e^{\left(2\,x\right)}-\left(e^{\left(2\,x\right)}-1\right)\sin\left(x\right)+\cos\left(x\right)\right)e^{\left(-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*(cos(x)*e^(2*x) - (e^(2*x) - 1)*sin(x) + cos(x))*e^(-x)

Sympy [A]
time = 0.18, size = 32, normalized size = 0.78

$$\frac{e^{x}\sin\left(x\right)}{2}-\frac{e^{x}\cos\left(x\right)}{2}-\frac{e^{-x}\sin\left(x\right)}{2}-\frac{e^{-x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2 - exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

Giac [A]
time = 0.39, size = 23, normalized size = 0.56

$$-\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{\left(-x\right)}-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

Mupad [B]
time = 2.41, size = 31, normalized size = 0.76

$$-\mathrm{e}^{-x}\left(\frac{\cos\left(x\right)}{2}+\frac{\sin\left(x\right)}{2}+\frac{\mathrm{e}^{2\,x}\,\cos\left(x\right)}{2}-\frac{\mathrm{e}^{2\,x}\,\sin\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x) + exp(-x)*sin(x),x)
```

```
[Out] -exp(-x)*(cos(x)/2 + sin(x)/2 + (exp(2*x)*cos(x))/2 - (exp(2*x)*sin(x))/2)
```

3.53 $\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] I*F^(b*x+a)/b/e/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F
)/d],I*exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4547, 4527, 2225, 2283}

$$\frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4547

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx &= -\frac{\int F^{a+bx}\tan\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{e}\\ &= -\frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)\,dx}{e}\\ &= \frac{i\int F^{a+bx}\,dx}{e}-\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx}{e}\\ &= \frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{be\log(F)}\end{aligned}$$

Mathematica [A]
time = 1.61, size = 64, normalized size = 0.78

$$-\frac{iF^{a+bx}\left(-1+2\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*
Log[F])/d, I*E^(I*(c + d*x))]))/(b*e*Log[F])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{F^{bx+a}\cos\left(dx+c\right)}{e+e\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

```
[Out] int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) +
(F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 + (F^a*b^2*log(F)^2 + F
^a*d^2)*F^(b*x)*sin(d*x + c)^2 - (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) - 2*(
F^a*b^3*d*e*log(F)^3 + F^a*b*d^3*e*log(F) + (F^a*b^3*d*e*log(F)^3 + F^a*b*d
^3*e*log(F))*cos(d*x + c)^2 + (F^a*b^3*d*e*log(F)^3 + F^a*b*d^3*e*log(F))*s
in(d*x + c)^2 + 2*(F^a*b^3*d*e*log(F)^3 + F^a*b*d^3*e*log(F))*sin(d*x + c))
*integrate((2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F)*sin(2*d*x +
2*c) - F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) + F^(b*x)*d)/(
b^2*e*log(F)^2 + (b^2*e*log(F)^2 + d^2*e)*cos(2*d*x + 2*c)^2 + 4*(b^2*e*log
(F)^2 + d^2*e)*cos(d*x + c)^2 + d^2*e + 4*(b^2*e*log(F)^2 + d^2*e)*cos(d*x
+ c)*sin(2*d*x + 2*c) + (b^2*e*log(F)^2 + d^2*e)*sin(2*d*x + 2*c)^2 + 4*(b^
2*e*log(F)^2 + d^2*e)*sin(d*x + c)^2 - 2*(b^2*e*log(F)^2 + d^2*e + 2*(b^2*e
*log(F)^2 + d^2*e)*sin(d*x + c))*cos(2*d*x + 2*c) + 4*(b^2*e*log(F)^2 + d^2
*e)*sin(d*x + c)), x))/(b^3*e*log(F)^3 + b*d^2*e*log(F) + (b^3*e*log(F)^3 +
 b*d^2*e*log(F))*cos(d*x + c)^2 + (b^3*e*log(F)^3 + b*d^2*e*log(F))*sin(d*x
 + c)^2 + 2*(b^3*e*log(F)^3 + b*d^2*e*log(F))*sin(d*x + c))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^a F^{bx} \cos(c+dx)}{\sin(c+dx)+1}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

```
[Out] Integral(F**a*F**(b*x)*cos(c + d*x)/(sin(c + d*x) + 1), x)/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{a+b\,x}\,\cos\left(c+d\,x\right)}{e+e\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(a + b*x)*cos(c + d*x))/(e + e*sin(c + d*x)),x)
```

```
[Out] int((F^(a + b*x)*cos(c + d*x))/(e + e*sin(c + d*x)), x)
```

3.54 $\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-ie^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] -I*F^(b*x+a)/b/e/ln(F)+2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(
F)/d],-I*exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4547, 4527, 2225, 2283}

$$\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-ie^{i(c+dx)}\right)}{be\log(F)}-\frac{iF^{a+bx}}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, (-I)*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4547

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx &= \frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{e} \\
&= \frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx}{e} \\
&= -\frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx}{e} \\
&= -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-ie^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}$$

Mathematica [A]
time = 1.92, size = 64, normalized size = 0.78

$$\frac{iF^{a+bx}\left(-1+2\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
[Out] (I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Log
[F])/d, (-I)*E^(I*(c + d*x))]))/(b*e*Log[F])
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{F^{bx+a}\cos\left(dx+c\right)}{e-e\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
[Out] int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) -
(F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 - (F^a*b^2*log(F)^2 + F
^a*d^2)*F^(b*x)*sin(d*x + c)^2 + (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) + 2*(
F^a*b^3*d*e*log(F)^3 + F^a*b*d^3*e*log(F) + (F^a*b^3*d*e*log(F)^3 + F^a*b*d
^3*e*log(F))*cos(d*x + c)^2 + (F^a*b^3*d*e*log(F)^3 + F^a*b*d^3*e*log(F))*s
in(d*x + c)^2 - 2*(F^a*b^3*d*e*log(F)^3 + F^a*b*d^3*e*log(F))*sin(d*x + c))
*integrate(-(2*F^(b*x)*b*cos(d*x + c)*log(F) - F^(b*x)*b*log(F)*sin(2*d*x +
 2*c) + F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) - F^(b*x)*d)/
(b^2*e*log(F)^2 + (b^2*e*log(F)^2 + d^2*e)*cos(2*d*x + 2*c)^2 + 4*(b^2*e*lo
g(F)^2 + d^2*e)*cos(d*x + c)^2 + d^2*e - 4*(b^2*e*log(F)^2 + d^2*e)*cos(d*x
 + c)*sin(2*d*x + 2*c) + (b^2*e*log(F)^2 + d^2*e)*sin(2*d*x + 2*c)^2 + 4*(b
^2*e*log(F)^2 + d^2*e)*sin(d*x + c)^2 - 2*(b^2*e*log(F)^2 + d^2*e - 2*(b^2*
e*log(F)^2 + d^2*e)*sin(d*x + c))*cos(2*d*x + 2*c) - 4*(b^2*e*log(F)^2 + d^
2*e)*sin(d*x + c)), x))/(b^3*e*log(F)^3 + b*d^2*e*log(F) + (b^3*e*log(F)^3
+ b*d^2*e*log(F))*cos(d*x + c)^2 + (b^3*e*log(F)^3 + b*d^2*e*log(F))*sin(d*
x + c)^2 - 2*(b^3*e*log(F)^3 + b*d^2*e*log(F))*sin(d*x + c))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{F^{a} F^{bx} \cos{\left(c + dx \right)}}{\sin{\left(c + dx \right)} - 1}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

```
[Out] -Integral(F**a*F**(b*x)*cos(c + d*x)/(sin(c + d*x) - 1), x)/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{a+b\,x}\,\cos\left(c+d\,x\right)}{e-e\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(a + b*x)*cos(c + d*x))/(e - e*sin(c + d*x)),x)
[Out] int((F^(a + b*x)*cos(c + d*x))/(e - e*sin(c + d*x)), x)
```

3.55 $\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-e^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] -I*F^(b*x+a)/b/e/ln(F)+2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(
F)/d],-exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4548, 4527, 2225, 2283}

$$\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-e^{i(c+dx)}\right)}{be\log(F)}-\frac{iF^{a+bx}}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, -E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4548

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx &= \frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,dx}{e}\\
&= \frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)\,dx}{e}\\
&= -\frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{e}\\
&= -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-e^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}$$

Mathematica [A]
time = 0.68, size = 68, normalized size = 0.85

$$\frac{iF^{a+bx}\left(-1+2\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-\cos(c+dx)-i\sin(c+dx)\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
[Out] (I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Log
[F])/d, -Cos[c + d*x] - I*Sin[c + d*x]]))/(b*e*Log[F])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{F^{bx+a}\sin(dx+c)}{e+e\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
[Out] int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*x)*F^a*b*log(F)*sin(d*x + c) - F^(b*x)*F^a*d*cos(d*x + c) - F^(b*x)
*F^a*d + (F^a*b^2*d*e*log(F)^2 + F^a*d^3*e + (F^a*b^2*d*e*log(F)^2 + F^a*d^
3*e)*cos(d*x + c)^2 + (F^a*b^2*d*e*log(F)^2 + F^a*d^3*e)*sin(d*x + c)^2 + 2
*(F^a*b^2*d*e*log(F)^2 + F^a*d^3*e)*cos(d*x + c))*integrate((F^(b*x)*b*cos(
2*d*x + 2*c)*log(F) + 2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F) +
F^(b*x)*d*sin(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c))/(b^2*e*log(F)^2 + (b
^2*e*log(F)^2 + d^2*e)*cos(2*d*x + 2*c)^2 + 4*(b^2*e*log(F)^2 + d^2*e)*cos(
d*x + c)^2 + d^2*e + (b^2*e*log(F)^2 + d^2*e)*sin(2*d*x + 2*c)^2 + 4*(b^2*e
*log(F)^2 + d^2*e)*sin(2*d*x + 2*c)*sin(d*x + c) + 4*(b^2*e*log(F)^2 + d^2*
e)*sin(d*x + c)^2 + 2*(b^2*e*log(F)^2 + d^2*e + 2*(b^2*e*log(F)^2 + d^2*e)*
cos(d*x + c))*cos(2*d*x + 2*c) + 4*(b^2*e*log(F)^2 + d^2*e)*cos(d*x + c)),
x))/(b^2*e*log(F)^2 + (b^2*e*log(F)^2 + d^2*e)*cos(d*x + c)^2 + d^2*e + (b^
2*e*log(F)^2 + d^2*e)*sin(d*x + c)^2 + 2*(b^2*e*log(F)^2 + d^2*e)*cos(d*x +
 c))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*x + a)*sin(d*x + c)/(cos(d*x + c)*e + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{a}F^{bx}\sin{\left(c+dx \right)}}{\cos{\left(c+dx \right)}+1}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

```
[Out] Integral(F**a*F**(b*x)*sin(c + d*x)/(cos(c + d*x) + 1), x)/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{a+b\,x}\,\sin\left(c+d\,x\right)}{e+e\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(a + b*x)*sin(c + d*x))/(e + e*cos(c + d*x)),x)
```

```
[Out] int((F^(a + b*x)*sin(c + d*x))/(e + e*cos(c + d*x)), x)
```

3.56 $\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};e^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] I*F^(b*x+a)/b/e/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F
)/d],exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4549, 4528, 2225, 2283}

$$\frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};e^{i(c+dx)}\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 4549

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Cot[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx &= \frac{\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\,dx}{e}\\ &= -\frac{i\int\left(-F^{a+bx}-\frac{2F^{a+bx}}{-1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)dx}{e}\\ &= \frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{-1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{e}\\ &= \frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};e^{i(c+dx)}\right)}{be\log(F)}\end{aligned}$$

Mathematica [A]
time = 0.63, size = 66, normalized size = 0.85

$$-\frac{iF^{a+bx}\left(-1+2\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};\cos(c+dx)+i\sin(c+dx)\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
[Out] ((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*
Log[F])/d, Cos[c + d*x] + I*Sin[c + d*x]]))/(b*e*Log[F])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{F^{bx+a}\sin(dx+c)}{e-e\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
[Out] int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*x)*F^a*b*log(F)*sin(d*x + c) - F^(b*x)*F^a*d*cos(d*x + c) + F^(b*x)
*F^a*d - (F^a*b^2*d*e*log(F)^2 + F^a*d^3*e + (F^a*b^2*d*e*log(F)^2 + F^a*d^
3*e)*cos(d*x + c)^2 + (F^a*b^2*d*e*log(F)^2 + F^a*d^3*e)*sin(d*x + c)^2 - 2
*(F^a*b^2*d*e*log(F)^2 + F^a*d^3*e)*cos(d*x + c))*integrate((F^(b*x)*b*cos(
2*d*x + 2*c)*log(F) - 2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F) +
F^(b*x)*d*sin(2*d*x + 2*c) - 2*F^(b*x)*d*sin(d*x + c))/(b^2*e*log(F)^2 + (b
^2*e*log(F)^2 + d^2*e)*cos(2*d*x + 2*c)^2 + 4*(b^2*e*log(F)^2 + d^2*e)*cos(
d*x + c)^2 + d^2*e + (b^2*e*log(F)^2 + d^2*e)*sin(2*d*x + 2*c)^2 - 4*(b^2*e
*log(F)^2 + d^2*e)*sin(2*d*x + 2*c)*sin(d*x + c) + 4*(b^2*e*log(F)^2 + d^2*
e)*sin(d*x + c)^2 + 2*(b^2*e*log(F)^2 + d^2*e - 2*(b^2*e*log(F)^2 + d^2*e)*
cos(d*x + c))*cos(2*d*x + 2*c) - 4*(b^2*e*log(F)^2 + d^2*e)*cos(d*x + c)),
x))/(b^2*e*log(F)^2 + (b^2*e*log(F)^2 + d^2*e)*cos(d*x + c)^2 + d^2*e + (b^
2*e*log(F)^2 + d^2*e)*sin(d*x + c)^2 - 2*(b^2*e*log(F)^2 + d^2*e)*cos(d*x +
 c))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-F^(b*x + a)*sin(d*x + c)/(cos(d*x + c)*e - e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{F^{a} F^{bx} \sin{\left(c + dx \right)}}{\cos{\left(c + dx \right)} - 1}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

```
[Out] -Integral(F**a*F**(b*x)*sin(c + d*x)/(cos(c + d*x) - 1), x)/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{a+b\,x}\,\sin\left(c+d\,x\right)}{e-e\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F^(a + b*x)*sin(c + d*x))/(e - e*cos(c + d*x)),x)
```

```
[Out] int((F^(a + b*x)*sin(c + d*x))/(e - e*cos(c + d*x)), x)
```

3.57 $\int e^{x^2} \sin(bx)\, dx$

Optimal. Leaf size=69

$$\frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\, \text{Erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\, \text{Erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*I*exp(1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(1/4*b^2)*erfi(1/2*I*
b+x)*Pi^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4560, 2266, 2235}

$$\frac{1}{4} i \sqrt{\pi}\, e^{\frac{b^2}{4}} \text{Erfi}\left(\frac{1}{2}(2x-ib)\right) - \frac{1}{4} i \sqrt{\pi}\, e^{\frac{b^2}{4}} \text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[b*x],x]
```

```
[Out] (I/4)*E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(b^2/4)*Sqrt[Pi]*
Erfi[(I*b + 2*x)/2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2} \sin(bx)\,dx = \int \left(\frac{1}{2} i e^{-ibx+x^2} - \frac{1}{2} i e^{ibx+x^2}\right) dx$$
$$= \frac{1}{2} i \int e^{-ibx+x^2}\,dx - \frac{1}{2} i \int e^{ibx+x^2}\,dx$$
$$= \frac{1}{2}\left(i e^{\frac{b^2}{4}}\right) \int e^{\frac{1}{4}(-ib+2x)^2}\,dx - \frac{1}{2}\left(i e^{\frac{b^2}{4}}\right) \int e^{\frac{1}{4}(ib+2x)^2}\,dx$$
$$= \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\,\operatorname{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\,\operatorname{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.62

$$\frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi} \left(\operatorname{Erf}\left(\frac{b}{2} - ix\right) + \operatorname{Erf}\left(\frac{b}{2} + ix\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Erf[b/2 - I*x] + Erf[b/2 + I*x]))/4
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.61

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}} \operatorname{erf}\left(-ix+\frac{b}{2}\right)}{4} + \frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}} \operatorname{erf}\left(ix+\frac{b}{2}\right)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2)*erf(I*x
+1/2*b)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.54

$$\frac{1}{4} \sqrt{\pi} \left(\operatorname{erf}\left(\frac{1}{2} b + i x\right) e^{\left(\frac{1}{4} b^2\right)} - \operatorname{erf}\left(-\frac{1}{2} b + i x\right) e^{\left(\frac{1}{4} b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*(erf(1/2*b + I*x)*e^(1/4*b^2) - erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A]
time = 1.86, size = 30, normalized size = 0.43

$$\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(\frac{1}{2}b+ix\right)-\operatorname{erf}\left(-\frac{1}{2}b+ix\right)\right)e^{\left(\frac{1}{4}b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="fricas")
[Out] 1/4*sqrt(pi)*(erf(1/2*b + I*x) - erf(-1/2*b + I*x))*e^(1/4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sin(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(b*x),x)
[Out] Integral(exp(x**2)*sin(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="giac")
[Out] integrate(e^(x^2)*sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{x^2}\,\sin(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x),x)
[Out] int(exp(x^2)*sin(b*x), x)
```

3.58 $\int e^{x^2} \cos(bx)\, dx$

Optimal. Leaf size=65

$$\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*exp(1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(1/4*b^2)*erfi(1/2*I*b+x)
*Pi^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4561, 2266, 2235}

$$\frac{1}{4}\sqrt{\pi}\,e^{\frac{b^2}{4}}\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}\,e^{\frac{b^2}{4}}\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(b^2/4)*Sqrt[Pi]*Erfi[(I
*b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\cos(bx)\,dx &= \int \left(\frac{1}{2}e^{-ibx+x^2}+\frac{1}{2}e^{ibx+x^2}\right)\,dx \\
&= \frac{1}{2}\int e^{-ibx+x^2}\,dx+\frac{1}{2}\int e^{ibx+x^2}\,dx \\
&= \frac{1}{2}e^{\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx \\
&= \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 47, normalized size = 0.72

$$
\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\text{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\text{Erfi}\left(\frac{1}{2}(ib+2x)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Erfi[((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2]))/4
```

Maple [**A**]
time = 0.10, size = 44, normalized size = 0.68

method	result	size
risch	$-\frac{i\sqrt{\pi}\ \mathrm{e}^{\frac{b^2}{4}}\ \mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\ \mathrm{e}^{\frac{b^2}{4}}\ \mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x),x,method=_RETURNVERBOSE)
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf
(-I*x+1/2*b)
```

Maxima [**A**]
time = 0.27, size = 38, normalized size = 0.58

$$
-\frac{1}{4}\sqrt{\pi}\left(i\ \mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}+i\ \mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*(I*erf(1/2*b + I*x)*e^(1/4*b^2) + I*erf(-1/2*b + I*x)*e^(1/4*
b^2))
```

Fricas [A]
time = 1.55, size = 32, normalized size = 0.49

$$\frac{1}{4}\sqrt{\pi}\left(-i\,\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)-i\,\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="fricas")
[Out] 1/4*sqrt(pi)*(-I*erf(1/2*b + I*x) - I*erf(-1/2*b + I*x))*e^(1/4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cos\left(bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(b*x),x)
[Out] Integral(exp(x**2)*cos(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="giac")
[Out] integrate(cos(b*x)*e^(x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2}\,\cos\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x),x)
[Out] int(exp(x^2)*cos(b*x), x)
```

3.59 $\int e^{x^2} \sin(a+bx)\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*I*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(I*a+1/4*b^2)*er
fi(1/2*I*b+x)*Pi^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4560, 2266, 2235}

$$\frac{1}{4}i\sqrt{\pi}\,e^{\frac{b^2}{4}-ia}\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}\,e^{\frac{b^2}{4}+ia}\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + b*x],x]
```

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\sin(a+bx)\,dx &= \int \left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx \\
&= \frac{1}{2}i\int e^{-ia-ibx+x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+x^2}\,dx \\
&= \frac{1}{2}\left(ie^{-ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx-\frac{1}{2}\left(ie^{ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx \\
&= \frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 1.00

$$
\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\cos(a)\mathrm{Erf}\left(\frac{b}{2}+ix\right)+\left(\mathrm{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\mathrm{Erfi}\left(\frac{1}{2}(ib+2x)\right)\right)\sin(a)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi[
((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.64

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2
)*exp(-I*a)*erf(I*x+1/2*b)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.63

$$
\frac{1}{4}\sqrt{\pi}\left((\cos(a)-i\sin(a))\,\mathrm{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}-(\cos(a)+i\sin(a))\,\mathrm{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A]
time = 1.31, size = 45, normalized size = 0.56

$$-\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2+ia\right)}-\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2-ia\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1/
4*b^2 - I*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{x^2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(a + b*x),x)
```

```
[Out] int(exp(x^2)*sin(a + b*x), x)
```

3.60 $\int e^{x^2} \cos(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(I*a+1/4*b^2)*erfi(1
/2*I*b+x)*Pi^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4561, 2266, 2235}

$$\frac{1}{4}\sqrt{\pi}\,e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}\,e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cos(a+bx)\,dx &= \int\left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia-ibx+x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+x^2}\,dx\\ &= \frac{1}{2}e^{-ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\ &= \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 1.06

$$\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(ib+2x)\right)-\left(\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\mathrm{Erf}\left(\frac{b}{2}+ix\right)\right)\sin(a)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*x
)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.70

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/
4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.68

$$-\frac{1}{4}\sqrt{\pi}\left(\left(i\cos(a)+\sin(a)\right)\mathrm{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}+\left(i\cos(a)-\sin(a)\right)\mathrm{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a)
 - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A]
time = 1.67, size = 46, normalized size = 0.60

$$\frac{1}{4}\sqrt{\pi}\left(-i\ \mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-i\ \mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
[Out] 1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*e
^(1/4*b^2 - I*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(b*x+a),x)
[Out] Integral(exp(x**2)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*e^(x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\mathrm{e}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*exp(x^2),x)
[Out] int(cos(a + b*x)*exp(x^2), x)
```

3.61 $\int e^{2x^2} x \cos(2x^2)\, dx$

Optimal. Leaf size=35

$$\frac{1}{8}e^{2x^2}\cos\left(2x^2\right)+\frac{1}{8}e^{2x^2}\sin\left(2x^2\right)$$

```
[Out] 1/8*exp(2*x^2)*cos(2*x^2)+1/8*exp(2*x^2)*sin(2*x^2)
```

Rubi [A]
time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6847, 4518}

$$\frac{1}{8}e^{2x^2}\sin\left(2x^2\right)+\frac{1}{8}e^{2x^2}\cos\left(2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x^2)*x*Cos[2*x^2],x]
```

```
[Out] (E^(2*x^2)*Cos[2*x^2])/8 + (E^(2*x^2)*Sin[2*x^2])/8
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x^2}x\cos\left(2x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int e^{2x}\cos(2x)\,dx,x,x^2\right)\\ &= \frac{1}{8}e^{2x^2}\cos\left(2x^2\right)+\frac{1}{8}e^{2x^2}\sin\left(2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 24, normalized size = 0.69

$$\frac{1}{8}e^{2x^2}\left(\cos\left(2x^2\right)+\sin\left(2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x^2)*x*Cos[2*x^2],x]
[Out] (E^(2*x^2)*(Cos[2*x^2] + Sin[2*x^2]))/8
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.86

method	result	size
derivativedivides	$\frac{e^{2x^2}\cos(2x^2)}{8}+\frac{e^{2x^2}\sin(2x^2)}{8}$	30
default	$\frac{e^{2x^2}\cos(2x^2)}{8}+\frac{e^{2x^2}\sin(2x^2)}{8}$	30
risch	$\frac{e^{(2+2i)x^2}}{16}-\frac{ie^{(2+2i)x^2}}{16}+\frac{e^{(2-2i)x^2}}{16}+\frac{ie^{(2-2i)x^2}}{16}$	44
norman	$\frac{\frac{e^{2x^2}\tan(x^2)}{4}-\frac{e^{2x^2}\left(\tan^2(x^2)\right)}{8}+\frac{e^{2x^2}}{8}}{1+\tan^2(x^2)}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x^2)*x*cos(2*x^2),x,method=_RETURNVERBOSE)
[Out] 1/8*exp(2*x^2)*cos(2*x^2)+1/8*exp(2*x^2)*sin(2*x^2)
```

Maxima [A]
time = 0.27, size = 29, normalized size = 0.83

$$\frac{1}{8}\cos\left(2\,x^2\right)e^{\left(2\,x^2\right)}+\frac{1}{8}\,e^{\left(2\,x^2\right)}\sin\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="maxima")
[Out] 1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

Fricas [A]
time = 1.15, size = 29, normalized size = 0.83

$$\frac{1}{8}\cos\left(2\,x^2\right)e^{\left(2\,x^2\right)}+\frac{1}{8}\,e^{\left(2\,x^2\right)}\sin\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="fricas")
[Out] 1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

Sympy [A]
time = 1.23, size = 29, normalized size = 0.83

$$\frac{e^{2x^2}\sin\left(2x^2\right)}{8}+\frac{e^{2x^2}\cos\left(2x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x**2)*x*cos(2*x**2),x)
```

```
[Out] exp(2*x**2)*sin(2*x**2)/8 + exp(2*x**2)*cos(2*x**2)/8
```

Giac [A]
time = 0.40, size = 21, normalized size = 0.60

$$\frac{1}{8}\left(\cos\left(2\,x^2\right)+\sin\left(2\,x^2\right)\right)e^{\left(2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="giac")
```

```
[Out] 1/8*(cos(2*x^2) + sin(2*x^2))*e^(2*x^2)
```

Mupad [B]
time = 0.08, size = 21, normalized size = 0.60

$$\frac{\mathrm{e}^{2\,x^2}\,\left(\cos\left(2\,x^2\right)+\sin\left(2\,x^2\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(2*x^2)*cos(2*x^2),x)
```

```
[Out] (exp(2*x^2)*(cos(2*x^2) + sin(2*x^2)))/8
```

3.62 $\int e^x \sin(e^x)\,dx$

Optimal. Leaf size=6

$$-\cos(e^x)$$

[Out] -cos(exp(x))

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 2718}

$$-\cos(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Sin[E^x],x]

[Out] -Cos[E^x]

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \sin(e^x)\,dx &= \text{Subst}\left(\int \sin(x)\,dx, x, e^x\right)\\ &= -\cos(e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$-\cos(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[E^x],x]
```

```
[Out] -Cos[E^x]
```

Maple [A]
time = 0.03, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$-\cos\left(e^{x}\right)$	6
default	$-\cos\left(e^{x}\right)$	6
risch	$-\cos\left(e^{x}\right)$	6
norman	$-\frac{2}{1+\tan^{2}\left(\frac{e^{x}}{2}\right)}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(exp(x))
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.83

$$-\cos\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="maxima")
```

```
[Out] -cos(e^x)
```

Fricas [A]
time = 1.72, size = 5, normalized size = 0.83

$$-\cos\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="fricas")
```

```
[Out] -cos(e^x)
```

Sympy [A]
time = 0.09, size = 5, normalized size = 0.83

$$-\cos\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x)
```

```
[Out] -cos(exp(x))
```

Giac [A]
time = 0.40, size = 5, normalized size = 0.83

$$-\cos\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="giac")
```

```
[Out] -cos(e^x)
```

Mupad [B]
time = 2.21, size = 5, normalized size = 0.83

$$-\cos\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x))*exp(x),x)
```

```
[Out] -cos(exp(x))
```

3.63 $\int e^x \csc(e^x) \sec(e^x) \, dx$

Optimal. Leaf size=5

$$\log(\tan(e^x))$$

[Out] ln(tan(exp(x)))

Rubi [A]

time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 2700, 29}

$$\log(\tan(e^x))$$

Antiderivative was successfully verified.

[In] Int[E^x*Csc[E^x]*Sec[E^x],x]

[Out] Log[Tan[E^x]]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\int e^x \csc(e^x) \sec(e^x) \, dx &= \text{Subst}\left(\int \csc(x) \sec(x) \, dx, x, e^x\right) \\
&= \text{Subst}\left(\int \frac{1}{x} \, dx, x, \tan(e^x)\right) \\
&= \log(\tan(e^x))
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(5) = 10.
time = 0.02, size = 21, normalized size = 4.20

$$2\left(-\frac{1}{2}\log\left(\cos\left(e^{x}\right)\right)+\frac{1}{2}\log\left(\sin\left(e^{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csc[E^x]*Sec[E^x],x]
[Out] 2*(-1/2*Log[Cos[E^x]] + Log[Sin[E^x]]/2)
```

Maple [A]
time = 0.14, size = 5, normalized size = 1.00

method	result	size
derivativedivides	$\ln\left(\tan\left(e^{x}\right)\right)$	5
default	$\ln\left(\tan\left(e^{x}\right)\right)$	5
risch	$-\ln\left(e^{2ie^{x}}+1\right)+\ln\left(e^{2ie^{x}}-1\right)$	22
norman	$-\ln\left(\tan\left(\frac{e^{x}}{2}\right)-1\right)-\ln\left(\tan\left(\frac{e^{x}}{2}\right)+1\right)+\ln\left(\tan\left(\frac{e^{x}}{2}\right)\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csc(exp(x))*sec(exp(x)),x,method=_RETURNVERBOSE)
[Out] ln(tan(exp(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(4) = 8.
time = 0.28, size = 19, normalized size = 3.80

$$-\frac{1}{2}\log\left(\sin\left(e^{x}\right)^{2}-1\right)+\frac{1}{2}\log\left(\sin\left(e^{x}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="maxima")
[Out] -1/2*log(sin(e^x)^2 - 1) + 1/2*log(sin(e^x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(4) = 8.
time = 1.11, size = 21, normalized size = 4.20

$$-\frac{1}{2}\log\left(\cos\left(e^{x}\right)^{2}\right)+\frac{1}{2}\log\left(-\frac{1}{4}\cos\left(e^{x}\right)^{2}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(cos(e^x)^2) + 1/2*log(-1/4*cos(e^x)^2 + 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \csc(e^x) \sec(e^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x)
```

```
[Out] Integral(exp(x)*csc(exp(x))*sec(exp(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(4) = 8$.
time = 0.38, size = 17, normalized size = 3.40

$$-\frac{1}{2}\log\left(\left|\sin\left(e^x\right)^2 - 1\right|\right) + \log\left(\left|\sin\left(e^x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(sin(e^x)^2 - 1)) + log(abs(sin(e^x)))
```

Mupad [B]
time = 2.51, size = 43, normalized size = 8.60

$$-\ln\left(-16\,e^{2\,x} - 16\,e^{2\,x}\,e^{e^x\,2i}\right) + \ln\left(16\,e^{2\,x} - 16\,e^{2\,x}\,e^{e^x\,2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cos(exp(x))*sin(exp(x))),x)
```

```
[Out] log(16*exp(2*x) - 16*exp(2*x)*exp(exp(x)*2i)) - log(- 16*exp(2*x) - 16*exp(
2*x)*exp(exp(x)*2i))
```

3.64 $\int e^x \cos(e^x)\,dx$

Optimal. Leaf size=4

$$\sin(e^x)$$

```
[Out] sin(exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 2717}

$$\sin(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[E^x],x]
```

```
[Out] Sin[E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \cos(e^x)\,dx &= \text{Subst}\left(\int \cos(x)\,dx, x, e^x\right)\\ &= \sin(e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\sin(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[E^x],x]
```

```
[Out] Sin[E^x]
```

Maple [A]
time = 0.05, size = 4, normalized size = 1.00

method	result	size
derivativedivides	$\sin(e^{x})$	4
default	$\sin(e^{x})$	4
risch	$\sin(e^{x})$	4
norman	$\frac{2\tan\left(\frac{e^{x}}{2}\right)}{1+\tan^{2}\left(\frac{e^{x}}{2}\right)}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] sin(exp(x))
```

Maxima [A]
time = 0.28, size = 3, normalized size = 0.75

$$\sin(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="maxima")
```

```
[Out] sin(e^x)
```

Fricas [A]
time = 1.52, size = 3, normalized size = 0.75

$$\sin(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="fricas")
```

```
[Out] sin(e^x)
```

Sympy [A]
time = 0.10, size = 3, normalized size = 0.75

$$\sin(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x)
```

```
[Out] sin(exp(x))
```

Giac [A]
time = 0.40, size = 3, normalized size = 0.75

$$\sin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="giac")
```

```
[Out] sin(e^x)
```

Mupad [B]
time = 0.04, size = 3, normalized size = 0.75

$$\sin(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(exp(x))*exp(x),x)
```

```
[Out] sin(exp(x))
```

3.65 $\int e^{2x} \cos\left(e^{2x}\right)\, dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin\left(e^{2x}\right)$$

[Out] 1/2*sin(exp(2*x))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 2717}

$$\frac{1}{2}\sin\left(e^{2x}\right)$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)*Cos[E^(2*x)],x]

[Out] Sin[E^(2*x)]/2

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\cos\left(e^{2x}\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, e^{2x}\right)\\ &= \frac{1}{2}\sin\left(e^{2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[E^(2*x)],x]
```

```
[Out] Sin[E^(2*x)]/2
```

Maple [A]
time = 0.06, size = 8, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\sin\left(\mathrm{e}^{2x}\right)}{2}$	8
default	$\frac{\sin\left(\mathrm{e}^{2x}\right)}{2}$	8
risch	$\frac{\sin\left(\mathrm{e}^{2x}\right)}{2}$	8
norman	$\frac{\tan\left(\frac{\mathrm{e}^{2x}}{2}\right)}{1+\tan^{2}\left(\frac{\mathrm{e}^{2x}}{2}\right)}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(exp(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(exp(2*x))
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.70

$$\frac{1}{2}\sin\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*sin(e^(2*x))
```

Fricas [A]
time = 1.37, size = 7, normalized size = 0.70

$$\frac{1}{2}\sin\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*sin(e^(2*x))
```

Sympy [A]
time = 0.10, size = 7, normalized size = 0.70

$$\frac{\sin\left(e^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x)
```

```
[Out] sin(exp(2*x))/2
```

Giac [A]
time = 0.40, size = 7, normalized size = 0.70

$$\frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*sin(e^(2*x))
```

Mupad [B]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\sin\left(\mathrm{e}^{2\,x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(exp(2*x)),x)
```

```
[Out] sin(exp(2*x))/2
```

3.66 $\int e^{-2x}\cos\left(e^{-2x}\right)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

```
[Out] -1/2*sin(exp(-2*x))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 2717}

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[E^(-2*x)]/E^(2*x),x]
```

```
[Out] -1/2*Sin[E^(-2*x)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}\cos\left(e^{-2x}\right)\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int\cos(x)\,dx,x,e^{-2x}\right)\right)\\ &= -\frac{1}{2}\sin\left(e^{-2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[E^(-2*x)]/E^(2*x),x]
```

```
[Out] -1/2*Sin[E^(-2*x)]
```

Maple [A]
time = 0.06, size = 8, normalized size = 0.80

method	result	size
default	$-\frac{\sin\left(\mathrm{e}^{-2x}\right)}{2}$	8
risch	$-\frac{\sin\left(\mathrm{e}^{-2x}\right)}{2}$	8
norman	$-\frac{\tan\left(\frac{\mathrm{e}^{-2x}}{2}\right)}{1+\tan^{2}\left(\frac{\mathrm{e}^{-2x}}{2}\right)}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(exp(-2*x))/exp(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sin(exp(-2*x))
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.70

$$-\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sin(e^(-2*x))
```

Fricas [A]
time = 1.41, size = 7, normalized size = 0.70

$$-\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*sin(e^(-2*x))
```

Sympy [A]
time = 0.18, size = 10, normalized size = 1.00

$$-\frac{\sin\left(e^{-2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x)
```

```
[Out] -sin(exp(-2*x))/2
```

Giac [A]
time = 0.39, size = 7, normalized size = 0.70

$$-\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

```
[Out] -1/2*sin(e^(-2*x))
```

Mupad [B]
time = 2.20, size = 7, normalized size = 0.70

$$-\frac{\sin\left(\mathrm{e}^{-2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x)*cos(exp(-2*x)),x)
```

```
[Out] -sin(exp(-2*x))/2
```

3.67 $\int e^{2x} \cos(e^x)\, dx$

Optimal. Leaf size=13

$$\cos(e^x) + e^x \sin(e^x)$$

```
[Out] cos(exp(x))+exp(x)*sin(exp(x))
```

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 3377, 2718}

$$e^x \sin(e^x) + \cos(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[E^x],x]
```

```
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{2x} \cos(e^x)\, dx &= \text{Subst}\left(\int x \cos(x)\, dx, x, e^x\right)\\
&= e^x \sin(e^x) - \text{Subst}\left(\int \sin(x)\, dx, x, e^x\right)\\
&= \cos(e^x) + e^x \sin(e^x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$\cos(e^x) + e^x \sin(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[E^x],x]
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Maple [A]
time = 0.07, size = 11, normalized size = 0.85

method	result	size
risch	$\cos(\mathrm{e}^x) + \mathrm{e}^x \sin(\mathrm{e}^x)$	11
norman	$\frac{2\,\mathrm{e}^x \tan\left(\frac{\mathrm{e}^x}{2}\right)+2}{1+\tan^2\left(\frac{\mathrm{e}^x}{2}\right)}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(exp(x)),x,method=_RETURNVERBOSE)
[Out] cos(exp(x))+exp(x)*sin(exp(x))
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.77

$$e^x \sin(e^x) + \cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="maxima")
[Out] e^x*sin(e^x) + cos(e^x)
```

Fricas [A]
time = 1.19, size = 10, normalized size = 0.77

$$e^x \sin(e^x) + \cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="fricas")
[Out] e^x*sin(e^x) + cos(e^x)
```

Sympy [A]
time = 6.96, size = 12, normalized size = 0.92

$$e^x \sin(e^x) + \cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x)
```

```
[Out] exp(x)*sin(exp(x)) + cos(exp(x))
```

Giac [A]
time = 0.39, size = 10, normalized size = 0.77

$$e^x \sin\left(e^x\right) + \cos\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="giac")
```

```
[Out] e^x*sin(e^x) + cos(e^x)
```

Mupad [B]
time = 2.23, size = 10, normalized size = 0.77

$$\cos\left(\mathrm{e}^x\right) + \sin\left(\mathrm{e}^x\right)\,\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(exp(x))*exp(2*x),x)
```

```
[Out] cos(exp(x)) + sin(exp(x))*exp(x)
```

3.68 $\int e^{-1+3x} \cos\left(e^{-1+3x}\right) \sin\left(1+e^{-1+3x}\right) \, dx$

Optimal. Leaf size=30

$$-\frac{1}{12} \cos\left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x} \sin(1)$$

```
[Out] -1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2320, 4670, 2718}

$$\frac{1}{6}e^{3x-1} \sin(1)-\frac{1}{12} \cos\left(2e^{3x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

```
[Out] -1/12*Cos[1 + 2*E^(-1 + 3*x)] + (E^(-1 + 3*x)*Sin[1])/6
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{1}{3}\text{Subst}\left(\int\cos(x)\sin(1+x)\,dx,x,e^{-1+3x}\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int\left(\frac{\sin(1)}{2}+\frac{1}{2}\sin(1+2x)\right)\,dx,x,e^{-1+3x}\right)$$
$$=\frac{1}{6}e^{-1+3x}\sin(1)+\frac{1}{6}\text{Subst}\left(\int\sin(1+2x)\,dx,x,e^{-1+3x}\right)$$
$$=-\frac{1}{12}\cos\left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x}\sin(1)$$

Mathematica [A]
time = 0.09, size = 30, normalized size = 1.00

$$-\frac{1}{12}\cos\left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x}\sin(1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

```
[Out] -1/12*Cos[1 + 2*E^(-1 + 3*x)] + (E^(-1 + 3*x)*Sin[1])/6
```

Maple [A]
time = 0.34, size = 25, normalized size = 0.83

method	result
derivativedivides	$-\frac{\cos\left(1+2\,e^{-1+3x}\right)}{12}+\frac{e^{-1+3x}\sin(1)}{6}$
default	$-\frac{\cos\left(1+2\,e^{-1+3x}\right)}{12}+\frac{e^{-1+3x}\sin(1)}{6}$
risch	$-\frac{\cos\left(1+2\,e^{-1+3x}\right)}{12}+\frac{e^{-1+3x}\sin(1)}{6}$
norman	$\frac{\frac{2\tan\left(\frac{e^{-1+3x}}{2}\right)\tan\left(\frac{1}{2}+\frac{e^{-1+3x}}{2}\right)}{3}-\frac{e^{-1+3x}\tan\left(\frac{e^{-1+3x}}{2}\right)}{3}+\frac{e^{-1+3x}\tan\left(\frac{1}{2}+\frac{e^{-1+3x}}{2}\right)}{3}+\frac{e^{-1+3x}\tan\left(\frac{e^{-1+3x}}{2}\right)\left(\tan^{2}\left(\frac{1}{2}+\frac{e^{-}}{}\right.\right.}{3}}{\left(1+\tan^{2}\left(\frac{e^{-1+3x}}{2}\right)\right)\left(1+\tan^{2}\left(\frac{1}{2}+\frac{e^{-1+3x}}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{6}\,e^{(3\,x-1)}\sin\left(1\right)-\frac{1}{12}\,\cos\left(2\,e^{(3\,x-1)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="max
ima")
```

```
[Out] 1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

Fricas [**A**]
time = 1.48, size = 42, normalized size = 1.40

$$-\frac{1}{6}\cos\left(1\right)\cos\left(e^{(3x-1)}\right)^{2}+\frac{1}{6}\cos\left(e^{(3x-1)}\right)\sin\left(1\right)\sin\left(e^{(3x-1)}\right)+\frac{1}{6}e^{(3x-1)}\sin\left(1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="fri
cas")
```

```
[Out] -1/6*cos(1)*cos(e^(3*x - 1))^2 + 1/6*cos(e^(3*x - 1))*sin(1)*sin(e^(3*x - 1
)) + 1/6*e^(3*x - 1)*sin(1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 24, normalized size = 0.80

$$\frac{1}{6}e^{(3x-1)}\sin\left(1\right)-\frac{1}{12}\cos\left(2e^{(3x-1)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="gia
c")
```

```
[Out] 1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

Mupad [**B**]
time = 0.31, size = 24, normalized size = 0.80

$$\frac{\mathrm{e}^{3x-1}\sin\left(1\right)}{6}-\frac{\cos\left(2\mathrm{e}^{3x-1}+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x - 1)*sin(exp(3*x - 1) + 1)*cos(exp(3*x - 1)),x)
```

```
[Out] (exp(3*x - 1)*sin(1))/6 - cos(2*exp(3*x - 1) + 1)/12
```

3.69 $\int e^x \tan(e^x)\, dx$

Optimal. Leaf size=7

$$-\log(\cos(e^x))$$

[Out] -ln(cos(exp(x)))

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 3556}

$$-\log(\cos(e^x))$$

Antiderivative was successfully verified.

[In] Int[E^x*Tan[E^x],x]

[Out] -Log[Cos[E^x]]

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \tan(e^x)\, dx &= \text{Subst}\left(\int \tan(x)\, dx, x, e^x\right)\\ &= -\log(\cos(e^x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$-\log(\cos(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tan[E^x],x]
[Out] -Log[Cos[E^x]]
```

Maple [A]
time = 0.03, size = 7, normalized size = 1.00

method	result	size
derivativedivides	$-\ln(\cos(\mathrm{e}^{x}))$	7
default	$-\ln(\cos(\mathrm{e}^{x}))$	7
norman	$\frac{\ln(1+\tan^{2}(\mathrm{e}^{x}))}{2}$	11
risch	$i\mathrm{e}^{x}-\ln\left(\mathrm{e}^{2i\mathrm{e}^{x}}+1\right)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tan(exp(x)),x,method=_RETURNVERBOSE)
[Out] -ln(cos(exp(x)))
```

Maxima [A]
time = 0.27, size = 4, normalized size = 0.57

$$\log(\sec(e^{x}))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="maxima")
[Out] log(sec(e^x))
```

Fricas [A]
time = 1.74, size = 12, normalized size = 1.71

$$-\frac{1}{2}\log\left(\frac{1}{\tan(e^{x})^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(e^x)^2 + 1))
```

Sympy [A]
time = 0.08, size = 10, normalized size = 1.43

$$\frac{\log(\tan^{2}(e^{x})+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x)
```

```
[Out] log(tan(exp(x))**2 + 1)/2
```

Giac [A]
time = 0.39, size = 7, normalized size = 1.00

$$-\log\left(|\cos\left(e^{x}\right)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="giac")
```

```
[Out] -log(abs(cos(e^x)))
```

Mupad [B]
time = 2.65, size = 10, normalized size = 1.43

$$\frac{\ln\left(\tan(\mathrm{e}^{x})^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(exp(x))*exp(x),x)
```

```
[Out] log(tan(exp(x))^2 + 1)/2
```

3.70 $\int e^x \sec(e^x)\,dx$

Optimal. Leaf size=5

$$\tanh^{-1}(\sin(e^x))$$

```
[Out] arctanh(sin(exp(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 3855}

$$\tanh^{-1}(\sin(e^x))$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sec[E^x],x]
```

```
[Out] ArcTanh[Sin[E^x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \sec(e^x)\,dx &= \text{Subst}\left(\int \sec(x)\,dx, x, e^x\right)\\ &= \tanh^{-1}(\sin(e^x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\tanh^{-1}(\sin(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sec[E^x],x]
[Out] ArcTanh[Sin[E^x]]
```

Maple [A]
time = 0.03, size = 9, normalized size = 1.80

method	result	size
derivativedivides	$\ln \left(\sec \left({\mathrm e}^{x}\right)+\tan \left({\mathrm e}^{x}\right)\right)$	9
default	$\ln \left(\sec \left({\mathrm e}^{x}\right)+\tan \left({\mathrm e}^{x}\right)\right)$	9
norman	$-\ln \left(\tan \left(\frac{{\mathrm e}^{x}}{2}\right)-1\right)+\ln \left(\tan \left(\frac{{\mathrm e}^{x}}{2}\right)+1\right)$	20
risch	$\ln \left({\mathrm e}^{i {\mathrm e}^{x}}+i\right)-\ln \left({\mathrm e}^{i {\mathrm e}^{x}}-i\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sec(exp(x)),x,method=_RETURNVERBOSE)
[Out] ln(sec(exp(x))+tan(exp(x)))
```

Maxima [A]
time = 0.28, size = 8, normalized size = 1.60

$$\log\left(\sec\left(e^{x}\right) + \tan\left(e^{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="maxima")
[Out] log(sec(e^x) + tan(e^x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(4) = 8.
time = 1.54, size = 19, normalized size = 3.80

$$\frac{1}{2} \, \log\left(\sin\left(e^{x}\right) + 1\right) - \frac{1}{2} \, \log\left(-\sin\left(e^{x}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="fricas")
[Out] 1/2*log(sin(e^x) + 1) - 1/2*log(-sin(e^x) + 1)
```

Sympy [A]
time = 0.60, size = 10, normalized size = 2.00

$$\log{\left(\tan{\left(e^{x} \right)} + \sec{\left(e^{x} \right)} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x)
```

```
[Out] log(tan(exp(x)) + sec(exp(x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(4) = 8$.
time = 0.40, size = 29, normalized size = 5.80

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin\left(e^{x}\right)}+\sin\left(e^{x}\right)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin\left(e^{x}\right)}+\sin\left(e^{x}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(e^x) + sin(e^x) + 2)) - 1/4*log(abs(1/sin(e^x) + sin(e^x)
 - 2))
```

Mupad [B]
time = 2.79, size = 10, normalized size = 2.00

$$-\mathrm{atan}\left(\mathrm{e}^{\mathrm{e}^{x}\,1\mathrm{i}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cos(exp(x)),x)
```

```
[Out] -atan(exp(exp(x)*1i))*2i
```

3.71 $\int e^x \sec(e^x) \tan(e^x)\, dx$

Optimal. Leaf size=4

$$\sec(e^x)$$

```
[Out] sec(exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 2686, 8}

$$\sec(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sec[E^x]*Tan[E^x],x]
```

```
[Out] Sec[E^x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int e^x \sec(e^x) \tan(e^x)\, dx &= \text{Subst}\left(\int \sec(x)\tan(x)\, dx, x, e^x\right) \\
&= \text{Subst}\left(\int 1\, dx, x, \sec(e^x)\right) \\
&= \sec(e^x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\sec(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sec[E^x]*Tan[E^x],x]
```

```
[Out] Sec[E^x]
```

Maple [A]
time = 0.05, size = 4, normalized size = 1.00

method	result	size
derivativedivides	$\sec(e^x)$	4
default	$\sec(e^x)$	4
risch	$\frac{2\,e^{ie^x}}{e^{2ie^x}+1}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sec(exp(x))*tan(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] sec(exp(x))
```

Maxima [A]
time = 0.27, size = 5, normalized size = 1.25

$$\frac{1}{\cos(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/cos(e^x)
```

Fricas [A]
time = 1.71, size = 5, normalized size = 1.25

$$\frac{1}{\cos(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/cos(e^x)
```

Sympy [A]
time = 0.17, size = 3, normalized size = 0.75

$$\sec(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x)
[Out] sec(exp(x))
```

Giac [A]
time = 0.41, size = 5, normalized size = 1.25

$$\frac{1}{\cos(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="giac")
[Out] 1/cos(e^x)
```

Mupad [B]
time = 0.09, size = 5, normalized size = 1.25

$$\frac{1}{\cos(\mathrm{e}^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(exp(x))*exp(x))/cos(exp(x)),x)
[Out] 1/cos(exp(x))
```

3.72 $\int e^x \csc^2\left(e^x\right)\,dx$

Optimal. Leaf size=6

$$-\cot\left(e^x\right)$$

```
[Out] -cot(exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 3852, 8}

$$-\cot\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csc[E^x]^2,x]
```

```
[Out] -Cot[E^x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \int e^x \csc^2\left(e^x\right)\,dx &= \text{Subst}\left(\int \csc^2(x)\,dx, x, e^x\right)\\ &= -\text{Subst}\left(\int 1\,dx, x, \cot\left(e^x\right)\right)\\ &= -\cot\left(e^x\right) \end{aligned}$$

Mathematica [A]
time = 0.02, size = 6, normalized size = 1.00

$$-\cot(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csc[E^x]^2,x]
```

```
[Out] -Cot[E^x]
```

Maple [A]
time = 0.04, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$-\cot(\mathrm{e}^x)$	6
default	$-\cot(\mathrm{e}^x)$	6
risch	$-\frac{2i}{\mathrm{e}^{2i\mathrm{e}^x}-1}$	14
norman	$\frac{-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{\mathrm{e}^x}{2}\right)\right)}{2}}{\tan\left(\frac{\mathrm{e}^x}{2}\right)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csc(exp(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -cot(exp(x))
```

Maxima [A]
time = 0.26, size = 7, normalized size = 1.17

$$-\frac{1}{\tan(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="maxima")
```

```
[Out] -1/tan(e^x)
```

Fricas [A]
time = 3.28, size = 10, normalized size = 1.67

$$-\frac{\cos(e^x)}{\sin(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="fricas")
```

```
[Out] -cos(e^x)/sin(e^x)
```

Sympy [**A**]
time = 0.49, size = 5, normalized size = 0.83

$$-\cot(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))**2,x)
[Out] -cot(exp(x))
```

Giac [**A**]
time = 0.40, size = 7, normalized size = 1.17

$$-\frac{1}{\tan(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="giac")
[Out] -1/tan(e^x)
```

Mupad [**B**]
time = 2.27, size = 13, normalized size = 2.17

$$-\frac{2\mathrm{i}}{\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sin(exp(x))^2,x)
[Out] -2i/(exp(exp(x)*2i) - 1)
```

3.73 $\int e^x \sin(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{be^x\cos(a+bx)}{1+b^2}+\frac{e^x\sin(a+bx)}{1+b^2}$$

```
[Out] -b*exp(x)*cos(b*x+a)/(b^2+1)+exp(x)*sin(b*x+a)/(b^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4517}

$$\frac{e^x\sin(a+bx)}{b^2+1}-\frac{be^x\cos(a+bx)}{b^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[a + b*x],x]
[Out] -((b*E^x*Cos[a + b*x])/(1 + b^2)) + (E^x*Sin[a + b*x])/(1 + b^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(a+bx)\,dx=-\frac{be^x\cos(a+bx)}{1+b^2}+\frac{e^x\sin(a+bx)}{1+b^2}$$

Mathematica [A]
time = 0.07, size = 27, normalized size = 0.73

$$\frac{e^x(-b\cos(a+bx)+\sin(a+bx))}{1+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[a + b*x],x]
[Out] (E^x*(-(b*Cos[a + b*x]) + Sin[a + b*x]))/(1 + b^2)
```

Maple [A]
time = 0.06, size = 36, normalized size = 0.97

method	result	size
default	$-\frac{b\,\mathrm{e}^{x} \cos(bx+a)}{b^{2}+1}+\frac{\mathrm{e}^{x} \sin(bx+a)}{b^{2}+1}$	36
risch	$-\frac{\mathrm{e}^{x} \mathrm{e}^{ibx} \mathrm{e}^{ia}}{2(b-i)}-\frac{\mathrm{e}^{x} \mathrm{e}^{-ibx} \mathrm{e}^{-ia}}{2(i+b)}$	44
norman	$\frac{\frac{b\,\mathrm{e}^{x}\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^{2}+1}-\frac{b\,\mathrm{e}^{x}}{b^{2}+1}+\frac{2\,\mathrm{e}^{x} \tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^{2}+1}}{1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b*exp(x)*cos(b*x+a)/(b^2+1)+exp(x)*sin(b*x+a)/(b^2+1)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.76

$$-\frac{(b\cos(bx+a)-\sin(bx+a))e^{x}}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(b*cos(b*x + a) - sin(b*x + a))*e^x/(b^2 + 1)
```

Fricas [A]
time = 3.00, size = 30, normalized size = 0.81

$$-\frac{b\cos(bx+a)\,e^{x}-e^{x}\sin(bx+a)}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x + a)*e^x - e^x*sin(b*x + a))/(b^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.25, size = 116, normalized size = 3.14

$$\begin{cases} \frac{xe^{x}\sin(a-ix)}{2}+\frac{ixe^{x}\cos(a-ix)}{2}-\frac{ie^{x}\cos(a-ix)}{2} & \text{for } b=-i \\ \frac{xe^{x}\sin(a+ix)}{2}-\frac{ixe^{x}\cos(a+ix)}{2}+\frac{ie^{x}\cos(a+ix)}{2} & \text{for } b=i \\ -\frac{be^{x}\cos(a+bx)}{b^{2}+1}+\frac{e^{x}\sin(a+bx)}{b^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sin(a - I*x)/2 + I*x*exp(x)*cos(a - I*x)/2 - I*exp(x)*c
os(a - I*x)/2, Eq(b, -I)), (x*exp(x)*sin(a + I*x)/2 - I*x*exp(x)*cos(a + I*
x)/2 + I*exp(x)*cos(a + I*x)/2, Eq(b, I)), (-b*exp(x)*cos(a + b*x)/(b**2 +
1) + exp(x)*sin(a + b*x)/(b**2 + 1), True))
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.95

$$-\left(\frac{b\cos\left(bx+a\right)}{b^2+1}-\frac{\sin\left(bx+a\right)}{b^2+1}\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*cos(b*x + a)/(b^2 + 1) - sin(b*x + a)/(b^2 + 1))*e^x
```

Mupad [B]
time = 0.10, size = 26, normalized size = 0.70

$$\frac{\mathrm{e}^x\left(\sin\left(a+b\,x\right)-b\,\cos\left(a+b\,x\right)\right)}{b^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(a + b*x),x)
```

```
[Out] (exp(x)*(sin(a + b*x) - b*cos(a + b*x)))/(b^2 + 1)
```

3.74 $\int e^x \sin(a+cx^2)\,dx$

Optimal. Leaf size=115

$$\frac{(-1)^{3/4}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*a+1/c))*erf(1/2*(-1)^(1/4)*(1+2*I*c*x)/c^(1/2))
*Pi^(1/2)/c^(1/2)+1/4*(-1)^(3/4)*erfi(1/2*(-1)^(1/4)*(1-2*I*c*x)/c^(1/2))*P
i^(1/2)/exp(1/4*I*(4*a+1/c))/c^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4560, 2266, 2235, 2236}

$$\frac{(-1)^{3/4}\sqrt{\pi}\,e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}\,e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[a + c*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c*
x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
(2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x \sin\left(a+cx^2\right)\,dx &= \int \left(\frac{1}{2} i e^{-ia+x-icx^2} - \frac{1}{2} i e^{ia+x+icx^2}\right) dx \\
&= \frac{1}{2} i \int e^{-ia+x-icx^2}\,dx - \frac{1}{2} i \int e^{ia+x+icx^2}\,dx \\
&= \frac{1}{2}\left(i e^{-\frac{1}{4} i\left(4a+\frac{1}{c}\right)}\right) \int e^{\frac{i(1-2icx)^2}{4c}}\,dx - \frac{1}{2}\left(i e^{\frac{1}{4} i\left(4a+\frac{1}{c}\right)}\right) \int e^{-\frac{i(1+2icx)^2}{4c}}\,dx \\
&= \frac{(-1)^{3/4} e^{\frac{1}{4} i\left(4a+\frac{1}{c}\right)} \sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} e^{-\frac{1}{4} i\left(4a+\frac{1}{c}\right)} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}\,(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 108, normalized size = 0.94

$$
-\frac{\sqrt[4]{-1}\, e^{-\frac{i}{4}/c} \sqrt{\pi} \left(e^{\frac{i}{2}/c}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-i+2cx)}{2\sqrt{c}}\right)(\cos(a)+i\sin(a)) + \mathrm{Erfi}\left(\frac{(-1)^{3/4}(i+2cx)}{2\sqrt{c}}\right)(i\cos(a)+\sin(a))\right)}{4\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[a + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sq
rt[c])]*(Cos[a] + I*Sin[a]) + Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(I
*Cos[a] + Sin[a])))/(Sqrt[c]*E^((I/4)/c))
```

Maple [A]
time = 0.19, size = 88, normalized size = 0.77

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{-ic}\,x-\frac{1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{ic}\,x-\frac{1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*x-1/2/
(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^
(1/2)*x-1/2/(I*c)^(1/2))
```

Maxima [**A**]
time = 0.29, size = 100, normalized size = 0.87

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i+1)\,\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i-1)\,\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx-1}{2\,\sqrt{i\,c}}\right)+\left(-(i-1)\,\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i+1)\,\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx+1}{2\,\sqrt{-i\,c}}\right)\right)}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(1/4*(4*a*c + 1)/c) + (I - 1)*sin(1/4*(
4*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt(I*c)) + (-(I - 1)*cos(1/4*(4*a*c
+ 1)/c) + (I + 1)*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*c))
)/sqrt(c)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(73) = 146$.
time = 2.91, size = 193, normalized size = 1.68

$$\frac{i\,\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}\,(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+i\,\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{4i\,ac+i}{4\,c}\right)}\,\mathrm{C}\left(-\frac{\sqrt{2}\,(2\,cx-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\,\mathrm{S}\left(\frac{\sqrt{2}\,(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{4i\,ac+i}{4\,c}\right)}\,\mathrm{S}\left(-\frac{\sqrt{2}\,(2\,cx-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_cos(1/2*sqrt(
2)*(2*c*x + I)*sqrt(c/pi)/c) + I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)
/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/
pi)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)
/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)/c)*fresnel_sin(-1/2*sqrt(2
)*(2*c*x - I)*sqrt(c/pi)/c))/c
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\sin\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*sin(a + c*x**2), x)
```

Giac [A]
time = 0.42, size = 127, normalized size = 1.10

$$-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4iac+i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4iac-i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + I/c)*(I*c/abs(c) + 1)*sqrt(
abs(c)))*e^(-1/4*(4*I*a*c + I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) + 1/4*I*s
qrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x - I/c)*(-I*c/abs(c) + 1)*sqrt(abs(c))
)*e^(-1/4*(-4*I*a*c - I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^x \sin\left(c x^2 + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(a + c*x^2),x)
```

```
[Out] int(exp(x)*sin(a + c*x^2), x)
```

3.75 $\int e^x \sin(a+bx+cx^2)\,dx$

Optimal. Leaf size=144

$$\frac{(-1)^{3/4}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}e^{-ia+\frac{i(i+b)^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*a+(1+I*b)^2/c))*erf(1/2*(-1)^(1/4)*(1+I*b+2*I*c
*x)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(3/4)*exp(-I*a+1/4*I*(I+b)^2/c)*erfi
(1/2*(-1)^(1/4)*(1-I*b-2*I*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4560, 2266, 2235, 2236}

$$\frac{(-1)^{3/4}\sqrt{\pi}\,e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}\,e^{\frac{i(b+i)^2}{4c}-ia}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + I
*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*E^((-I)*a + ((I/4)
*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \sin\left(a+bx+cx^2\right) dx &= \int \left(\frac{1}{2} i e^{-ia+(1-ib)x-icx^2} - \frac{1}{2} i e^{ia+(1+ib)x+icx^2}\right) dx \\ &= \frac{1}{2} i \int e^{-ia+(1-ib)x-icx^2} dx - \frac{1}{2} i \int e^{ia+(1+ib)x+icx^2} dx \\ &= -\left(\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\right) \int e^{-\frac{i(1+ib+2icx)^2}{4c}} dx\right) + \frac{1}{2}\left(ie^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\right) \int e^{\frac{i(1-ib-}{4c}} \\ &= \frac{(-1)^{3/4} e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)} \sqrt{\pi}\, \text{erf}\left(\frac{\sqrt[4]{-1}\,(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}} \sqrt{\pi}}{4\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 134, normalized size = 0.93

$$-\frac{\sqrt[4]{-1}\, e^{-\frac{i\left(1-2ib+b^2\right)}{4c}} \sqrt{\pi} \left(e^{\frac{i}{2}/c}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(-i+b+2cx)}{2\sqrt{c}}\right)(\cos(a)+i\sin(a)) + e^{\frac{ib^2}{2c}}\text{Erfi}\left(\frac{(-1)^{3/4}(i+b+2cx)}{2\sqrt{c}}\right)(i\cos(a)+\sin(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[a + b*x + c*x^2],x]
[Out] -1/4*((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b + 2*c*x))/(
2*Sqrt[c])]*(Cos[a] + I*Sin[a]) + E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b
 + 2*c*x))/(2*Sqrt[c])]*(I*Cos[a] + Sin[a])))/(Sqrt[c]*E^(((I/4)*(1 - (2*I)
*b + b^2))/c))
```

Maple [A]
time = 0.19, size = 119, normalized size = 0.83

method	result	size
risch	$\frac{i\sqrt{\pi}\,e^{\frac{i\left(4ac-b^2+2ib+1\right)}{4c}}\,\text{erf}\left(-\sqrt{-ic}\,x+\frac{ib+1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}} + \frac{i\sqrt{\pi}\,e^{-\frac{i\left(4ac-b^2-2ib+1\right)}{4c}}\,\text{erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)/(-I*c)^(1/2)*erf(-(-I*c)^(
1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(-b^2-2*I*b+4*a*
c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2*(1-I*b)/(I*c)^(1/2))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i+1)\cos\left(-\frac{b^2-4ac-1}{4c}\right)-(i-1)\sin\left(-\frac{b^2-4ac-1}{4c}\right)\right)\operatorname{erf}\left(\frac{i(2icx+ib-1)\sqrt{ic}}{2c}\right)+\left(-(i-1)\cos\left(-\frac{b^2-4ac-1}{4c}\right)+(i+1)\sin\left(-\frac{b^2-4ac-1}{4c}\right)\right)\operatorname{erf}\left(\frac{i(2icx+ib+1)\sqrt{-ic}}{2c}\right)\right)e^{\left(-\frac{b}{2c}\right)}}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I + 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I - 1)*sin
(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2*I*c*x + I*b - 1)*sqrt(I*c)/c) + (-
(I - 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (I + 1)*sin(-1/4*(b^2 - 4*a*c - 1)/
c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(91) = 182$.
time = 2.44, size = 229, normalized size = 1.59

$$\frac{i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^2-4iac-2b-i}{4c}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2cx+b+i)\sqrt{\frac{c}{\pi}}}{2c}\right)+i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-ib^2+4iac-2b+i}{4c}\right)}\mathrm{C}\left(-\frac{\sqrt{2}(2cx+b-i)\sqrt{\frac{c}{\pi}}}{2c}\right)+\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^2-4iac-2b-i}{4c}\right)}\mathrm{S}\left(\frac{\sqrt{2}(2cx+b+i)\sqrt{\frac{c}{\pi}}}{2c}\right)-\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-ib^2+4iac-2b+i}{4c}\right)}\mathrm{S}\left(-\frac{\sqrt{2}(2cx+b-i)\sqrt{\frac{c}{\pi}}}{2c}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_
cos(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) + I*sqrt(2)*pi*sqrt(c/pi)*e^(
1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I
)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/
c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(
c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)*(2*c*
x + b - I)*sqrt(c/pi)/c))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \sin\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sin(a + b*x + c*x**2), x)
```

Giac [A]
time = 0.42, size = 147, normalized size = 1.02

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b-i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac+2b-i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b+i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac+2b+i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b - I)/c)*(-I*c/abs(c) + 1)
*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((-I*c/abs(c) + 1)*sq
rt(abs(c))) - 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b + I)/c)*(I*
c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((I*c/a
bs(c) + 1)*sqrt(abs(c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x + c*x^2)*exp(x),x)
```

```
[Out] int(sin(a + b*x + c*x^2)*exp(x), x)
```

3.76 $\int e^{x^2} \sin(a+bx)\,dx$

Optimal. Leaf size=81

$$\frac{1}{4} i e^{-ia+\frac{b^2}{4}} \sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{ia+\frac{b^2}{4}} \sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*I*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(I*a+1/4*b^2)*er
fi(1/2*I*b+x)*Pi^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4560, 2266, 2235}

$$\frac{1}{4} i \sqrt{\pi}\, e^{\frac{b^2}{4}-ia} \text{Erfi}\left(\frac{1}{2}(2x-ib)\right) - \frac{1}{4} i \sqrt{\pi}\, e^{\frac{b^2}{4}+ia} \text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + b*x],x]
```

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\sin(a+bx)\,dx = \int \left(\frac{1}{2}ie^{-ia-ibx+x^2} - \frac{1}{2}ie^{ia+ibx+x^2}\right)\,dx$$
$$= \frac{1}{2}i\int e^{-ia-ibx+x^2}\,dx - \frac{1}{2}i\int e^{ia+ibx+x^2}\,dx$$
$$= \frac{1}{2}\left(ie^{-ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx - \frac{1}{2}\left(ie^{ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx$$
$$= \frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 1.00

$$\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\cos(a)\mathrm{Erf}\left(\frac{b}{2}+ix\right)+\left(\mathrm{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\mathrm{Erfi}\left(\frac{1}{2}(ib+2x)\right)\right)\sin(a)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi[
((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

Maple [A]
time = 0.00, size = 52, normalized size = 0.64

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2
)*exp(-I*a)*erf(I*x+1/2*b)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.63

$$\frac{1}{4}\sqrt{\pi}\left((\cos(a)-i\sin(a))\,\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}-(\cos(a)+i\sin(a))\,\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A]
time = 2.29, size = 45, normalized size = 0.56

$$-\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2+ia\right)}-\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2-ia\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1/
4*b^2 - I*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{x^2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(a + b*x),x)
```

```
[Out] int(exp(x^2)*sin(a + b*x), x)
```

3.77 $\int e^{x^2} \sin(a + cx^2)\, dx$

Optimal. Leaf size=87

$$\frac{ie^{-ia}\sqrt{\pi}\,\text{Erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}} - \frac{ie^{ia}\sqrt{\pi}\,\text{Erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}$$

```
[Out] 1/4*I*erfi(x*(1-I*c)^(1/2))*Pi^(1/2)/exp(I*a)/(1-I*c)^(1/2)-1/4*I*exp(I*a)*
erfi(x*(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4560, 2235}

$$\frac{i\sqrt{\pi}\,e^{-ia}\text{Erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}} - \frac{i\sqrt{\pi}\,e^{ia}\text{Erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + c*x^2],x]
```

```
[Out] ((I/4)*Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(Sqrt[1 - I*c]*E^(I*a)) - ((I/4)*E^(
I*a)*Sqrt[Pi]*Erfi[Sqrt[1 + I*c]*x])/Sqrt[1 + I*c]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2} \sin\left(a + cx^2\right)\, dx &= \int \left(\frac{1}{2} i e^{-ia+(1-ic)x^2} - \frac{1}{2} i e^{ia+(1+ic)x^2}\right) dx \\
&= \frac{1}{2} i \int e^{-ia+(1-ic)x^2}\, dx - \frac{1}{2} i \int e^{ia+(1+ic)x^2}\, dx \\
&= \frac{ie^{-ia}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}} - \frac{ie^{ia}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}
\end{aligned}$$

Mathematica [**A**]
time = 0.26, size = 129, normalized size = 1.48

$$-\frac{\sqrt[4]{-1}\,\sqrt{\pi}\left(\sqrt{-i+c}\,(i+c)\mathrm{Erfi}\left(\sqrt[4]{-1}\,\sqrt{-i+c}\,x\right)(\cos(a)+i\sin(a))+\sqrt{i+c}\left(\mathrm{Erf}\left(\frac{(1+i)\sqrt{i+c}\,x}{\sqrt{2}}\right)\sin(a)+\mathrm{Erfi}\left((-1)^{3/4}\sqrt{i+c}\,x\right)(\cos(a)+ic\cos(a)+c\sin(a))\right)\right)}{4\,(1+c^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[Pi]*(Sqrt[-I + c]*(I + c)*Erfi[(-1)^(1/4)*Sqrt[-I + c
]*x]*(Cos[a] + I*Sin[a]) + Sqrt[I + c]*(Erf[((1 + I)*Sqrt[I + c]*x)/Sqrt[2]
]*Sin[a] + Erfi[(-1)^(3/4)*Sqrt[I + c]*x]*(Cos[a] + I*c*Cos[a] + c*Sin[a]))
))/(1 + c^2)
```

Maple [**A**]
time = 0.11, size = 62, normalized size = 0.71

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(\sqrt{-ic-1}\,x\right)}{4\sqrt{-ic-1}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(\sqrt{ic-1}\,x\right)}{4\sqrt{ic-1}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)+1/4*I*Pi^(1/2
)*exp(-I*a)/(-1+I*c)^(1/2)*erf((-1+I*c)^(1/2)*x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(53) = 106$.
time = 0.29, size = 137, normalized size = 1.57

$$\frac{\sqrt{\pi}\,\sqrt{2\,c^2+2}\,\left((\cos(a)-i\,\sin(a))\,\mathrm{erf}\left(\sqrt{i\,c-1}\,x\right)+(\cos(a)+i\,\sin(a))\,\mathrm{erf}\left(\sqrt{-i\,c-1}\,x\right)\right)\sqrt{\sqrt{c^2+1}+1}-\sqrt{\pi}\,\sqrt{2\,c^2+2}\,\left((-i\,\cos(a)-\sin(a))\,\mathrm{erf}\left(\sqrt{i\,c-1}\,x\right)+(i\,\cos(a)-\sin(a))\,\mathrm{erf}\left(\sqrt{-i\,c-1}\,x\right)\right)\sqrt{\sqrt{c^2+1}-1}}{8\,(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((cos(a) - I*sin(a))*erf(sqrt(I*c - 1)*x) + (
cos(a) + I*sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(pi
)*sqrt(2*c^2 + 2)*((-I*cos(a) - sin(a))*erf(sqrt(I*c - 1)*x) + (I*cos(a) -
sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Fricas [**A**]
time = 2.51, size = 66, normalized size = 0.76

$$\frac{\sqrt{\pi}\,(c+i)\sqrt{-i\,c-1}\,\mathrm{erf}\left(\sqrt{-i\,c-1}\,x\right)e^{(i\,a)}+\sqrt{\pi}\,(c-i)\sqrt{i\,c-1}\,\mathrm{erf}\left(\sqrt{i\,c-1}\,x\right)e^{(-i\,a)}}{4\,(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(c + I)*sqrt(-I*c - 1)*erf(sqrt(-I*c - 1)*x)*e^(I*a) + sqrt(p
i)*(c - I)*sqrt(I*c - 1)*erf(sqrt(I*c - 1)*x)*e^(-I*a))/(c^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2} \sin(a + cx^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{x^2}\, \sin\left(c\, x^2 + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(a + c*x^2),x)
```

```
[Out] int(exp(x^2)*sin(a + c*x^2), x)
```

3.78 $\int e^{x^2} \sin\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=155

$$-\frac{ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\text{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{ie^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\text{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

```
[Out] -1/4*I*erfi(1/2*(I*b-2*(1-I*c)*x)/(1-I*c)^(1/2))*Pi^(1/2)/exp(I*(a-b^2/(4*I
+4*c)))/(1-I*c)^(1/2)-1/4*I*exp(I*a+1/4*b^2/(1+I*c))*erfi(1/2*(I*b+2*(1+I*c
)*x)/(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4560, 2266, 2235}

$$-\frac{i\sqrt{\pi}\,e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{i\sqrt{\pi}\,e^{ia+\frac{b^2}{4(1+ic)}}\text{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] ((-1/4*I)*Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 -
 I*c]*E^(I*(a - b^2/(4*I + 4*c)))) - ((I/4)*E^(I*a + b^2/(4*(1 + I*c)))*Sqr
t[Pi]*Erfi[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/Sqrt[1 + I*c]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx = \int \left(\frac{1}{2}ie^{-ia-ibx+(1-ic)x^2}-\frac{1}{2}ie^{ia+ibx+(1+ic)x^2}\right)\,dx$$
$$= \frac{1}{2}i\int e^{-ia-ibx+(1-ic)x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+(1+ic)x^2}\,dx$$
$$= -\left(\frac{1}{2}\left(ie^{ia+\frac{b^2}{4(1+ic)}}\right)\int \exp\left(\frac{(ib+2(1+ic)x)^2}{4(1+ic)}\right)\,dx\right)+\frac{1}{2}\left(ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\right)\int e$$
$$= -\frac{ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{ie^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

Mathematica [A]
time = 0.62, size = 165, normalized size = 1.06

$$-\frac{(-1)^{3/4}e^{\frac{ib^2}{4i-4c}}\sqrt{\pi}\left((-i+c)\sqrt{i+c}\,e^{\frac{ib^2c}{2+2c^2}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(b+2(i+c)x)}{2\sqrt{i+c}}\right)(\cos(a)-i\sin(a))+\sqrt{-i+c}\,(i+c)\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(b+2(-i+c)x)}{2\sqrt{-i+c}}\right)(-i\cos(a)+\sin(a))\right)}{4\left(1+c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*((-I + c)*Sqrt[I + c]*E^(
(I*b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]
*(Cos[a] - I*Sin[a]) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I + c
)*x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(1 + c^2)
```

Maple [A]
time = 0.24, size = 129, normalized size = 0.83

method	result	siz
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-4ia-b^2}{4(ic+1)}}\,\mathrm{erf}\left(-\sqrt{-ic-1}\,x+\frac{ib}{2\sqrt{-ic-1}}\right)}{4\sqrt{-ic-1}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{4ac+4ia-b^2}{4ic-4}}\,\mathrm{erf}\left(\sqrt{ic-1}\,x+\frac{ib}{2\sqrt{ic-1}}\right)}{4\sqrt{ic-1}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(-1/4*(4*a*c-4*I*a-b^2)/(1+I*c))/(-I*c-1)^(1/2)*erf(-(-I*
c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))+1/4*I*Pi^(1/2)*exp(1/4*(4*a*c+4*I*a-b^
2)/(-1+I*c))/(-1+I*c)^(1/2)*erf((-1+I*c)^(1/2)*x+1/2*I*b/(-1+I*c)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(101) = 202.
time = 0.30, size = 475, normalized size = 3.06

$$\frac{\sqrt{\pi}\sqrt{2c^2+2}\left(\left(\left(\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}-ie^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\mathrm{erf}\left(-\frac{2(-ic+1)x-ib}{2\sqrt{ic-1}}\right)-\left(\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}+ie^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\mathrm{erf}\left(-\frac{2(-ic-1)x-ib}{2\sqrt{-ic-1}}\right)\right)\sqrt{\sqrt{c^2+1}+1}-\sqrt{\pi}\sqrt{2c^2+2}\left(\left(-i\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}-e^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\mathrm{erf}\left(-\frac{2(-ic+1)x-ib}{2\sqrt{ic-1}}\right)+\left(-i\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}+e^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\mathrm{erf}\left(-\frac{2(-ic-1)x-ib}{2\sqrt{-ic-1}}\right)\right)\sqrt{\sqrt{c^2+1}-1}}{8(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))
*e^(1/4*b^2/(c^2 + 1)) - I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2
- 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) - (cos(-1
/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + I*e^(1/4*b^2/
(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c -
 1)*x - I*b)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(pi)*sqrt(2*c^2
 + 2)*((-I*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)
) - e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(
-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) + (-I*cos(-1/4*(b^2*c - 4*a*c^2
- 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b
^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c - 1)*x - I*b)/sqrt(-I*c
 - 1)))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Fricas [A]
time = 2.51, size = 161, normalized size = 1.04

$$-\frac{\sqrt{\pi}\,(c-i)\sqrt{i\,c-1}\,\operatorname{erf}\left(-\frac{(bc+2\,(c^2+1)x-i\,b)\sqrt{i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\,(c^2+1)}\right)}-\sqrt{\pi}\,(c+i)\sqrt{-i\,c-1}\,\operatorname{erf}\left(\frac{(bc+2\,(c^2+1)x+i\,b)\sqrt{-i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{-i\,b^2c+4i\,ac^2+b^2+4i\,a}{4\,(c^2+1)}\right)}}{4\,(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c - I)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*s
qrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 + 1
)) - sqrt(pi)*(c + I)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)*sq
rt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^2 +
1)))/(c^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x + c*x^2)*exp(x^2),x)
```

```
[Out] int(sin(a + b*x + c*x^2)*exp(x^2), x)
```

3.79 $\int f^{a+bx} \sin(d + fx^2) \, dx$

Optimal. Leaf size=142

$$\frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Er}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)
*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(3/4)*f^(-1/2+a)*erfi(1/2*(-1
)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A]
time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[((
-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*f^(-1/2 + a
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^((I/
4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\sin\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-ifx^2}f^{a+bx}-\frac{1}{2}ie^{id+ifx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+bx}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}i\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx-\frac{1}{2}i\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx\\ &= \frac{1}{2}\left(ie^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(}{}}\\ &= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}e^{-}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 132, normalized size = 0.93

$$-\frac{1}{4}\sqrt[4]{-1}\,e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))+\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(i\cos(d)+\sin(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-
1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Erfi[((-1
)^(3/4)*(2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(I*Cos[d] + Sin[d])))/E^(((I/4)*
b^2*Log[f]^2)/f)
```

Maple [A]
time = 0.43, size = 116, normalized size = 0.82

method	result	size

risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{-i f}\, x+\frac{\ln(f) b}{2 \sqrt{-i f}}\right)}{4 \sqrt{-i f}}-\frac{i \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{i f}\, x+\frac{\ln(f) b}{2 \sqrt{i f}}\right)}{4 \sqrt{i f}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f
)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*
b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))
```

Maxima [A]
time = 0.28, size = 147, normalized size = 1.04

$$-\frac{\sqrt{2} \sqrt{\pi} \left(\left(-(i+1) f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)+(i-1) f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)\right) \operatorname{erf}\left(\frac{2 i f x-b \log(f)}{2 \sqrt{i f}}\right)+\left(-(i-1) f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)+(i+1) f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)\right) \operatorname{erf}\left(\frac{2 i f x+b \log(f)}{2 \sqrt{-i f}}\right)\right)}{8 \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I
 - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x - b*log(f))/s
qrt(I*f)) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I + 1)*f^a*s
in(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x + b*log(f))/sqrt(-I*f)))
/sqrt(f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(98) = 196.
time = 3.21, size = 265, normalized size = 1.87

$$\frac{i \sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+4 a f \log(f)-4 i d f}{4 f}\right)} \operatorname{C}\left(\frac{\sqrt{2} (2 f x+i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)+i \sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+4 a f \log(f)+4 i d f}{4 f}\right)} \operatorname{C}\left(-\frac{\sqrt{2} (2 f x-i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)+\sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+4 a f \log(f)-4 i d f}{4 f}\right)} \operatorname{S}\left(\frac{\sqrt{2} (2 f x+i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)-\sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+4 a f \log(f)+4 i d f}{4 f}\right)} \operatorname{S}\left(-\frac{\sqrt{2} (2 f x-i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d
*f)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + I*sqrt(
2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresne
l_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/p
i)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqr
t(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b
^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x -
I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sin\left(d+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(98) = 196.
time = 0.45, size = 300, normalized size = 2.11

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x-\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^{2}b^{2}\operatorname{sgn}(f)}{8f}+\frac{\pi b^{2}\log(|f|)\operatorname{sgn}(f)}{4f}-\frac{i\pi^{2}b^{2}}{8f}-\frac{\pi b^{2}\log(|f|)}{4f}+\frac{ib^{2}\log(|f|)^{2}}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a+a\log(|f|)+id\right)}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x+\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^{2}b^{2}\operatorname{sgn}(f)}{8f}-\frac{\pi b^{2}\log(|f|)\operatorname{sgn}(f)}{4f}+\frac{i\pi^{2}b^{2}}{8f}+\frac{\pi b^{2}\log(|f|)}{4f}-\frac{ib^{2}\log(|f|)^{2}}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a+a\log(|f|)-id\right)}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
+ 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f
))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(a
bs(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sqrt(pi)*erf
(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f
) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*s
gn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f
))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((I*f/abs(f)
 + 1)*sqrt(abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sin(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sin(d + f*x^2), x)
```

3.80 $\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=157

$$\left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+(1/16+1/16*I)*exp(2*I*d+1/8*I*b^2*ln(f)^2/f)*f^(-1/2+
a)*erf((1/4+1/4*I)*(4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)+(1/16+1/16*I)*f^(-1/
2+a)*erfi((1/4+1/4*I)*(4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/8*I*(16*d+b
^2*ln(f)^2/f))
```

Rubi [A]
time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4560, 2225, 2325, 2266, 2235, 2236}

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\text{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] + ((1/16 + I/16)*f^(-1/2 +
a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(1
6*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+bx}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx}\right)\,dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\int e^{-2id-2ifx^2+a\log(f)+bx\log(f)}\,dx-\frac{1}{4}\int e^{2id+2ifx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\left(e^{2id+\frac{ib^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{i(4ifx+b\log(f))^2}{8f}}\,dx-\frac{1}{4}\left(e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{a}\right.\\
&= \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)+\left(\frac{1}{16}+\right.
\end{aligned}
$$

Mathematica [A]
time = 1.15, size = 156, normalized size = 0.99

$$
\frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)}-\frac{(1-i)e^{-\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\,\text{Erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)-i\sin(d))^2}{\sqrt{f}}-\frac{(1-i)e^{\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\,\text{Erfi}\left(\frac{(4+4i)fx+(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)+i\sin(d))^2}{\sqrt{f}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) - ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)/f
```

```
)*Sqrt[f]) - ((1 - I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I)*f
*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] + I*Sin[d])^2)/Sqrt[f]))/16
```

Maple [A]
time = 0.68, size = 139, normalized size = 0.89

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{i f}}\right)}{16 \sqrt{i f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \operatorname{erf}\left(-\sqrt{-2 i f}\, x+\frac{b \ln}{2 \sqrt{-}} \right.}{8 \sqrt{-2 i f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*er
f(-2^(1/2)*(I*f)^(1/2)*x+1/4*b*ln(f)*2^(1/2)/(I*f)^(1/2))+1/8*Pi^(1/2)*f^a*
exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*
b*ln(f)/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [A]
time = 0.50, size = 186, normalized size = 1.18

$$\frac{4^{\frac{1}{4}} \sqrt{2} \sqrt{\pi} \left(\left(\left(i-1\right) b f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+16 d f}{8 f}\right) \log\left(f\right)+\left(i+1\right) b f^{a} \log\left(f\right) \sin\left(\frac{b^{2} \log(f)^{2}+16 d f}{8 f}\right)\right) \operatorname{erf}\left(\frac{4 i f x-b \log(f)}{2 \sqrt{2 i f}}\right)+\left(\left(i+1\right) b f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+16 d f}{8 f}\right) \log\left(f\right)+\left(i-1\right) b f^{a} \log\left(f\right) \sin\left(\frac{b^{2} \log(f)^{2}+16 d f}{8 f}\right)\right) \operatorname{erf}\left(\frac{4 i f x+b \log(f)}{2 \sqrt{-2 i f}}\right)\right) f^{\frac{3}{2}}+16 f^{b x} f^{a+2}}{32 b f^{2} \log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*d
*f)/f)*log(f) + (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f)/f))*er
f(1/2*(4*I*f*x - b*log(f))/sqrt(2*I*f)) + ((I + 1)*b*f^a*cos(1/8*(b^2*log(f
)^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d
*f)/f))*erf(1/2*(4*I*f*x + b*log(f))/sqrt(-2*I*f)))*f^(3/2) + 16*f^(b*x)*f^
(a + 2))/(b*f^2*log(f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(103) = 206.
time = 2.17, size = 270, normalized size = 1.72

$$-\frac{\pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+8 a f \log(f)-16 i d f}{8 f}\right)} \mathrm{C}\left(\frac{\left(4 f x+i b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2 f}\right) \log\left(f\right)-\pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+8 a f \log(f)+16 i d f}{8 f}\right)} \mathrm{C}\left(-\frac{\left(4 f x-i b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2 f}\right) \log\left(f\right)-i \pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+8 a f \log(f)-16 i d f}{8 f}\right)} \mathrm{S}\left(\frac{\left(4 f x+i b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2 f}\right) \log\left(f\right)-i \pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+8 a f \log(f)+16 i d f}{8 f}\right)} \mathrm{S}\left(-\frac{\left(4 f x-i b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2 f}\right) \log\left(f\right)-4 f f^{b x+a}}{8 b f \log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)
*fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - pi*b*sqrt(f/pi
```

```
)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2*(4*
f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*l
og(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f))*
sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log
(f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(
f) - 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(103) = 206.
time = 0.46, size = 521, normalized size = 3.32

$$\left(\frac{2\,b\cos\left(-\frac{1}{2}\,\pi bx\operatorname{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\operatorname{sgn}(f)+\frac{1}{2}\,\pi a\right)\log\left(|f|\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\operatorname{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\,\pi bx\operatorname{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\operatorname{sgn}(f)+\frac{1}{2}\,\pi a\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\operatorname{sgn}(f)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\operatorname{sgn}(f)-\frac{1}{2}i\,\pi a\right)}}{2i\,\pi b\operatorname{sgn}\left(f\right)-2i\,\pi b+4\,b\log\left(|f|\right)}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\operatorname{sgn}(f)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\operatorname{sgn}(f)+\frac{1}{2}i\,\pi a\right)}}{-2i\,\pi b\operatorname{sgn}\left(f\right)+2i\,\pi b+4\,b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{f}\left(8\,x-\frac{\pi b\operatorname{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(-\frac{i\,f}{|f|}+1\right)\right)e^{\left(\frac{i\,\pi^2b^2\operatorname{sgn}(f)}{16\,f}+\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{8\,f}-\frac{i\,\pi^2b^2}{16\,f}-\frac{\pi b^2\log(|f|)}{8\,f}+\frac{i\,b^2\log(|f|)^2}{8\,f}-\frac{1}{2}i\,\pi a\operatorname{sgn}(f)+\frac{1}{2}i\,\pi a+a\log(|f|)+2i\,d\right)}}{8\sqrt{f}\left(-\frac{i\,f}{|f|}+1\right)}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{f}\left(8\,x+\frac{\pi b\operatorname{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(\frac{i\,f}{|f|}+1\right)\right)e^{\left(-\frac{i\,\pi^2b^2\operatorname{sgn}(f)}{16\,f}-\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{8\,f}+\frac{i\,\pi^2b^2}{16\,f}+\frac{\pi b^2\log(|f|)}{8\,f}-\frac{i\,b^2\log(|f|)^2}{8\,f}-\frac{1}{2}i\,\pi a\operatorname{sgn}(f)+\frac{1}{2}i\,\pi a+a\log(|f|)-2i\,d\right)}}{8\sqrt{f}\left(\frac{i\,f}{|f|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) + 1/8*sqr
t(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(
-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs(f))*sgn(
f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*log(abs(f))
^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*d)/(sqrt(f)*(-I
*f/abs(f) + 1)) + 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (pi*b*sgn(f) - pi*b
+ 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b^2*sgn(f)/f - 1/
8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*b^2*log(abs(f))/
f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(
f)) - 2*I*d)/(sqrt(f)*(I*f/abs(f) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sin(d + f*x^2)^2,x)
[Out] int(f^(a + b*x)*sin(d + f*x^2)^2, x)
```

3.81 $\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=298

$$\frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\text{Erf}$$

```
[Out] (1/96-1/96*I)*exp(3*I*d+1/12*I*b^2*ln(f)^2/f)*f^(-1/2+a)*erf((1/12+1/12*I)*
(6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+(-1/96+1/96*I)*f^(-1/2+
a)*erfi((1/12+1/12*I)*(6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)/e
xp(1/12*I*(36*d+b^2*ln(f)^2/f))+3/16*(-1)^(3/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/
f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*
(-1)^(3/4)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/
2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A]
time = 0.25, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}\,f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)-\frac{3}{16}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}\,f^{a-\frac{1}{2}}e^{-\frac{1}{12}i\left(\frac{b^2\log^2(f)}{f}+36d\right)}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[
((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/16)*E^((3*
I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*((
6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(3/4)*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4*
d + (b^2*Log[f]^2)/f))) - ((1/16 - I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/2
 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2*
Log[f]^2)/f))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx &= \int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}ie^{3id+3ifx^2}f^{a-}\right.\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+bx}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+bx}\,dx+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+bx}\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2+a\log(f)+bx\log(f)}\,dx+\\
&= \frac{1}{8}\left(ie^{3id+\frac{ib^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{i(6ifx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\left(3ie^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-}{}}\\
&= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\right.
\end{aligned}
$$

Mathematica [A]
time = 0.90, size = 268, normalized size = 0.90

$$
\frac{1}{48}(-1)^{3/4}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-9\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+9ie^{\frac{ib^2\log^2(f)}{2f}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))+\sqrt{3}\,e^{\frac{ib^2\log^2(f)}{6f}}\left(\mathrm{Erfi}\left(\frac{(-1)^{3/4}(6fx+ib\log(f))}{2\sqrt{3}\,\sqrt{f}}\right)(\cos(3d)-i\sin(3d))+e^{\frac{ib^2\log^2(f)}{6f}}\mathrm{Erfi}\left(\frac{(6+6i)fx+(1-i)b\log(f)}{2\sqrt{6}\,\sqrt{f}}\right)(-i\cos(3d)+\sin(3d))\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(3/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f])
)/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + (9*I)*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[
((-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Sqrt[3
```

]*E^(((I/6)*b^2*Log[f]^2)/f)*(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2*Sqr
t[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d]) + E^(((I/6)*b^2*Log[f]^2)/f)*Erfi[((
6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[
3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))

Maple [A]
time = 0.96, size = 239, normalized size = 0.80

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+36 d f\right)}{12 f}} \operatorname{erf}\left(-\sqrt{-3 i f}\, x+\frac{\ln(f) b}{2\sqrt{-3 i f}}\right)}{16\sqrt{-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36 d f\right)}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\,\sqrt{i f}\, x+\frac{\ln(\ldots}{6\ldots}\right.}{48\sqrt{i f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(
-(-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-3*I*f)^(1/2))+1/48*I*Pi^(1/2)*f^a*exp(-1/12
*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1
/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2
+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))+3/16*I*P
i^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)
*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

Maxima [A]
time = 0.52, size = 302, normalized size = 1.01

$$\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\, f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+36\, df}{12\, f}\right)+(i-1)\, f^{a}\sin\left(\frac{b^{2}\log(f)^{2}+36\, df}{12\, f}\right)\right)\operatorname{erf}\left(\frac{6i\, fx-b\log(f)}{2\sqrt{3i\, f}}\right)+\left(-(i-1)\, f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+36\, df}{12\, f}\right)+(i+1)\, f^{a}\sin\left(\frac{b^{2}\log(f)^{2}+36\, df}{12\, f}\right)\right)\operatorname{erf}\left(\frac{6i\, fx+b\log(f)}{2\sqrt{-3i\, f}}\right)\right)f^{\frac{3}{2}}-9\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\, f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+4\, df}{4\, f}\right)+(i-1)\, f^{a}\sin\left(\frac{b^{2}\log(f)^{2}+4\, df}{4\, f}\right)\right)\operatorname{erf}\left(\frac{2i\, fx-b\log(f)}{2\sqrt{i\, f}}\right)+\left(-(i-1)\, f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+4\, df}{4\, f}\right)+(i+1)\, f^{a}\sin\left(\frac{b^{2}\log(f)^{2}+4\, df}{4\, f}\right)\right)\operatorname{erf}\left(\frac{2i\, fx+b\log(f)}{2\sqrt{-i\, f}}\right)\right)f^{\frac{3}{2}}}{96\, f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d
*f)/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x
- b*log(f))/sqrt(3*I*f)) + (-(I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*f)/f
) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x + b*l
og(f))/sqrt(-3*I*f)))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/4*(
b^2*log(f)^2 + 4*d*f)/f) + (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*e
rf(1/2*(2*I*f*x - b*log(f))/sqrt(I*f)) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^
2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I
*f*x + b*log(f))/sqrt(-I*f)))*f^(3/2))/f^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(196) = 392$.
time = 3.04, size = 525, normalized size = 1.76

$$\frac{-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i b^{2}\log(f)^{2}+12 a f\log(f)-36 i df}{12 f}\right)}\mathrm{C}\left(\frac{\sqrt{6}(6fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{6f}\right)-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i b^{2}\log(f)^{2}+12 a f\log(f)+36 i df}{12 f}\right)}\mathrm{C}\left(-\frac{\sqrt{6}(6fx-ib\log(f))\sqrt{\frac{f}{\pi}}}{6f}\right)+9i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i b^{2}\log(f)^{2}+4 a f\log(f)-4 i df}{4 f}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)+9i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i b^{2}\log(f)^{2}+4 a f\log(f)+4 i df}{4 f}\right)}\mathrm{C}\left(-\frac{\sqrt{2}(2fx-ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)-\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i b^{2}\log(f)^{2}+12 a f\log(f)-36 i df}{12 f}\right)}\mathrm{S}\left(\frac{\sqrt{6}(6fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{6f}\right)+\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i b^{2}\log(f)^{2}+12 a f\log(f)+36 i df}{12 f}\right)}\mathrm{S}\left(-\frac{\sqrt{6}(6fx-ib\log(f))\sqrt{\frac{f}{\pi}}}{6f}\right)+9\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i b^{2}\log(f)^{2}+4 a f\log(f)-4 i df}{4 f}\right)}\mathrm{S}\left(\frac{\sqrt{2}(2fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)-9\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i b^{2}\log(f)^{2}+4 a f\log(f)+4 i df}{4 f}\right)}\mathrm{S}\left(-\frac{\sqrt{2}(2fx-ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)}{48 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 3
6*I*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*
sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f
)*fresnel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)
*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel
_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f
/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*s
qrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(
-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6*f
*x + I*b*log(f))*sqrt(f/pi)/f) + sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f
)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*lo
g(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a
*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f
/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4
*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2)**3, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(196) = 392.
time = 0.50, size = 595, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
 + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(
f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(
abs(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*I*sqrt(6)*sqrt(pi)*e
rf(-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f
)*(-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(f))*
```

```
sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*log(a
bs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/(sqrt(
f)*(-I*f/abs(f) + 1)) + 1/48*I*sqrt(6)*sqrt(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(
12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/
24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2
/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sg
n(f) + 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(I*f/abs(f) + 1)) - 3/1
6*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*lo
g(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f -
1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))
/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs
(f)) - I*d)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sin(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sin(d + f*x^2)^3, x)
```

3.82 $\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=162

$$\frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(
1/4)*(I*e+2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(3/4)*exp(-I*d+1/4*I*
(e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(
1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*E
^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1)^(
1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+bx}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+bx}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+bx}\,dx\\
&= \frac{1}{2}i\int\exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id+\right.\\
&= \frac{1}{2}\left(ie^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right.\\
&= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)-
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 162, normalized size = 1.00

$$-\frac{1}{4}\sqrt[4]{-1}\,e^{-\frac{i\left(e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(e^{\frac{ib^2\log^2(f)}{2f}}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))+e^{\frac{ie^2}{2f}}\text{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(i\cos(d)+\sin(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/
f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[
d]) + E^(((I/2)*e^2)/f)*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[
f])]*(I*Cos[d] + Sin[d])))/E^(((I/4)*(e^2 + b^2*Log[f]^2))/f)
```

Maple [A]
time = 0.41, size = 152, normalized size = 0.94

method	result

risch	$\frac{i\sqrt{\pi}\, f^a \mathrm{e}^{\frac{i\left(\ln(f)^2 b^2+2i\ln(f)be-e^2+4df\right)}{4f}} \operatorname{erf}\left(-\sqrt{-if}\, x+\frac{ie+b\ln(f)}{2\sqrt{-if}}\right)}{4\sqrt{-if}} - \frac{i\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{i\left(\ln(f)^2 b^2-2i\ln(f)be-e^2+4df\right)}{4f}} \operatorname{erf}\left(-\sqrt{i}\right.}{4\sqrt{if}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f
)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/4*I*Pi^(1/2)*
f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I
*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))
```

Maxima [A]
time = 0.29, size = 189, normalized size = 1.17

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i+1)\, f^a \cos\left(\frac{b^2\log(f)^2+4\,df-e^2}{4\,f}\right)-(i-1)\, f^a \sin\left(\frac{b^2\log(f)^2+4\,df-e^2}{4\,f}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,fx-b\log(f)+i\,e)\sqrt{i\,f}}{2\,f}\right)+\left(-(i-1)\, f^a \cos\left(\frac{b^2\log(f)^2+4\,df-e^2}{4\,f}\right)+(i+1)\, f^a \sin\left(\frac{b^2\log(f)^2+4\,df-e^2}{4\,f}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,fx+b\log(f)+i\,e)\sqrt{-i\,f}}{2\,f}\right)\right)}{8\sqrt{f}\, f^{\frac{be}{2\,f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f)
 - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))*erf(1/2*I*(2*I*f*x
- b*log(f) + I*e)*sqrt(I*f)/f) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*
f - e^2)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))*erf(1/2*
I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))/(sqrt(f)*f^(1/2*b*e/f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(113) = 226.
time = 2.19, size = 321, normalized size = 1.98

$$\frac{i\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\,b^2\log(f)^2-4i\,df+2\,(2\,af-be)\log(f)+i\,e^2}{4\,f}\right)} \mathrm{C}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)+i\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\,b^2\log(f)^2+4i\,df+2\,(2\,af-be)\log(f)-i\,e^2}{4\,f}\right)} \mathrm{C}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)+\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\,b^2\log(f)^2-4i\,df+2\,(2\,af-be)\log(f)+i\,e^2}{4\,f}\right)} \mathrm{S}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\,b^2\log(f)^2+4i\,df+2\,(2\,af-be)\log(f)-i\,e^2}{4\,f}\right)} \mathrm{S}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 - 4*I*d*f + 2*(2*a*f -
 b*e)*log(f) + I*e^2)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*s
qrt(f/pi)/f) + I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*I*d*f + 2
*(2*a*f - b*e)*log(f) - I*e^2)/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log
(f) + e)*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 - 4*
I*d*f + 2*(2*a*f - b*e)*log(f) + I*e^2)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x +
 I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)
^2 + 4*I*d*f + 2*(2*a*f - b*e)*log(f) - I*e^2)/f)*fresnel_sin(-1/2*sqrt(2)*
(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(109) = 218.
time = 0.46, size = 378, normalized size = 2.33

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x-\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)-2e}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\operatorname{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{i\pi be\operatorname{sgn}(f)}{4f}+\frac{1}{2}i\pi a-\frac{i\pi be}{4f}+a\log(|f|)-\frac{be\log(|f|)}{2f}+id-\frac{ie^2}{4f}\right)}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x+\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)+2e}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\operatorname{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^2\log(|f|)}{4f}-\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{i\pi be\operatorname{sgn}(f)}{4f}+\frac{1}{2}i\pi a-\frac{i\pi be}{4f}+a\log(|f|)-\frac{be\log(|f|)}{2f}-id+\frac{ie^2}{4f}\right)}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn
(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log
(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*s
gn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))
/f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sq
rt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*
e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b
^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4
*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*
pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I
*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\sin\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sin(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sin(d + e*x + f*x^2), x)
```

3.83 $\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=179

$$\left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)e^{-2id+\frac{i(2e+ib\log(f}{8f}}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+(1/16+1/16*I)*exp(2*I*d+1/8*I*(2*I*e+b*ln(f))^2/f)*f^
(-1/2+a)*erf((1/4+1/4*I)*(2*I*e+4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)+(1/16+1/
16*I)*exp(-2*I*d+1/8*I*(2*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/4+1/4*I)*(2*
I*e+4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4560, 2225, 2325, 2266, 2235, 2236}

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\mathrm{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*S
qrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] + (1/16
 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*x
)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+bx}-\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx}-\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx}\right)dx \\
&= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx}\,dx\right)-\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a-} \\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\int \exp\left(-2id-2ifx^2+a\log(f)-x(2ie-b\log(f))\right)dx- \\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\exp\left(-2id+a\log(f)-\frac{i(-2ie+b\log(f))^2}{8f}\right)\int e^{\frac{i(-2ie-4ifx+}{8f}} \\
&= \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f)}{\sqrt{f}}\right.
\end{aligned}$$

Mathematica [A]
time = 1.19, size = 244, normalized size = 1.36

$$\frac{e^{-\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{a-\frac{be+f}{2f}}\left(8e^{\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+\sqrt[4]{-1}\,be^{\frac{ib^2\log^2(f)}{4f}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2i(e+2fx)+b\log(f))}{\sqrt{f}}\right)\log(f)(\cos(2d)+i\sin(2d))+\sqrt[4]{-1}\,be^{\frac{ie^2}{f}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2e+4fx+ib\log(f))}{\sqrt{f}}\right)\log(f)(i\cos(2d)+\sin(2d))\right)}{16b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 + b
*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erf[((
1/4 + I/4)*((2*I)*(e + 2*f*x) + b*Log[f]))/Sqrt[f]]*Log[f]*(Cos[2*d] + I*Si
n[2*d]) + (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e + 4*f
```

```
*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^(((I/8)
*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

Maple [A]
time = 0.73, size = 175, normalized size = 0.98

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}-4 i \ln(f) b e-4 e^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{(b \ln(f)-2 i e) \sqrt{2}}{4 \sqrt{i f}}\right)}{16 \sqrt{i f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4 i \ln(f) b e-4 e^{2} \ldots\right.}{8 f}} \ldots}{8 \ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2-4*I*ln(f)*b*e-4*e^2+16*d*f)/f)*2^
(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)/(I
*f)^(1/2))+1/8*Pi^(1/2)*f^a*exp(1/8*I*(ln(f)^2*b^2+4*I*ln(f)*b*e-4*e^2+16*d
*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I*f)^(1
/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [A]
time = 0.49, size = 240, normalized size = 1.34

$$\frac{4^{\frac{1}{4}} \sqrt{2} \sqrt{\pi} \left(\left(-(i-1)\, b f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right) \log(f)-(i+1)\, b f^{a} \log(f) \sin\left(\frac{b^{2} \log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right)\right) \operatorname{erf}\left(\frac{i\,(4 i\, f x-b \log(f)+2 i\, e) \sqrt{2 i\, f}}{4\, f}\right)+\left((i+1)\, b f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right) \log(f)+(i-1)\, b f^{a} \log(f) \sin\left(\frac{b^{2} \log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right)\right) \operatorname{erf}\left(\frac{i\,(4 i\, f x+b \log(f)+2 i\, e) \sqrt{-2 i\, f}}{4\, f}\right)\right) f^{\frac{3}{2}}+16\, f^{a+2} e^{\left(b x \log(f)+\frac{b e \log(f)}{2\, f}\right)}}{32\, b f^{2} f^{\frac{b e}{2\, f}} \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*
d*f - 4*e^2)/f)*log(f) - (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*
f - 4*e^2)/f))*erf(1/4*I*(4*I*f*x - b*log(f) + 2*I*e)*sqrt(2*I*f)/f) + ((I
+ 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*d*f - 4*e^2)/f)*log(f) + (I - 1)*b*f^
a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f - 4*e^2)/f))*erf(1/4*I*(4*I*f*x + b
*log(f) + 2*I*e)*sqrt(-2*I*f)/f))*f^(3/2) + 16*f^(a + 2)*e^(b*x*log(f) + 1/
2*b*e*log(f)/f))/(b*f^2*f^(1/2*b*e/f)*log(f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(120) = 240$.
time = 2.71, size = 334, normalized size = 1.87

$$-\frac{\pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\, b^{2} \log(f)^{2}-16 i\, d f+4\,(2\, a f-b e) \log(f)+4 i\, e^{2}}{8\, f}\right)} \mathrm{C}\left(\frac{(4\, f x+i\, b \log(f)+2\, e) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log(f)-\pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\, b^{2} \log(f)^{2}+16 i\, d f+4\,(2\, a f-b e) \log(f)-4 i\, e^{2}}{8\, f}\right)} \mathrm{C}\left(-\frac{(4\, f x-i\, b \log(f)+2\, e) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log(f)-i\, \pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\, b^{2} \log(f)^{2}-16 i\, d f+4\,(2\, a f-b e) \log(f)+4 i\, e^{2}}{8\, f}\right)} \mathrm{S}\left(\frac{(4\, f x+i\, b \log(f)+2\, e) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log(f)-i\, \pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\, b^{2} \log(f)^{2}+16 i\, d f+4\,(2\, a f-b e) \log(f)-4 i\, e^{2}}{8\, f}\right)} \mathrm{S}\left(-\frac{(4\, f x-i\, b \log(f)+2\, e) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log(f)-4\, f f^{b x+a} \ldots}{8\, b f \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

[Out] -1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 - 16*I*d*f + 4*(2*a*f - b*e)*
log(f) + 4*I*e^2)/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi)/
f)*log(f) - pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 16*I*d*f + 4*(2*a*f -
b*e)*log(f) - 4*I*e^2)/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*sqrt(
f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 - 16*I*d*f + 4*
(2*a*f - b*e)*log(f) + 4*I*e^2)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f) + 2*
e)*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 16*I*d
f + 4(2*a*f - b*e)*log(f) - 4*I*e^2)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log
(f) + 2*e)*sqrt(f/pi)/f)*log(f) - 4*f*f^(b*x + a))/(b*f*log(f))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**2,x)

[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**2, x)

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(116) = 232.
time = 0.48, size = 599, normalized size = 3.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")

[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) + 1/8*sqr
t(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) - 4*e
)/f)*(-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs(f)
)*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*log(a
bs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I
*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d - 1/2*I*e^2/f)/(s
qrt(f)*(-I*f/abs(f) + 1)) + 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (pi*b*sgn(
f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b
^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*

```
b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*p
i*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(
abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sin(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*sin(d + e*x + f*x^2)^2, x)
```

3.84 $\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=340

$$\frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}}$$

```
[Out] (1/96-1/96*I)*exp(3*I*d+1/12*I*(3*I*e+b*ln(f))^2/f)*f^(-1/2+a)*erf((1/12+1/
12*I)*(3*I*e+6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+(-1/96+1/96
*I)*exp(-3*I*d+1/12*I*(3*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/12+1/12*I)*(3
*I*e+6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+3/16*(-1)^(3/4)*exp
(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(I*e+2*I*f*x+
b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*(-1)^(3/4)*exp(-I*d+1/4*I*(e+I*b*ln(f))^2/f
)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}\,f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)-\frac{3}{16}(-1)^{3/4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}\,f^{a-\frac{1}{2}}e^{\frac{i(3e+ib\log(f))^2}{12f}-3id}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*
Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/
16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]
*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3
*(-1)^(3/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]
*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 -
I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx = \int\left(-\frac{1}{8}ie^{-3i\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+\right.\right.\right.$$

$$= -\left(\frac{1}{8}i\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx}\,dx\right)+\frac{1}{8}i\int\exp\left(6id+6iex+6ifx^2-3i\left(\right.\right.$$

$$= -\left(\frac{1}{8}i\int\exp\left(-3id-3ifx^2+a\log(f)-x(3ie-b\log(f))\right)\,dx\right)+\frac{1}{8}i\int$$

$$= -\left(\frac{1}{8}\left(i\exp\left(-3id+a\log(f)-\frac{i(-3ie+b\log(f))^2}{12f}\right)\right)\int e^{\frac{i(-3ie-6ifx+b\log}{12f}}$$

$$= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right.$$

Mathematica [A]
time = 1.62, size = 323, normalized size = 0.95

$$\frac{1}{48}(-1)^{3/4}e^{-\frac{i\left(3e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(9ie^{\frac{i\left(e^2+b^2\log^2(f)\right)}{2f}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))+e^{\frac{ie^2}{f}}\left(-9\mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+\sqrt{3}\,e^{\frac{i\left(3e^2+b^2\log^2(f)\right)}{6f}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(3e+6fx+ib\log(f))}{2\sqrt{3}\sqrt{f}}\right)(\cos(3d)-i\sin(3d))\right)+\sqrt{3}\,e^{\frac{ib^2\log^2(f)}{3f}}\mathrm{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3e+6fx-ib\log(f))}{\sqrt{6}\sqrt{f}}\right)(-i\cos(3d)+\sin(3d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(3/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*((9*I)*E^(((I/2)*(e^2 + b^2*Lo
g[f]^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d]
 + I*Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))
```

```
/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f]^2
))/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Co
s[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2 + I/2
)*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d])
))/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

Maple [A]
time = 1.12, size = 311, normalized size = 0.91

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{i\left(\ln\left(f\right)^{2} b^{2}+6 i \ln\left(f\right) b e -9 e^{2}+36 d f\right)}{12 f}} \operatorname{erf}\left(-\sqrt{-3 i f}\, x +\frac{3 i e +b \ln\left(f\right)}{2 \sqrt{-3 i f}}\right)}{16 \sqrt{-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{i\left(\ln\left(f\right)^{2} b^{2}-6 i \ln\left(f\right) b e -9 e^{2}+36 d f\right)}{12 f}} \sqrt{}}{48 \sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+6*I*ln(f)*b*e-9*e^2+36*d*f)/f)
/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))+1
/48*I*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2-6*I*ln(f)*b*e-9*e^2+36*d*f)/f)*
3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(-3*I*e+b*ln(f))*3^(1/2)
/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2
+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))+3/
16*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f)
^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

Maxima [A]
time = 0.53, size = 374, normalized size = 1.10

$$\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right) f^{a}\cos\left(\frac{b^{2}\log\left(f\right)^{2}+36\, df-9\, e^{2}}{12\, f}\right)-\left(i-1\right) f^{a}\sin\left(\frac{b^{2}\log\left(f\right)^{2}+36\, df-9\, e^{2}}{12\, f}\right)\right)\operatorname{erf}\left(\frac{i\left(6i\, fx-b\log\left(f\right)+3i\, e\right)\sqrt{3i\, f}}{6\, f}\right)+\left(-\left(i-1\right) f^{a}\cos\left(\frac{b^{2}\log\left(f\right)^{2}+36\, df-9\, e^{2}}{12\, f}\right)+\left(i+1\right) f^{a}\sin\left(\frac{b^{2}\log\left(f\right)^{2}+36\, df-9\, e^{2}}{12\, f}\right)\right)\operatorname{erf}\left(\frac{i\left(6i\, fx+b\log\left(f\right)+3i\, e\right)\sqrt{-3i\, f}}{6\, f}\right)\right)f^{\frac{3}{2}}-9\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right) f^{a}\cos\left(\frac{b^{2}\log\left(f\right)^{2}+4\, df-e^{2}}{4\, f}\right)-\left(i-1\right) f^{a}\sin\left(\frac{b^{2}\log\left(f\right)^{2}+4\, df-e^{2}}{4\, f}\right)\right)\operatorname{erf}\left(\frac{i\left(2i\, fx-b\log\left(f\right)+i\, e\right)\sqrt{i\, f}}{2\, f}\right)+\left(-\left(i-1\right) f^{a}\cos\left(\frac{b^{2}\log\left(f\right)^{2}+4\, df-e^{2}}{4\, f}\right)+\left(i+1\right) f^{a}\sin\left(\frac{b^{2}\log\left(f\right)^{2}+4\, df-e^{2}}{4\, f}\right)\right)\operatorname{erf}\left(\frac{i\left(2i\, fx+b\log\left(f\right)+i\, e\right)\sqrt{-i\, f}}{2\, f}\right)\right)f^{\frac{3}{2}}}{96\, f^{2} f^{\frac{be}{2\, f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*
f - 9*e^2)/f) - (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f - 9*e^2)/f))*er
f(1/6*I*(6*I*f*x - b*log(f) + 3*I*e)*sqrt(3*I*f)/f) + (-(I - 1)*f^a*cos(1/1
2*(b^2*log(f)^2 + 36*d*f - 9*e^2)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 +
 36*d*f - 9*e^2)/f))*erf(1/6*I*(6*I*f*x + b*log(f) + 3*I*e)*sqrt(-3*I*f)/f)
)*f^(3/2) - 9*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f
- e^2)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))*erf(1/2*I*
(2*I*f*x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)
^2 + 4*d*f - e^2)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))
*erf(1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))*f^(3/2))/(f^2*f^(1/2*b
*e/f))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(228) = 456$.
time = 2.12, size = 645, normalized size = 1.90

$$\frac{-i\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i b^2\log(f)^2-36 i d f+6(2af-be)\log(f)+9 i e^2}{12f}\right)}C\left(\frac{\sqrt{6}(6fx+ib\log(f)+3e)\sqrt{\frac{f}{\pi}}}{6f}\right)-i\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i b^2\log(f)^2+36 i d f+6(2af-be)\log(f)-9 i e^2}{12f}\right)}C\left(-\frac{\sqrt{6}(6fx-ib\log(f)+3e)\sqrt{\frac{f}{\pi}}}{6f}\right)+9i\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i b^2\log(f)^2-4 i d f+2(2af-be)\log(f)+i e^2}{4f}\right)}C\left(\frac{\sqrt{2}(2fx+ib\log(f)+e)\sqrt{\frac{f}{\pi}}}{2f}\right)+9i\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i b^2\log(f)^2+4 i d f+2(2af-be)\log(f)-i e^2}{4f}\right)}C\left(-\frac{\sqrt{2}(2fx-ib\log(f)+e)\sqrt{\frac{f}{\pi}}}{2f}\right)-\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i b^2\log(f)^2-36 i d f+6(2af-be)\log(f)+9 i e^2}{12f}\right)}S\left(\frac{\sqrt{6}(6fx+ib\log(f)+3e)\sqrt{\frac{f}{\pi}}}{6f}\right)+\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i b^2\log(f)^2+36 i d f+6(2af-be)\log(f)-9 i e^2}{12f}\right)}S\left(-\frac{\sqrt{6}(6fx-ib\log(f)+3e)\sqrt{\frac{f}{\pi}}}{6f}\right)+9\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i b^2\log(f)^2-4 i d f+2(2af-be)\log(f)+i e^2}{4f}\right)}S\left(\frac{\sqrt{2}(2fx+ib\log(f)+e)\sqrt{\frac{f}{\pi}}}{2f}\right)-9\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i b^2\log(f)^2+4 i d f+2(2af-be)\log(f)-i e^2}{4f}\right)}S\left(-\frac{\sqrt{2}(2fx-ib\log(f)+e)\sqrt{\frac{f}{\pi}}}{2f}\right)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 - 36*I*d*f + 6*(2*a
*f - b*e)*log(f) + 9*I*e^2)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f)
+ 3*e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 36
*I*d*f + 6*(2*a*f - b*e)*log(f) - 9*I*e^2)/f)*fresnel_cos(-1/6*sqrt(6)*(6*f
*x - I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I
*b^2*log(f)^2 - 4*I*d*f + 2*(2*a*f - b*e)*log(f) + I*e^2)/f)*fresnel_cos(1/
2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi
)*e^(1/4*(I*b^2*log(f)^2 + 4*I*d*f + 2*(2*a*f - b*e)*log(f) - I*e^2)/f)*fre
snel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(6)*pi*s
qrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 - 36*I*d*f + 6*(2*a*f - b*e)*log(f) + 9*
I*e^2)/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)/f)
+ sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 36*I*d*f + 6*(2*a*f - b*e
)*log(f) - 9*I*e^2)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f) + 3*e)*
sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 - 4*I*d*f +
 2*(2*a*f - b*e)*log(f) + I*e^2)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*lo
g(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 +
4*I*d*f + 2*(2*a*f - b*e)*log(f) - I*e^2)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*
x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**3, x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(220) = 440$.
time = 0.55, size = 751, normalized size = 2.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sg
n(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*lo
g(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*
sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f)
)/f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*I*sqrt(6)*
sqrt(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(
abs(f)) - 6*e)/f)*(-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*
b^2*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f +
1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/
2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 3*I*d -
 3/4*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) + 1/48*I*sqrt(6)*sqrt(pi)*erf(-1/
24*sqrt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 6*e)/f
)*(I*f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*
sgn(f)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(a
bs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I
*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/4*I*e^2/f)/(s
qrt(f)*(I*f/abs(f) + 1)) - 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x +
(pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs
(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I
*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*
pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(a
bs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(
abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sin(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sin(d + e*x + f*x^2)^3, x)
```

3.85 $\int f^{a+cx^2} \sin(d+ex)\,dx$

Optimal. Leaf size=151

$$-\frac{ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(
f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*I*exp(I*d+1/4*e^2/c/ln(f))*f^a*e
rfi(1/2*(I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4560, 2325, 2266, 2235}

$$-\frac{i\sqrt{\pi}\,f^a e^{\frac{e^2}{4c\log(f)}-id}\mathrm{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{i\sqrt{\pi}\,f^a e^{\frac{e^2}{4c\log(f)}+id}\mathrm{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x],x]
```

```
[Out] ((-1/4*I)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - ((I/4)*E^(I*d + e^2
/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f
]])])/(Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin(d+ex)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex}f^{a+cx^2}-\frac{1}{2}ie^{id+iex}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex}f^{a+cx^2}\,dx\\ &= \frac{1}{2}i\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx-\frac{1}{2}i\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx\\ &= \frac{1}{2}\left(ie^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{2}\left(ie^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\ &= -\frac{ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f}}\right.}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 119, normalized size = 0.79

$$\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(i\mathrm{Erfi}\left(\frac{-ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+\mathrm{Erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x],x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(I*Erfi[((-I)*e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.39, size = 123, normalized size = 0.81

method	result
risch	$\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{\frac{4id\ln(f)c+e^2}{4\ln(f)c}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{ie}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}+\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{4id\ln(f)c-e^2}{4\ln(f)c}}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x+\right.}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))+1/4*I*Pi^(1/2)*f^a*exp(-1/4
*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I
*e/(-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 206, normalized size = 1.36

$$-\frac{\sqrt{\pi}\left(f^{a}(i\cos(d)+\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}+f^{a}(-i\cos(d)+\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}+f^{a}(i\cos(d)-\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)+ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}+f^{a}(-i\cos(d)-\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)-ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}\right)\sqrt{-c\log(f)}}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) + 1
/2*I*conjugate(1/sqrt(-c*log(f)))*e)*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(d
) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*conjugate(1/sqrt(-c*lo
g(f)))*e)*e^(1/4*e^2/(c*log(f))) + f^a*(I*cos(d) - sin(d))*erf(1/2*(2*c*x*l
og(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(d) - sin
(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))))*
sqrt(-c*log(f))/(c*log(f))
```

Fricas [A]
time = 2.34, size = 144, normalized size = 0.95

$$\frac{i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^2+4icd\log(f)+e^2}{4c\log(f)}\right)}-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^2-4icd\log(f)+e^2}{4c\log(f)}\right)}}{4c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) -
I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*log
(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sin\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + e*x),x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x), x)
```

3.86 $\int f^{a+cx^2} \sin^2(d+ex)\,dx$

Optimal. Leaf size=171

$$\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] -1/8*exp(-2*I*d+e^2/c/ln(f))*f^a*erfi((-I*e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))
*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(2*I*d+e^2/c/ln(f))*f^a*erfi((I*e+c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(
1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4560, 2235, 2325, 2266}

$$\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{c\log(f)}-2id}\mathrm{Erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{c\log(f)}+2id}\mathrm{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx = \int\left(\frac{1}{2}f^{a+cx^2}-\frac{1}{4}e^{-2id-2iex}f^{a+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+cx^2}\right)dx$$

$$= -\left(\frac{1}{4}\int e^{-2id-2iex}f^{a+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{1}{4}\int e^{-2id-2iex+a\log(f)+cx^2\log(f)}\,dx-\frac{1}{4}\int e^{2id+2iex+\cdots}$$

$$= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{1}{4}\left(e^{-2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{4}\left(e^{2id\cdots}\right.$$

$$= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{2id+\frac{\cdots}{c\log\cdots}}\cdots}{\cdots}$$

Mathematica [A]
time = 0.28, size = 132, normalized size = 0.77

$$\frac{f^a\sqrt{\pi}\left(2\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)-e^{\frac{e^2}{c\log(f)}}\left(\text{Erfi}\left(\frac{-ie+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+\text{Erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(2d)+i\sin(2d))\right)\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] - E^(e^2/(c*Log[f]))*(Erfi[((
-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erfi[
(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.42, size = 145, normalized size = 0.85

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c - e^{2}}{\ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x + \frac{i e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}} + \frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c + e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x + \right.}{8 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-
c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c+
e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2)
)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 236, normalized size = 1.38

$$-\frac{\sqrt{\pi}\left(f^{a}(\cos(2d) - i \sin(2d)) \operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}} + i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^2}{c\log(f)}\right)} + f^{a}(\cos(2d) + i \sin(2d)) \operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}} - i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^2}{c\log(f)}\right)} - f^{a}(\cos(2d) + i \sin(2d)) \operatorname{erf}\left(\frac{cx\log(f)+ie}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)} - f^{a}(\cos(2d) - i \sin(2d)) \operatorname{erf}\left(\frac{cx\log(f)-ie}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)} - 2 f^{a} \operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}\right) - 2 f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)\right)}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*(f^a*(cos(2*d) - I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))
) + I*conjugate(1/sqrt(-c*log(f)))*e)*e^(e^2/(c*log(f))) + f^a*(cos(2*d) +
I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))) - I*conjugate(1/sqrt(-c*log(f)
))*e)*e^(e^2/(c*log(f))) - f^a*(cos(2*d) + I*sin(2*d))*erf((c*x*log(f) + I*
e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) - I*sin(2*d))*erf((c
*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - 2*f^a*erf(x*conjugat
e(sqrt(-c*log(f)))) - 2*f^a*erf(sqrt(-c*log(f))*x))/sqrt(-c*log(f))
```

Fricas [A]
time = 2.66, size = 161, normalized size = 0.94

$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x\right) - \sqrt{\pi}\sqrt{-c\log(f)} \operatorname{erf}\left(\frac{(cx\log(f)+ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^2+2icd\log(f)+e^2}{c\log(f)}\right)} - \sqrt{\pi}\sqrt{-c\log(f)} \operatorname{erf}\left(\frac{(cx\log(f)-ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^2-2icd\log(f)+e^2}{c\log(f)}\right)}}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) - sqrt(pi)*sqrt
(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(
f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) - sqrt(pi)*sqrt(-c*log(f))*erf((c*
x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*log(
f) + e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \sin^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sin\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x)^2, x)
```

3.87 $\int f^{a+cx^2}\sin^3(d+ex)\,dx$

Optimal. Leaf size=301

$$-\frac{3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{3ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{}}{16\sqrt{}}$$

```
[Out] 3/16*I*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln
(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*I*exp(-3*I*d+9/4*e^2/c/ln(f))*
f^a*erfi(1/2*(-3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(
f)^(1/2)-3/16*I*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(I*e+2*c*x*ln(f))/c^(
1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*I*exp(3*I*d+9/4*e^2/c/l
n(f))*f^a*erfi(1/2*(3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4560, 2325, 2266, 2235}

$$-\frac{3i\sqrt{\pi}\,f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{i\sqrt{\pi}\,f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}\,f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{i\sqrt{\pi}\,f^a e^{\frac{9e^2}{4c\log(f)}+3id}\text{Erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((-3*I
)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + e^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*
Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin^3(d+ex)\,dx &= \int \left(\frac{3}{8}ie^{-id-iex}f^{a+cx^2}-\frac{3}{8}ie^{id+iex}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}ie^{3id+3iex}f^{a+cx^2}\right)\\
&= -\left(\frac{1}{8}i\int e^{-3id-3iex}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3iex}f^{a+cx^2}\,dx+\frac{3}{8}i\int e^{-id-iex}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{8}i\int e^{-3id-3iex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{8}i\int e^{3id+3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}i\\
&= \frac{1}{8}\left(3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{8}\left(3ie^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,d\\
&= -\frac{3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-2cx\,\mathrm{l}}{2\sqrt{c}\sqrt{\mathrm{lo}}}\right.}{16\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [**A**]
time = 0.46, size = 224, normalized size = 0.74

$$\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(3i\mathrm{Erfi}\left(\frac{-ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+3\mathrm{Erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))-ie^{\frac{2e^2}{c\log(f)}}\left(\mathrm{Erfi}\left(\frac{-3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(3d)-i\sin(3d))-\mathrm{Erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(3d)+i\sin(3d))\right)\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*((3*I)*Erfi[((-I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + 3*Erfi[((-I)*e + 2*c*x*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - I*E^((2*e^2)/(c*Log[f]))*(E
rfi[((-3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3
*d]) - Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] +
I*Sin[3*d]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.76, size = 246, normalized size = 0.82

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 i d \ln(f) c+\frac{9 e^{2}}{4}}{c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c-3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\right.}{16 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2
)*erf(-(-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))-1/16*I*Pi^(1/2)*f^a*exp
(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*
x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2)
)+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)
*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 412, normalized size = 1.37

$$\frac{\sqrt{\pi}\left(f^{a}(i\cos(3d)+\sin(3d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{3}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}+f^{a}(-i\cos(3d)+\sin(3d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{3}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}+f^{a}(i\cos(3d)-\sin(3d))\operatorname{erf}\left(\frac{2cx\log(f)+3ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}+f^{a}(-i\cos(3d)-\sin(3d))\operatorname{erf}\left(\frac{2cx\log(f)-3ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}-3f^{a}(i\cos(d)+\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-3f^{a}(-i\cos(d)+\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-3f^{a}(i\cos(d)-\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)+ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-3f^{a}(-i\cos(d)-\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)-ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}\right)\sqrt{-c\log(f)}}{32c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(pi)*(f^a*(I*cos(3*d) + sin(3*d))*erf(x*conjugate(sqrt(-c*log(f)))
 + 3/2*I*conjugate(1/sqrt(-c*log(f)))*e)*e^(9/4*e^2/(c*log(f))) + f^a*(-I*c
os(3*d) + sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))) - 3/2*I*conjugate(1/sq
rt(-c*log(f)))*e)*e^(9/4*e^2/(c*log(f))) + f^a*(I*cos(3*d) - sin(3*d))*erf(
1/2*(2*c*x*log(f) + 3*I*e)/sqrt(-c*log(f)))*e^(9/4*e^2/(c*log(f))) + f^a*(-
I*cos(3*d) - sin(3*d))*erf(1/2*(2*c*x*log(f) - 3*I*e)/sqrt(-c*log(f)))*e^(9
/4*e^2/(c*log(f))) - 3*f^a*(I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(
f))) + 1/2*I*conjugate(1/sqrt(-c*log(f)))*e)*e^(1/4*e^2/(c*log(f))) - 3*f^a
*(-I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*conjugate(1/
sqrt(-c*log(f)))*e)*e^(1/4*e^2/(c*log(f))) - 3*f^a*(I*cos(d) - sin(d))*erf(
1/2*(2*c*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(-
I*cos(d) - sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2
/(c*log(f))))*sqrt(-c*log(f))/(c*log(f))
```

Fricas [A]
time = 2.02, size = 282, normalized size = 0.94

$$\frac{-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+3ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}+12icd\log(f)+9e^{2}}{4c\log(f)}\right)}+3i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}+4icd\log(f)+e^{2}}{4c\log(f)}\right)}-3i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}-4icd\log(f)+e^{2}}{4c\log(f)}\right)}+i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-3ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}-12icd\log(f)+9e^{2}}{4c\log(f)}\right)}}{16c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(
f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))
) - 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f
))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) +
 I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/
(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/
(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin^3\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\sin\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + e*x)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x)^3, x)
```

3.88 $\int f^{a+cx^2} \sin(d+fx^2)\,dx$

Optimal. Leaf size=107

$$\frac{ie^{-id}f^a\sqrt{\pi}\operatorname{Erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id}f^a\sqrt{\pi}\operatorname{Erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*I*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)-
1/4*I*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4560, 2325, 2236, 2235}

$$\frac{i\sqrt{\pi}\,e^{-id}f^a\operatorname{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}\,e^{id}f^a\operatorname{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] ((I/4)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c*Log[
f]]) - ((I/4)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/Sqrt[I*f +
 c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}i\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx-\frac{1}{2}i\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\ &= \frac{ie^{-id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 170, normalized size = 1.59

$$-\frac{\sqrt[4]{-1}\,f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\sqrt[4]{-1}\,x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\,(f+ic\log(f))(\cos(d)+i\sin(d))+\sqrt{f+ic\log(f)}\left(c\mathrm{Erf}\left(\frac{(1+i)x\sqrt{f+ic\log(f)}}{\sqrt{2}}\right)\log(f)\sin(d)+\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{f+ic\log(f)}\right)(\cos(d)(if+c\log(f))+f\sin(d))\right)\right)}{4\left(f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt
[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[f]
]*(c*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Log[f]*Sin[d] + Erfi[(-1
)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*(Cos[d]*(I*f + c*Log[f]) + f*Sin[d]))))/(f^
2 + c^2*Log[f]^2)
```

Maple [A]
time = 0.31, size = 84, normalized size = 0.79

method	result	size
risch	$-\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{id}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-if}\,x\right)}{4\sqrt{-c\ln(f)-if}}+\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-id}\,\mathrm{erf}\left(x\sqrt{if-c\ln(f)}\right)}{4\sqrt{if-c\ln(f)}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)*
x)+1/4*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/
2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(73) = 146.
time = 0.28, size = 209, normalized size = 1.95

$$\frac{\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+2\,f^2}\left(f^a(\cos(d)-i\,\sin(d))\operatorname{erf}\left(\sqrt{-c\log(f)+i\,f}\,x\right)+f^a(\cos(d)+i\,\sin(d))\operatorname{erf}\left(\sqrt{-c\log(f)-i\,f}\,x\right)\right)\sqrt{c\log(f)+\sqrt{c^2\log(f)^2+f^2}}-\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+2\,f^2}\left(f^a(-i\,\cos(d)-\sin(d))\operatorname{erf}\left(\sqrt{-c\log(f)+i\,f}\,x\right)+f^a(i\,\cos(d)-\sin(d))\operatorname{erf}\left(\sqrt{-c\log(f)-i\,f}\,x\right)\right)\sqrt{-c\log(f)+\sqrt{c^2\log(f)^2+f^2}}}{8\left(c^2\log(f)^2+f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(f^a*(cos(d) - I*sin(d))*erf(sqr
t(-c*log(f) + I*f)*x) + f^a*(cos(d) + I*sin(d))*erf(sqrt(-c*log(f) - I*f)*x
))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^2
 + 2*f^2)*(f^a*(-I*cos(d) - sin(d))*erf(sqrt(-c*log(f) + I*f)*x) + f^a*(I*c
os(d) - sin(d))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log
(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Fricas [A]
time = 2.79, size = 107, normalized size = 1.00

$$\frac{\sqrt{\pi}\,(i\,c\log(f)+f)\sqrt{-c\log(f)-i\,f}\;\operatorname{erf}\left(\sqrt{-c\log(f)-i\,f}\,x\right)e^{(a\log(f)+i\,d)}+\sqrt{\pi}\,(-i\,c\log(f)+f)\sqrt{-c\log(f)+i\,f}\;\operatorname{erf}\left(\sqrt{-c\log(f)+i\,f}\,x\right)e^{(a\log(f)-i\,d)}}{4\left(c^2\log(f)^2+f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I
*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*log(f) + I*f
)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sin\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + f*x^2), x)
```

3.89 $\int f^{a+cx^2} \sin^2\left(d+fx^2\right) dx$

Optimal. Leaf size=140

$$\frac{f^a\sqrt{\pi}\,\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id}f^a\sqrt{\pi}\,\text{Erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id}f^a\sqrt{\pi}\,\text{Erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*f^a*er
f(x*(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*I*d)/(2*I*f-c*ln(f))^(1/2)-1/8*ex
p(2*I*d)*f^a*erfi(x*(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4560, 2235, 2325, 2236}

$$-\frac{\sqrt{\pi}\,e^{-2id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}\,e^{2id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) - (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+cx^2}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{1}{4}\int \exp\left(-2id+a\log(f)-x^2(2if-c\log(f))\right.\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id}f^a\sqrt{\,}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.88, size = 188, normalized size = 1.34

$$\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\text{Erf}\left(\sqrt[4]{-1}\,x\sqrt{2f+ic\log(f)}\right)\sqrt{2f+ic\log(f)}\,(2if+c\log(f))(\cos(2d)-i\sin(2d))+\text{Erf}\left((-1)^{3/4}x\sqrt{2f-ic\log(f)}\right)\sqrt{2f-ic\log(f)}\,(2f+ic\log(f))(\cos(2d)+i\sin(2d))\right)}{4f^2+c^2\log^2(f)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(Erf[(-1)^(1/4)*x*Sqrt[2*f + I*c*Log[f]]]*Sqrt[2*f + I*c*Log[f]]*
((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]) + Erf[(-1)^(3/4)*x*Sqrt[2*f -
I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*
d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A]
time = 0.41, size = 107, normalized size = 0.76

method	result
risch	$-\frac{\sqrt{\pi}\,f^{a}e^{-2id}\,\text{erf}\left(x\sqrt{2if-c\ln(f)}\right)}{8\sqrt{2if-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}e^{2id}\,\text{erf}\left(\sqrt{-c\ln(f)-2if}\,x\right)}{8\sqrt{-c\ln(f)-2if}}+\frac{f^{a}\sqrt{\pi}\,\text{erf}\left(\sqrt{-c\ln(f}\right.}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I*
f)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 315, normalized size = 2.25

$$\frac{\sqrt{\pi}\,\sqrt{2\,c^2\log\left(f\right)^2+8\,f^2}\,\left(f^a\left(i\,\cos\left(2\,d\right)+\sin\left(2\,d\right)\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2i\,f}\,x\right)+f^a\left(-i\,\cos\left(2\,d\right)+\sin\left(2\,d\right)\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2i\,f}\,x\right)\right)\sqrt{c\log\left(f\right)+\sqrt{c^2\log\left(f\right)^2+4\,f^2}}\,\sqrt{-c\log\left(f\right)}-\sqrt{\pi}\,\sqrt{2\,c^2\log\left(f\right)^2+8\,f^2}\,\left(f^a\left(\cos\left(2\,d\right)-i\,\sin\left(2\,d\right)\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2i\,f}\,x\right)+f^a\left(\cos\left(2\,d\right)+i\,\sin\left(2\,d\right)\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2i\,f}\,x\right)\right)\sqrt{-c\log\left(f\right)+\sqrt{c^2\log\left(f\right)^2+4\,f^2}}\,\sqrt{-c\log\left(f\right)}+2\,\sqrt{\pi}\left(\left(c^2f^a\log\left(f\right)^2+4\,f^{a+2}\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log\left(f\right)}}\right)+\left(c^2f^a\log\left(f\right)^2+4\,f^{a+2}\right)\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}\,x\right)\right)}{16\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(I*cos(2*d) + sin(2*d))*er
f(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(-I*cos(2*d) + sin(2*d))*erf(sqrt(-c*log
(f) - 2*I*f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)
) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(cos(2*d) - I*sin(2*d))*erf(
sqrt(-c*log(f) + 2*I*f)*x) + f^a*(cos(2*d) + I*sin(2*d))*erf(sqrt(-c*log(f)
 - 2*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f))
+ 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(sqrt(-c*log(
f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)))/((c^2*log
(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Fricas [A]
time = 2.95, size = 169, normalized size = 1.21

$$-\frac{2\,\sqrt{\pi}\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}\,f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}\,x\right)-\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2i\,f}\,x\right)e^{\left(a\log\left(f\right)+2i\,d\right)}-\sqrt{\pi}\left(c^2\log\left(f\right)^2+2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)+2i\,f}\,\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2i\,f}\,x\right)e^{\left(a\log\left(f\right)-2i\,d\right)}}{8\left(c^3\log\left(f\right)^3+4\,cf^2\log\left(f\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) - sqrt(pi)*(c^2*log(f)^
2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)*x)*
e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sin\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + f*x^2)^2, x)
```

3.90 $\int f^{a+cx^2} \sin^3(d+fx^2)\,dx$

Optimal. Leaf size=213

$$\frac{3ie^{-id}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}$$

```
[Out] 3/16*I*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)
-1/16*I*f^a*erf(x*(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*I*d)/(3*I*f-c*ln(f)
)^(1/2)-3/16*I*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(
f))^(1/2)+1/16*I*exp(3*I*d)*f^a*erfi(x*(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I
*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4560, 2325, 2236, 2235}

$$\frac{3i\sqrt{\pi}\,e^{-id}f^a\mathrm{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}\,e^{-3id}f^a\mathrm{Erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}\,e^{id}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}\,e^{3id}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+3if}\right)}{16\sqrt{c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c
*Log[f]]) - ((I/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(E^((3*I)
*d)*Sqrt[(3*I)*f - c*Log[f]]) - (((3*I)/16)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqr
t[I*f + c*Log[f]]])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d)*f^a*Sqrt[Pi]
*Erfi[x*Sqrt[(3*I)*f + c*Log[f]]])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx &= \int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}ie^{3id+3ifx^2}f\right.\\ &= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+}\\ &= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)-x^2(3if-c\log(f))\right)\,dx\right)+\frac{1}{8}i\int \exp\left(3id+a\right.\\ &= \frac{3ie^{-id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}-\end{aligned}$$

Mathematica [A]
time = 2.53, size = 386, normalized size = 1.81

$$\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(-3\mathrm{Erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)+ic^3\log^3(f)\right)(\cos(d)+i\sin(d))+(f-ic\log(f))\left(\mathrm{Erfi}\left(\sqrt[4]{-1}x\sqrt{3f-ic\log(f)}\right)\sqrt{3f-ic\log(f)}\left(3f^2+4icf\log(f)-c^2\log^2(f)\right)(\cos(3d)+i\sin(3d))+(3f-ic\log(f))\left(3\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{f+ic\log(f)}\right)\sqrt{f+ic\log(f)}\left(c\cos(d)\log(f)-3f\sin(d)\right)+3\mathrm{Erf}\left(\frac{(1+i)x\sqrt{f+ic\log(f)}}{\sqrt{2}}\right)\sqrt{f+ic\log(f)}\left(3f\cos(d)+c\log(f)\sin(d)\right)+\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{3f+ic\log(f)}\right)(f+ic\log(f))\sqrt{3f+ic\log(f)}\left(i\cos(3d)+\sin(3d)\right)\right)\right)\right)}{16\left(9f^4+10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
 - I*c*Log[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^
3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(Erfi[(-1)^(1/4)*x*Sqrt[3*f - I*c
*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*
(Cos[3*d] + I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*Erfi[(-1)^(3/4)*x*Sqrt[f +
I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(c*Cos[d]*Log[f] - 3*f*Sin[d]) + 3*Erf[((
1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*(3*f*Cos[d] +
c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f + I*c*Log[f
])*Sqrt[3*f + I*c*Log[f]]*(I*Cos[3*d] + Sin[3*d])))))/(16*(9*f^4 + 10*c^2*f
^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A]
time = 0.75, size = 166, normalized size = 0.78

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} e^{3id} \operatorname{erf}\left(\sqrt{-c\ln\left(f\right)-3if}\, x\right)}{16\sqrt{-c\ln\left(f\right)-3if}}-\frac{i\sqrt{\pi}\, f^{a} e^{-3id} \operatorname{erf}\left(x\sqrt{3if-c\ln\left(f\right)}\right)}{16\sqrt{3if-c\ln\left(f\right)}}+\frac{3i\sqrt{\pi}\, f^{a} e^{-id} \operatorname{erf}\left(x\sqrt{if}\right.}{16\sqrt{if-c\ln}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*I*f)^
(1/2)*x)-1/16*I*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f
-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I
*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-
c*ln(f)-I*f)^(1/2)*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(145) = 290$.
time $= 0.31$, size $= 661$, normalized size $= 3.10$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*cos(3*d) - I*c^2*sin(3
*d))*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) - I*sin(3*d)))*erf(sqrt(-c*log(f) +
 3*I*f)*x) + ((c^2*cos(3*d) + I*c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(cos
(3*d) + I*sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqrt(c*log(f) + sqrt(c
^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2*cos(
d) - I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(cos(d) - I*sin(d)))*erf(sqrt
(-c*log(f) + I*f)*x) + ((c^2*cos(d) + I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a +
 2)*(cos(d) + I*sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(c*log(f) + sqrt
(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*cos
(3*d) + c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(I*cos(3*d) + sin(3*d)))*erf
(sqrt(-c*log(f) + 3*I*f)*x) + ((-I*c^2*cos(3*d) + c^2*sin(3*d))*f^a*log(f)^
2 + f^(a + 2)*(-I*cos(3*d) + sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqr
t(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2
+ 2*f^2)*(((-I*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(-I*cos(
d) - sin(d)))*erf(sqrt(-c*log(f) + I*f)*x) + ((I*c^2*cos(d) - c^2*sin(d))*f
^a*log(f)^2 + 9*f^(a + 2)*(I*cos(d) - sin(d)))*erf(sqrt(-c*log(f) - I*f)*x)
)*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*lo
g(f)^2 + 9*f^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(145) = 290$.

time = 1.84, size = 317, normalized size = 1.49

$$\frac{\sqrt{\pi}\left(-i c^{3} \log (f)^{3}-3 c^{2} f \log (f)^{2}-i c f^{2} \log (f)-3 f^{3}\right) \sqrt{-c \log (f)-3 i f} \operatorname{erf}\left(\sqrt{-c \log (f)-3 i f} x\right) e^{(a \log (f)+3 i d)}-3 \sqrt{\pi}\left(-i c^{3} \log (f)^{3}-c^{2} f \log (f)^{2}-9 i c f^{2} \log (f)-9 f^{3}\right) \sqrt{-c \log (f)-i f} \operatorname{erf}\left(\sqrt{-c \log (f)-i f} x\right) e^{(a \log (f)+i d)}-3 \sqrt{\pi}\left(i c^{3} \log (f)^{3}-c^{2} f \log (f)^{2}+9 i c f^{2} \log (f)-9 f^{3}\right) \sqrt{-c \log (f)+i f} \operatorname{erf}\left(\sqrt{-c \log (f)+i f} x\right) e^{(a \log (f)-i d)}+\sqrt{\pi}\left(i c^{3} \log (f)^{3}-3 c^{2} f \log (f)^{2}+i c f^{2} \log (f)-3 f^{3}\right) \sqrt{-c \log (f)+3 i f} \operatorname{erf}\left(\sqrt{-c \log (f)+3 i f} x\right) e^{(a \log (f)-3 i d)}}{16\left(c^{4} \log (f)^{4}+10 c^{2} f^{2} \log (f)^{2}+9 f^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*I*
d) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9*f^
3)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d) -
3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f^2*log(f) - 9*f^3)*sqr
t(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) + sqrt(p
i)*(I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log
(f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(c^4*log(
f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\sin\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + f*x^2)^3, x)
```

3.91 $\int f^{a+cx^2} \sin\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=187

$$\frac{ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*exp(I*d+e^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\frac{i\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\text{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((I/4)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((I/4)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}i\int\exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id+\right.\\ &= \frac{1}{2}\left(ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx-\frac{1}{2}\left(ie\right.\\ &= \frac{ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}}{4\sqrt{if+c}}\end{aligned}$$

Mathematica [A]
time = 0.99, size = 216, normalized size = 1.16

$$-\frac{(-1)^{3/4}e^{\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\left(e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+2icx\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(d)-i\sin(d))+\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-2icx\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}(-if+c\log(f))(\cos(d)+i\sin(d))\right)}{4\left(f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(E^(((I/2)*e^2
*f)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/
(2*Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I
*Sin[d]) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c
*Log[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d])))/(
f^2 + c^2*Log[f]^2)
```

Maple [A]
time = 0.63, size = 169, normalized size = 0.90

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 i d \ln(f) c-4 d f+e^{2}}{4 i f+4 c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i f}\, x+\frac{i e}{2 \sqrt{-c \ln(f)-i f}}\right)}{4 \sqrt{-c \ln(f)-i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c+4 d f-e^{2}}{4(-i f+c \ln(f))}} \operatorname{erf}\left(x \ldots\right.}{4 \sqrt{\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(
f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))+1/4
*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*l
n(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 748 vs. 2(135) = 270.
time = 0.33, size = 748, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*f^(1/4*c*e^2/(c^2*log(f)^
2 + f^2))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)
) - I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*c^2*d*log(f)^2 + 4*
d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/
sqrt(-c*log(f) + I*f)) + (f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4
*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + I*f^a*f^(1/4*c*e
^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*
log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f
)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 2*f^2)*((I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*c^2*d*log(
f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + f^a*f^(1/4*c*e^2/(c^2*log(f
)^2 + f^2))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^
2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + (-I*f^a*
f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*
e^2)/(c^2*log(f)^2 + f^2)) + f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4
*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*
log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*lo
g(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(135) = 270$.
time = 1.47, size = 299, normalized size = 1.60

$$\frac{\sqrt{\pi}\,(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^{2}x\log(f)^{2}+2\,f^{2}x+i\,ce\log(f)+fe\right)\sqrt{-c\log\left(f\right)-i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(\frac{4\,ac^{2}\log(f)^{3}+4i\,c^{2}d\log(f)^{2}+4i\,df^{2}-i\,fe^{2}+\left(4\,af^{2}+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)}+\sqrt{\pi}\,(-i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)+i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^{2}x\log(f)^{2}+2\,f^{2}x-i\,ce\log(f)+fe\right)\sqrt{-c\log\left(f\right)+i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(\frac{4\,ac^{2}\log(f)^{3}-4i\,c^{2}d\log(f)^{2}-4i\,df^{2}+i\,fe^{2}+\left(4\,af^{2}+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)}}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f
)^2 + 2*f^2*x + I*c*e*log(f) + f*e)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f
^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 + 4*I*d*f^2 - I*f*e^2 +
(4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f) + f
)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*log(f)
+ f*e)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3
 - 4*I*c^2*d*log(f)^2 - 4*I*d*f^2 + I*f*e^2 + (4*a*f^2 + c*e^2)*log(f))/(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^{2}}\sin\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^{2}+a}\,\sin\left(f\,x^{2}+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x + f*x^2), x)
```

3.92 $\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=211

$$\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(-\right.}{8\sqrt{2if+c\log}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(-2
*I*d-e^2/(2*I*f-c*ln(f)))*f^a*erf((I*e+x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(
1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^(1/2)-1/8*exp(2*I*d+e^2/(2*I*f+c*ln(f)))*f^a
*erfi((I*e+x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f
))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4560, 2235, 2325, 2266, 2236}

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\mathrm{Erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\mathrm{Erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+cx^2} - \frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+cx^2} - \frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+cx^2}\right)dx \\ &= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+cx^2}\,dx\right) - \frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+cx^2}\,dx + \frac{1}{2}\int \\ &= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}} - \frac{1}{4}\int \exp\left(-2id-2iex+a\log(f)-x^2(2i\right. \\ &= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}} - \frac{1}{4}\left(e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\right)\int \exp\left(\frac{(-2ie+2}{4(-}\right. \\ &= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}} - \frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+x(2if-c\log(f)}{\sqrt{2if-c\log(}}\right.}{8\sqrt{2if-c\log(f)}} \end{aligned}$$

Mathematica [A]
time = 2.38, size = 251, normalized size = 1.19

$$\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{\sqrt{c}\,\sqrt{\log(f)}} + \frac{\sqrt[4]{-1}\left(e^{\frac{e^2}{2if+c\log(f)}}\mathrm{Erf}\left(\frac{(-1)^{3/4}(e+2fx-icx\log(f))}{\sqrt{2f-ic\log(f)}}\right)\sqrt{2f-ic\log(f)}\,(2f+ic\log(f))(\cos(2d)+i\sin(2d)) + e^{\frac{e^2}{-2if+c\log(f)}}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(e+2fx+icx\log(f))}{\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}\,(i\cos(2d)+\sin(2d))\right)}{4f^2+c^2\log^2(f)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(E^(e^2/((2*I)*f + c*Log[f]))*Erf[((-1)^(3/4)*(e + 2*f*x - I*c*x*
```

Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*
(Cos[2*d] + I*Sin[2*d]) + E^(e^2/((-2*I)*f + c*Log[f]))*Erf[((-1)^(1/4)*(e
+ 2*f*x + I*c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*
f + I*c*Log[f]]*(I*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8

Maple [A]
time = 0.71, size = 191, normalized size = 0.91

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c+4 d f-e^{2}}{-2 i f+c \ln(f)}} \operatorname{erf}\left(x \sqrt{2 i f-c \ln(f)}+\frac{i e}{\sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c-4 d f+e^{2}}{2 i f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{\right.}{8\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c
*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))+1/8*Pi
^(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^
(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*Pi^
(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 851, normalized size = 4.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")

[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(c*e^2/(c^2*log(f)^2
 + 4*f^2))*cos(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2))
 + f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*sin(2*(c^2*d*log(f)^2 + 4*d*f^2 - f
*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log
(f) + 2*I*f)) + (-I*f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*cos(2*(c^2*d*log(f
)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(c*e^2/(c^2*log(f)^2
 + 4*f^2))*sin(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2))
)*erf(((c*log(f) + 2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f)
+ sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 8*f^2)*((f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*cos(2*(c^2*d*log(f)^2 + 4
*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)) - I*f^a*f^(c*e^2/(c^2*log(f)^2 + 4*
f^2))*sin(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf
(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(f) + 2*I*f)) + (f^a*f^(c*e^2/(c^2
*log(f)^2 + 4*f^2))*cos(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2
+ 4*f^2)) + I*f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*sin(2*(c^2*d*log(f)^2 +

```
4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2))*erf(((c*log(f) + 2*I*f)*x + I*e)/
sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt
(-c*log(f)) - 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(
sqrt(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)
))/((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(155) = 310$.
time = 2.35, size = 363, normalized size = 1.72

$$-\frac{2\sqrt{\pi}\left(c^2\log(f)^2+4f^2\right)\sqrt{-c\log(f)}\,f^a\operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)-\sqrt{\pi}\left(c^2\log(f)^2-2icf\log(f)\right)\sqrt{-c\log(f)-2if}\operatorname{erf}\left(\frac{\left(c^2x\log(f)^2+4f^2x+ice\log(f)+2fe\right)\sqrt{-c\log(f)-2if}}{c^2\log(f)^2+4f^2}\right)e^{\left(\frac{ac^2\log(f)^3+2ic^2d\log(f)^2+8idf^2-2ife^2+\left(4af^2+ce^2\right)\log(f)}{c^2\log(f)^2+4f^2}\right)}-\sqrt{\pi}\left(c^2\log(f)^2+2icf\log(f)\right)\sqrt{-c\log(f)+2if}\operatorname{erf}\left(\frac{\left(c^2x\log(f)^2+4f^2x-ice\log(f)+2fe\right)\sqrt{-c\log(f)+2if}}{c^2\log(f)^2+4f^2}\right)e^{\left(\frac{ac^2\log(f)^3-2ic^2d\log(f)^2-8idf^2+2ife^2+\left(4af^2+ce^2\right)\log(f)}{c^2\log(f)^2+4f^2}\right)}}{8\left(c^3\log(f)^3+4cf^2\log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*f*e)*sqrt(-c*log(f) - 2*I*
f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 + 8*I*d*
f^2 - 2*I*f*e^2 + (4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) - sqrt(
pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*x*log(
f)^2 + 4*f^2*x - I*c*e*log(f) + 2*f*e)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^
2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 - 8*I*d*f^2 + 2*I*f*e^2
 + (4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^3 + 4*c*f
^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^2, x)
```

3.93 $\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=377

$$\frac{3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id+}\cdots}{}$$

```
[Out] 3/16*I*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/
(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*I*exp(-3*I*d-9/4*e^2
/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)-3/16*I*exp(I*d+e^2/(4*I*f+4*c*ln(f)))*
f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*l
n(f))^(1/2)+1/16*I*exp(3*I*d+9/4*e^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+2
*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$-\frac{i\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\mathrm{Erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3i\sqrt{\pi}f^ae^{-\frac{e^2}{-4c\log(f)+4if}-id}\mathrm{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{3i\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)+4if}+id}\mathrm{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}f^ae^{\frac{9e^2}{4(c\log(f)+3if)}+3id}\mathrm{Erfi}\left(\frac{2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e +
 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((
I/16)*E^((-3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*
I)*e + 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*
f - c*Log[f]] - (((3*I)/16)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[P
i]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
c*Log[f]] + ((I/16)*E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])
])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{8}ie^{-3i(d+ex+fx^2)}f^{a+cx^2}+\frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i(d+ex\right.\right.\\ &= -\left(\frac{1}{8}i\int e^{-3i(d+ex+fx^2)}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int\exp\left(6id+6iex+6ifx^2-3i\right.\\ &= -\left(\frac{1}{8}i\int\exp\left(-3id-3iex+a\log(f)-x^2(3if-c\log(f))\right)dx\right)+\frac{1}{8}i\\ &= \frac{1}{8}\left(3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx-\frac{1}{8}\\ &= \frac{3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a}{16\sqrt{3}}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3003 vs. $2(377) = 754$.
time = 7.06, size = 3003, normalized size = 7.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-27*(-1)^(3/4)*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*
I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e
^2)/(f - I*c*Log[f])) + (27*(-1)^(1/4)*c*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2
*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log
[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) - (3*(-1)^(3/4)*c^2*f*Cos[d]*Erfi[((
-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (3*(-1)^(1/4)*c^
3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*L
og[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) +
(3*(-1)^(3/4)*f^3*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]
))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]])/E^((((9*I)/4)*e^2)/(
3*f - I*c*Log[f])) - ((-1)^(1/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f
*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*L
og[f]])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f])) + (3*(-1)^(3/4)*c^2*f*Cos[3*
d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f
])) - ((-1)^(1/4)*c^3*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]])/E^((((9
*I)/4)*e^2)/(3*f - I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*e^2)/(f + I*c*Log
[f]))*f^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f
 + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]] - 27*(-1)^(3/4)*c*E^(((I/4)*e^2)/(f +
 I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(
2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]] + 3*(-1)^(1/4)*c^2*E^(
((I/4)*e^2)/(f + I*c*Log[f]))*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]] - 3*(-
1)^(3/4)*c^3*E^(((I/4)*e^2)/(f + I*c*Log[f]))*Cos[d]*Erfi[((-1)^(3/4)*(e +
2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*
Log[f]] - 3*(-1)^(1/4)*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f^3*Cos[3*d]*
Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]
])]*Sqrt[3*f + I*c*Log[f]] + (-1)^(3/4)*c*E^((((9*I)/4)*e^2)/(3*f + I*c*Log
[f]))*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sq
rt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]] - 3*(-1)^(1/4)*c^2*E^(
(((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*
x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*
Log[f]] + (-1)^(3/4)*c^3*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*Cos[3*d]*Er
fi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
]*Log[f]^3*Sqrt[3*f + I*c*Log[f]] + (27*(-1)^(1/4)*f^3*Erfi[((-1)^(1/4)*(e
+ 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]
*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Erfi[((-1)
^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqr
t[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (3*(-1)^(1/4)*
c^2*f*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[
f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])
) + (3*(-1)^(3/4)*c^3*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*S
qrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/
```

```
(f - I*c*Log[f])) - 27*(-1)^(3/4)*E^(((I/4)*e^2)/(f + I*c*Log[f]))*f^3*Erfi
[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt
[f + I*c*Log[f]]*Sin[d] - 27*(-1)^(1/4)*c*E^(((I/4)*e^2)/(f + I*c*Log[f]))*
f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]
])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d] - 3*(-1)^(3/4)*c^2*E^(((I/4)*e^2)/(f
 + I*c*Log[f]))*f*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[
f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d] - 3*(-1)^(1/4)*c^3*E
^(((I/4)*e^2)/(f + I*c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log
[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d] - (3*(
-1)^(1/4)*f^3*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*
f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/4)*e^2)/(3*f
- I*c*Log[f])) - ((-1)^(3/4)*c*f^2*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*
x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*
d])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Erfi[((-1)
^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]
^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f]))
- ((-1)^(3/4)*c^3*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqr
t[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/
4)*e^2)/(3*f - I*c*Log[f])) + 3*(-1)^(3/4)*E^((((9*I)/4)*e^2)/(3*f + I*c*Lo
g[f]))*f^3*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f +
 I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d] + (-1)^(1/4)*c*E^((((9*I)/4)
*e^2)/(3*f + I*c*Log[f]))*f^2*Erfi[((-1)^(3/4)*...
```

Maple [A]
time = 1.22, size = 338, normalized size = 0.90

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} e^{\frac{3 i d \ln(f) c-9 d f+\frac{9 e^{2}}{4}}{3 i f+c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3 i f}\, x+\frac{3 i e}{2\sqrt{-c\ln(f)-3 i f}}\right)}{16\sqrt{-c\ln(f)-3 i f}}-\frac{i\sqrt{\pi}\, f^{a} e^{-\frac{3\left(4 i d\ln(f) c+12 d f\right.}{4(-3 i f+c\ln(\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/
(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*
f)^(1/2))-1/16*I*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(-3*I*f
+c*ln(f)))/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f
-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I
*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*l
n(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*l
n(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I
*f)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2135 vs. $2(269) = 538$.

time = 0.33, size = 2135, normalized size = 5.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*f^(9/4*c*e^2/(c^2*l
og(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))
)*cos(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) +
 (-I*c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - I*f^(a + 2)*f^
(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*sin(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 -
9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/
sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*l
og(f)^2 + f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(4*c^2*d*
log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^a*f^(9/4*
c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + I*f^(a + 2)*f^(9/4*c*e^2/(c^2*log(
f)^2 + 9*f^2)))*sin(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)
^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I
*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*l
og(f)^2 + 2*f^2)*(((c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9
*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*c^2*d*log(f)^2 +
4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*f^(1/4*c*e^2/(c^2*log(
f)^2 + f^2))*log(f)^2 - 9*I*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*s
in(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*
(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + ((c^2*f^a*f^(1/4*c*e^
2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 +
 f^2)))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2))
+ (I*c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*I*f^(a + 2)*f^
(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log
(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(
f)^2 + I*f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(4*c^2*d*l
og(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*f^(9/4*c*e
^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2
+ 9*f^2)))*sin(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 +
9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/sqrt(-c*log(f) + 3*I*f))
+ ((-I*c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - I*f^(a + 2)*
f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2
- 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 +
9*f^2))*log(f)^2 + f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*sin(3/4*
(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2
*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f) + s
qrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((I*c
```

```
^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*I*f^(a + 2)*f^(1/4*c
*e^2/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c
^2*log(f)^2 + f^2)) + (c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2
+ 9*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*sin(1/4*(4*c^2*d*log(f)^2
 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x -
I*e)/sqrt(-c*log(f) + I*f)) + ((-I*c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2
))*log(f)^2 - 9*I*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*
c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^a*f^(1/4*c
*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^
2 + f^2)))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2
)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*lo
g(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f
^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(269) = 538$.
time = 2.44, size = 713, normalized size = 1.89

$$\frac{\sqrt{\pi}\left(-i c^3\log(f)^3 - 3c^2 f\log(f)^2 - i c f^2\log(f) - 3f^3\right)\sqrt{-c\log(f)-3if}\,\operatorname{erf}\left(\frac{(2c^2x\log(f)^2+18f^2x+3ice\log(f)+9fe)\sqrt{-c\log(f)-3if}}{2(c^2\log(f)^2+9f^2)}\right)e^{\left(\frac{4ac^2\log(f)^3+12ic^2d\log(f)^2+108idf^2-27ife^2+9(4af^2+ce^2)\log(f)}{4(c^2\log(f)^2+9f^2)}\right)} + \sqrt{\pi}\left(ic^3\log(f)^3-3c^2f\log(f)^2+icf^2\log(f)-3f^3\right)\sqrt{-c\log(f)+3if}\,\operatorname{erf}\left(\frac{(2c^2x\log(f)^2+18f^2x-3ice\log(f)+9fe)\sqrt{-c\log(f)+3if}}{2(c^2\log(f)^2+9f^2)}\right)e^{\left(\frac{4ac^2\log(f)^3-12ic^2d\log(f)^2-108idf^2+27ife^2+9(4af^2+ce^2)\log(f)}{4(c^2\log(f)^2+9f^2)}\right)} - 3\sqrt{\pi}\left(-ic^3\log(f)^3-c^2f\log(f)^2-9icf^2\log(f)-9f^3\right)\sqrt{-c\log(f)-if}\,\operatorname{erf}\left(\frac{(2c^2x\log(f)^2+2f^2x+ice\log(f)+fe)\sqrt{-c\log(f)-if}}{2(c^2\log(f)^2+f^2)}\right)e^{\left(\frac{4ac^2\log(f)^3+4ic^2d\log(f)^2+4idf^2-ife^2+(4af^2+ce^2)\log(f)}{4(c^2\log(f)^2+f^2)}\right)} - 3\sqrt{\pi}\left(ic^3\log(f)^3-c^2f\log(f)^2+9icf^2\log(f)-9f^3\right)\sqrt{-c\log(f)+if}\,\operatorname{erf}\left(\frac{(2c^2x\log(f)^2+2f^2x-ice\log(f)+fe)\sqrt{-c\log(f)+if}}{2(c^2\log(f)^2+f^2)}\right)e^{\left(\frac{4ac^2\log(f)^3-4ic^2d\log(f)^2-4idf^2+ife^2+(4af^2+ce^2)\log(f)}{4(c^2\log(f)^2+f^2)}\right)}}{16\left(c^4\log(f)^4+10c^2f^2\log(f)^2+9f^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*lo
g(f) + 9*f*e)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c
^2*log(f)^3 + 12*I*c^2*d*log(f)^2 + 108*I*d*f^2 - 27*I*f*e^2 + 9*(4*a*f^2 +
 c*e^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*
f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(2*c^2
*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*f*e)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^2 - 108*
I*d*f^2 + 27*I*f*e^2 + 9*(4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + 9*f^2))
- 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9*f^3)*
sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e*log(f) +
f*e)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3 +
 4*I*c^2*d*log(f)^2 + 4*I*d*f^2 - I*f*e^2 + (4*a*f^2 + c*e^2)*log(f))/(c^2*
log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f^2*
log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x -
 I*c*e*log(f) + f*e)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*
a*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 - 4*I*d*f^2 + I*f*e^2 + (4*a*f^2 + c*e^
2)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f
^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^3, x)
```

3.94 $\int f^{a+bx+cx^2} \sin(d+ex)\,dx$

Optimal. Leaf size=176

$$-\frac{ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*
x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*I*exp(I*d+1/
4*(e-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4560, 2325, 2266, 2235}

$$-\frac{i\sqrt{\pi}\,f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{i\sqrt{\pi}\,f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

```
[Out] ((-1/4*I)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[
f]]) - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(
I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log
[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sin(d+ex)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+iex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}i\int\exp\left(-id+a\log(f)+cx^2\log(f)-x(ie-b\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id+\right.\\ &= -\left(\frac{1}{2}\left(ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\right)+\frac{1}{2}\left(ie\right.\\ &= -\frac{ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\right.}{4\sqrt{c}\sqrt{\log(}}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 155, normalized size = 0.88

$$\frac{e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(i\mathrm{Erfi}\left(\frac{-ie-(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+e^{\frac{ibe}{c}}\mathrm{Erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

```
[Out] (E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(I*Er
fi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[
d]) + E^((I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.45, size = 170, normalized size = 0.97

method	result

risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+2 i \ln(f) b e-4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{i e+b \ln(f)}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}} - \frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ldots}{4 \ln(f) c}} \ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/l
n(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f
))^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f
)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e
)/(-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 362, normalized size = 2.06

$$-\frac{\sqrt{\pi}\left(f^{a}\left(-i\cos\left(\frac{2cd-be}{2c}\right)+\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}\left(b\log(f)+ie\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}\left(i\cos\left(\frac{2cd-be}{2c}\right)+\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}\left(b\log(f)-ie\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}\left(-i\cos\left(\frac{2cd-be}{2c}\right)+\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+b\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}\left(i\cos\left(\frac{2cd-be}{2c}\right)+\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+b\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}\right)\sqrt{-c\log(f)}}{8cf^{\frac{b^{2}}{4c}}\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(-I*cos(1/2*(2*c*d - b*e)/c) + sin(1/2*(2*c*d - b*e)/c))
*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-
c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(I*cos(1/2*(2*c*d - b*e)/c) + sin(
1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - I*
e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(1/2*(
2*c*d - b*e)/c) + sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f
) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(I*cos(1/
2*(2*c*d - b*e)/c) + sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*lo
g(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt(-c*log
(f))/(c*f^(1/4*b^2/c)*log(f))
```

Fricas [A]
time = 2.15, size = 180, normalized size = 1.02

$$\frac{-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^{2}-4ac)\log(f)^{2}+2(2icd-ibe)\log(f)-e^{2}}{4c\log(f)}\right)}+i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^{2}-4ac)\log(f)^{2}+2(-2icd+ibe)\log(f)-e^{2}}{4c\log(f)}\right)}}{4c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 2*(2*I*c*d - I*b*e)*
log(f) - e^2)/(c*log(f))) + I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)
```

```
*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
+ 2*(-2*I*c*d + I*b*e)*log(f) - e^2)/(c*log(f))))/(c*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d),x)
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x),x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x), x)
```

3.95 $\int f^{a+bx+cx^2} \sin^2(d+ex)\,dx$

Optimal. Leaf size=231

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{}}{8\sqrt{}}$$

```
[Out] -1/8*exp(-2*I*d+1/4*(2*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*I*e+b*ln(f)
+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(2*I
*d-1/4*(2*I*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*I*e+b*ln(f)+2*c*x*ln(f))/
c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(
1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4560, 2266, 2235, 2325}

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}\right)\,dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2iex}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,\text{(}\\
&= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)+cx^2\log(f)-x(2ie-b\log(f))\right)\,dx\right)-\frac{1}{4}\int \text{ex}\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{1}{4}\left(\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\,\text{J}\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erfi}\Big(\frac{2ie}{2}}{8\sqrt{c}\,\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 0.72, size = 204, normalized size = 0.88

$$-\frac{e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-2e^{\frac{ibe}{c}}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\text{Erfi}\left(\frac{-2ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+e^{\frac{e^2}{c\log(f)}}\text{Erfi}\left(\frac{2ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(2d)+i\sin(2d))\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] -1/8*(f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[L
og[f]])/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)
*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d])
+ E^(e^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Lo
g[f]])]*(Cos[2*d] + I*Sin[2*d])))/(Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

Maple [A]
time = 0.58, size = 217, normalized size = 0.94

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c-4 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-2 i e}{2 \sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) b e-8 i d}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c-4*e^2)/l
n(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*ln
(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f
)*c-4*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*l
n(f))/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 399, normalized size = 1.73

$$-\frac{\sqrt{\pi}\left(f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)+i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}\left(b\log(f)+2ie\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)-i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}\left(b\log(f)-2ie\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)+i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(\frac{\left(2cx\log(f)+b\log(f)+2ie\right)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)-i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(\frac{\left(2cx\log(f)+b\log(f)-2ie\right)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}-2f^{a}\operatorname{erf}\left(-\frac{1}{2}b\overline{\frac{1}{\sqrt{-c\log(f)}}}\log(f)+x\overline{\sqrt{-c\log(f)}}\right)+2f^{a}\operatorname{erf}\left(\frac{2cx\log(f)+b\log(f)}{2\sqrt{-c\log(f)}}\right)\right)}{16\sqrt{-c\log(f)}\,f^{\frac{b^{2}}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*(f^a*(cos((2*c*d - b*e)/c) + I*sin((2*c*d - b*e)/c))*erf(x*c
onjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 2*I*e)*conjugate(1/sqrt(-c*log(
f))))*e^(e^2/(c*log(f))) + f^a*(cos((2*c*d - b*e)/c) - I*sin((2*c*d - b*e)/
c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - 2*I*e)*conjugate(1/s
qrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos((2*c*d - b*e)/c) + I*sin((2*
c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + 2*I*e)*sqrt(-c*log(f))/(c
*log(f)))*e^(e^2/(c*log(f))) + f^a*(cos((2*c*d - b*e)/c) - I*sin((2*c*d - b
*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - 2*I*e)*sqrt(-c*log(f))/(c*log(f)
))*e^(e^2/(c*log(f))) - 2*f^a*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)))*log(f
) + x*conjugate(sqrt(-c*log(f)))) + 2*f^a*erf(1/2*(2*c*x*log(f) + b*log(f))
/sqrt(-c*log(f))))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [A]
time = 2.65, size = 226, normalized size = 0.98

$$\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{\left((2cx+b)\log(f)-2ie\right)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{\left(b^{2}-4ac\right)\log(f)^{2}+4\left(2icd-ibe\right)\log(f)-4e^{2}}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{\left((2cx+b)\log(f)+2ie\right)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{\left(b^{2}-4ac\right)\log(f)^{2}+4\left(-2icd+ibe\right)\log(f)-4e^{2}}{4c\log(f)}\right)}-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{\frac{b^{2}-4ac}{4c}}}}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 2*I*e)*sqrt(-c*
log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*(2*I*c*d - I*b*e)*l
og(f) - 4*e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*
log(f) + 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
 + 4*(-2*I*c*d + I*b*e)*log(f) - 4*e^2)/(c*log(f))) - 2*sqrt(pi)*sqrt(-c*lo
g(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c*lo
g(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin^2{(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\sin\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x)^2, x)
```

3.96 $\int f^{a+bx+cx^2} \sin^3(d+ex)\,dx$

Optimal. Leaf size=354

$$-\frac{3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{ie^{-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{3i}{}$$

```
[Out] 3/16*I*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c
*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*I*exp(-3*I
*d+1/4*(3*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*I*e+b*ln(f)+2*c*x*ln(f))
/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/16*I*exp(I*d+1/4*(e-I*
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*I*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/c/
ln(f))*f^a*erfi(1/2*(3*I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/
2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4560, 2325, 2266, 2235}

$$-\frac{3i\sqrt{\pi}\,f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{i\sqrt{\pi}\,f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}\,f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{i\sqrt{\pi}\,f^a e^{3id-\frac{(b\log(f)+3ie)^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+3ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[L
og[f]]) + ((I/16)*E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(S
qrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f])
)*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]]
)])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx &= \int \left(\frac{3}{8}ie^{-id-iex}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+iex}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3iex}f^{a+bx+cx^2}+\frac{1}{8}ie^{3id}\cdots\right.\\
&= -\left(\frac{1}{8}i\int e^{-3id-3iex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3iex}f^{a+bx+cx^2}\,dx+\frac{3}{8}i\int e^{-id-iex}\cdots\\
&= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)+cx^2\log(f)-x(3ie-b\log(f))\right)\,dx\right)+\frac{1}{8}i\int\cdots\\
&= -\left(\frac{1}{8}\left(3ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx\right)+\frac{1}{8}\left(\cdots\right.\\
&= -\frac{3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\exp\left(-3id+\frac{(3e+ib\log(f\cdots}{4c\log(f)}\cdots\right.}{16\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.03, size = 391, normalized size = 1.10

$$
\frac{e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-ie^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\cos(3d)\mathrm{Erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+ie^{\frac{2e^2}{c\log(f)}}\cos(3d)\mathrm{Erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3ie^{\frac{ibe}{c}}\mathrm{Erfi}\left(\frac{-ie-(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+3e^{\frac{2ibe}{c}}\mathrm{Erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))-e^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)\sin(3d)-e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)\sin(3d)\right)}{16\sqrt{c}\sqrt{\log(f)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*((-I)
*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2
*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + I*E^((2*e^2)/(c*Log[f]))*Cos[3*d]
*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + (3*I)*E^((
```

```
I*b*e)/c)*Erfi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos
[d] + I*Sin[d]) + 3*E^(((2*I)*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - E^((e*(2*e + (3*I)*b*Log[f]))
/(c*Log[f]))*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*Sin[3*d] - E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.90, size = 338, normalized size = 0.95

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6 i \ln(f) b e-12 i d \ln(f) c-9 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3 i e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6 i \ln(f) b}{4 \ln}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c-9*e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*I*e+b*ln(f))/(
-c*ln(f))^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12
*I*d*ln(f)*c-9*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(
-3*I*e+b*ln(f))/(-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2
-2*I*ln(f)*b*e+4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))
^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(
ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(
-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 696, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(pi)*(f^a*(-I*cos(3/2*(2*c*d - b*e)/c) + sin(3/2*(2*c*d - b*e)/c))
*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 3*I*e)*conjugate(1/sqrt
(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(I*cos(3/2*(2*c*d - b*e)/c) + si
n(3/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) -
3*I*e)*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(-I*cos(3
/2*(2*c*d - b*e)/c) + sin(3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*l
og(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f))) + f^a*(I*
cos(3/2*(2*c*d - b*e)/c) + sin(3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
+ b*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f))) - 3*
```

```
f^a*(-I*cos(1/2*(2*c*d - b*e)/c) + sin(1/2*(2*c*d - b*e)/c))*erf(x*conjugat
e(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-c*log(f))))*e^(
1/4*e^2/(c*log(f))) - 3*f^a*(I*cos(1/2*(2*c*d - b*e)/c) + sin(1/2*(2*c*d -
b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - I*e)*conjugate(
1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(-I*cos(1/2*(2*c*d - b*e
)/c) + sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + I*e)*sq
rt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(I*cos(1/2*(2*c*d
- b*e)/c) + sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - I*
e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt(-c*log(f))/(c*f
^(1/4*b^2/c)*log(f))
```

Fricas [**A**]
time = 2.65, size = 350, normalized size = 0.99

$$\frac{i\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)-3ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2+6(2icd-ibe)\log(f)-9e^2}{4c\log(f)}\right)}-3i\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2+2(2icd-ibe)\log(f)-e^2}{4c\log(f)}\right)}-i\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)+3ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2+6(-2icd+ibe)\log(f)-9e^2}{4c\log(f)}\right)}+3i\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2+2(-2icd+ibe)\log(f)-e^2}{4c\log(f)}\right)}}{16c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(
-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 6*(2*I*c*d - I*b*e
)*log(f) - 9*e^2)/(c*log(f))) - 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*
x + b)*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log
(f)^2 + 2*(2*I*c*d - I*b*e)*log(f) - e^2)/(c*log(f))) - I*sqrt(pi)*sqrt(-c*
log(f))*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^
(-1/4*((b^2 - 4*a*c)*log(f)^2 + 6*(-2*I*c*d + I*b*e)*log(f) - 9*e^2)/(c*log
(f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqr
t(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 2*(-2*I*c*d + I*
b*e)*log(f) - e^2)/(c*log(f))))/(c*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^3, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x)^3, x)
```

3.97 $\int f^{a+bx+cx^2} \sin(d+fx^2)\,dx$

Optimal. Leaf size=193

$$-\frac{ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] -1/4*I*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*
f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*exp(I*d
-b^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I
*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$-\frac{i\sqrt{\pi}\,f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\mathrm{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}\,f^ae^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] ((-1/4*I)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*E
rf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f -
c*Log[f]] - ((I/4)*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt
[I*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+bx+cx^2}\right)\,dx\\
&= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}i\int\exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)\,dx-\frac{1}{2}i\int\exp\left(\right.\\
&= \frac{1}{2}\left(ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)\,dx-\frac{1}{2}\\
&= -\frac{ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{e}}{4\sqrt{if-}}
\end{aligned}$$

Mathematica [A]
time = 1.04, size = 230, normalized size = 1.19

$$-\frac{\sqrt[4]{-1}\,e^{\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)\sqrt{f+ic\log(f)}\,(if+c\log(f))(\cos(d)-i\sin(d))+e^{\frac{ib^2f\log^2(f)}{2\left(f^2+c^2\log^2(f)\right)}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(2fx-i(b+2cx)\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\,(f+ic\log(f))(\cos(d)+i\sin(d))\right)}{4\left(f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(Er
fi[((-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sq
rt[f + I*c*Log[f]]*(I*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^(((I/2)*b^2*f*L
og[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*Log[
```

```
f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d
] + I*Sin[d])))/(f^2 + c^2*Log[f]^2)
```

Maple [A]
time = 0.55, size = 180, normalized size = 0.93

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i d \ln(f) c+4 d f}{4(i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i f}\, x+\frac{\ln(f) b}{2 \sqrt{-c \ln(f)-i f}}\right)}{4 \sqrt{-c \ln(f)-i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i d \ln(f) c-}{4(-i f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f))
)/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*
f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(-I
*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*
f-c*ln(f))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(145) = 290.
time = 0.30, size = 647, normalized size = 3.35

$$-\frac{\sqrt{\pi}\sqrt{2c^{2}\log(f)^{2}+2f^{2}}\left(\left(f^{a}\cos\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)-if^{a}\sin\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)-if)x+b\log(f)}{2\sqrt{-c\log(f)+if}}\right)+\left(f^{a}\cos\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)+if^{a}\sin\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)+if)x+b\log(f)}{2\sqrt{-c\log(f)-if}}\right)\right)\sqrt{c\log(f)+\sqrt{c^{2}\log(f)^{2}+f^{2}}}+\sqrt{\pi}\sqrt{2c^{2}\log(f)^{2}+2f^{2}}\left(\left(if^{a}\cos\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)+f^{a}\sin\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)-if)x+b\log(f)}{2\sqrt{-c\log(f)+if}}\right)+\left(-if^{a}\cos\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)+f^{a}\sin\left(\frac{4df^{2}+(4c^{2}d+b^{2}f)\log(f)^{2}}{4(c^{2}\log(f)^{2}+f^{2})}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)+if)x+b\log(f)}{2\sqrt{-c\log(f)-if}}\right)\right)\sqrt{-c\log(f)+\sqrt{c^{2}\log(f)^{2}+f^{2}}}}{8\left(c^{2}e^{\left(\frac{b^{2}c\log(f)^{3}}{4(c^{2}\log(f)^{2}+f^{2})}\right)}\log(f)^{2}+f^{2}e^{\left(\frac{b^{2}c\log(f)^{3}}{4(c^{2}\log(f)^{2}+f^{2})}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*cos(1/4*(4*d*f^2 + (4*c^2
*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^a*sin(1/4*(4*d*f^2 + (4*c
^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*
x + b*log(f))/sqrt(-c*log(f) + I*f)) + (f^a*cos(1/4*(4*d*f^2 + (4*c^2*d + b
^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^a*sin(1/4*(4*d*f^2 + (4*c^2*d +
 b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*
log(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) +
 sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*cos(1/4*(4*d*f^2 + (4*c^2*d
+ b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^2*d
+ b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b
*log(f))/sqrt(-c*log(f) + I*f)) + (-I*f^a*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(
f))/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^
2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^2*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + f^2)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(145) = 290$.
time = 2.19, size = 309, normalized size = 1.60

$$\frac{\sqrt{\pi}\,(i\,c\log(f)+f)\sqrt{-c\log(f)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^2x-i\,bf\log(f)+(2\,c^2x+bc)\log(f)^2\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,af^2\log(f)-\left(b^2c-4\,ac^2\right)\log(f)^3+4i\,df^2+\left(4i\,c^2d+i\,b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)}+\sqrt{\pi}\,(-i\,c\log(f)+f)\sqrt{-c\log(f)+i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^2x+i\,bf\log(f)+(2\,c^2x+bc)\log(f)^2\right)\sqrt{-c\log(f)+i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,af^2\log(f)-\left(b^2c-4\,ac^2\right)\log(f)^3-4i\,df^2+\left(-4i\,c^2d-i\,b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)}}{4\left(c^2\log(f)^2+f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b
*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
 f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4
*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f)
 + f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x + b*
c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*lo
g(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*f)*log(
f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sin(d + f*x^2), x)
```

3.98 $\int f^{a+bx+cx^2} \sin^2(d+fx^2)\, dx$

Optimal. Leaf size=245

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{I}}{8\sqrt{2if}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)+1/8*exp(-2*I*d+b^2*ln(f)^2/(8*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*
ln(f)-2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^
(1/2)-1/8*exp(2*I*d-b^2*ln(f)^2/(8*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*
x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4560, 2266, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\mathrm{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}\,f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}\right)\,dx \\
&= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+b} \\
&= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)+bx\log(f)-x^2(2if-c\log(f))\right)\,dx\right)-\frac{1}{4} \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{1}{4}\left(e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log}{}\right. \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2}{2\sqrt{2if-}}\right.}{8\sqrt{2if-c\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 3.30, size = 299, normalized size = 1.22

$$\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\,e^{\frac{b^2\log^2(f)}{8if-4c\log(f)}}\left(\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(4fx+i(b+2cx)\log(f))}{2\sqrt{2f+ic\log(f)}}\right)\sqrt{2f+ic\log(f)}\,(2if+c\log(f))(\cos(2d)-i\sin(2d))+e^{\frac{ib^2f\log^2(f)}{4f^2+c^2\log^2(f)}}\mathrm{Erf}\left(\frac{(-1)^{3/4}(4fx-i(b+2cx)\log(f))}{2\sqrt{2f-ic\log(f)}}\right)\sqrt{2f-ic\log(f)}\,(2f+ic\log(f))(\cos(2d)+i\sin(2d))\right)}{4f^2+c^2\log^2(f)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
```

```
g[f]))*(Erf[((-1)^(1/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*c*L
og[f]])]*Sqrt[2*f + I*c*Log[f]]*((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]
) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erf[((-1)^(3/4)*(4*f*x -
I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log[f]]*(
2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A]
time = 0.66, size = 227, normalized size = 0.93

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+8 i d \ln(f) c+16 d f}{4(-2 i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{2 i f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-8 i d \ln(f) c+16}{4(2 i f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(-2*I*f+c*ln(f
)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-c
*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*f)
/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*
ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^
(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 997, normalized size = 4.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(1/4*b^2/c)*cos(1/2*
(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1/
4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*
f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*log(f) + 2*I*f))
 + (-I*f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c
^2*log(f)^2 + 4*f^2)) + f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^
2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b
*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^
2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((f^a*f^(1/4*b^
2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)
) - I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^
2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*l
og(f) + 2*I*f)) + (f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*
```

```
log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 +
 (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f)
+ 2*I*f)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*
log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - 2*sqrt(pi)*((c^2*f^a*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)))*log(f) + x*
conjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
+ 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^
2)))*erf(1/2*(2*c*x*log(f) + b*log(f))/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 + 4*f^2*e^(1
/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f
)))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(185) = 370$.
time = 2.31, size = 402, normalized size = 1.64

$$\frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2 - 2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right) - 2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x - 2i\,bf\log(f) + \left(2\,c^2x + bc\right)\log(f)^2\right)\sqrt{-c\log\left(f\right) - 2i\,f}}{2\left(c^2\log(f)^2 + 4\,f^2\right)}\right)e^{\left(\frac{16\,af^2\log(f) - \left(b^2c - 4\,ac^2\right)\log(f)^3 + 32i\,df^2 - 2\left(-4i\,c^2d - i\,b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2 + 4\,f^2\right)}\right)} + \sqrt{\pi}\left(c^2\log\left(f\right)^2 + 2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right) + 2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x + 2i\,bf\log(f) + \left(2\,c^2x + bc\right)\log(f)^2\right)\sqrt{-c\log\left(f\right) + 2i\,f}}{2\left(c^2\log(f)^2 + 4\,f^2\right)}\right)e^{\left(\frac{16\,af^2\log(f) - \left(b^2c - 4\,ac^2\right)\log(f)^3 - 32i\,df^2 - 2\left(4i\,c^2d + i\,b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2 + 4\,f^2\right)}\right)} - \frac{2\sqrt{\pi}\left(c^2\log(f)^2 + 4\,f^2\right)\sqrt{-c\log\left(f\right)}\,\operatorname{erf}\left(\frac{(2\,cx + b)\sqrt{-c\log\left(f\right)}}{2\,c}\right)}{f^{\frac{b^2 - 4\,ac}{4\,c}}}}{8\left(c^3\log\left(f\right)^3 + 4\,cf^2\log\left(f\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1
/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - 2
*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2)*l
og(f)^3 + 32*I*d*f^2 - 2*(-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4
*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*e
rf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f)
 + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^
2)*log(f)^3 - 32*I*d*f^2 - 2*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2
+ 4*f^2)) - 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*
x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*
log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^2, x)
```

3.99 $\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=386

$$-\frac{3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}-\cdots$$

```
[Out] -3/16*I*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I
*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*I*exp(-
3*I*d+b^2*ln(f)^2/(12*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*I*f-c*ln(
f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)-3/16*I*exp(I*d-b
^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f
+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*I*exp(3*I*d-1/4*b^2*ln(f
)^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln
(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$-\frac{3i\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{i\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{i\sqrt{\pi}\,f^a\exp\left(3id-\frac{b^2\log^2(f)}{4(c\log(f)+3if)}\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}-\frac{3i\sqrt{\pi}\,f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi
]*Erf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f
 - c*Log[f]] + ((I/16)*E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f -
c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/16)*E^(I*d - (b^2*Log[f]^2)
/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f])
)/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d - (b
^2*Log[f]^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((
3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx &= \int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx+cx^2}+\right.\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}i\int e^{-i}\\
&= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)+bx\log(f)-x^2(3if-c\log(f))\right)\,dx\right)+\frac{1}{8}i\\
&= \frac{1}{8}\left(3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)\,dx-\frac{1}{8}\\
&= -\frac{3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{}}{16\sqrt{3}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3291 vs. 2(386) = 772.
time = 7.03, size = 3291, normalized size = 8.53

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3
*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f
 - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Log[f
]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2
*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*
Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f
 - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4
)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-1)^(1/4)
*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[
3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f])
)*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2
*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*
E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4)*(6*
f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^2*
Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*
Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))
/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]] + (27*(-1)^(1/
4)*f^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I
*c*Log[f])) - (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log
[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (3*(-1)^(1/4)*c^2*f*Cos[d]
*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*
Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) - (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (
2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/
E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*f^3*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ ((-1)^(3/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^((
(I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f])) + ((-1)^(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] +
(2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*
b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] -
(2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] +
27*(-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^2*Erfi[((-1)^(1
```

```
/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[
f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(
f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f])
)/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(
3/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*f*x
- I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f
 - I*c*Log[f]]*Sin[d] - (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(2*f*x + I*b*Lo
g[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*Si
n[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (27*(-1)^(1/4)*c*f^2*Erfi
[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) - (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d]
)/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f]
)) - 3*(-1)^(1/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Erfi[((-1
)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3
*f - I*c*Log[f]))*f^2*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - 3
*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Erfi[((-1)^(1
/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Lo
g[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(...
```

Maple [A]
time = 1.09, size = 358, normalized size = 0.93

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-12 i d \ln(f) c+36 d f}{4(3 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3 i f}\, x+\frac{\ln(f) b}{2\sqrt{-c\ln(f)-3 i f}}\right)}{16\sqrt{-c\ln(f)-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+1\ldots}{4(-3i\ldots}}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*I*d*ln(f)*c+36*d*f)/(3*I*f+c*
ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-
c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f
)*c+36*d*f)/(-3*I*f+c*ln(f)))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^
(1/2)+1/2*ln(f)*b/(3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f
)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*
f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-
1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*e
rf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. 2(289) = 578.
time = 0.33, size = 2451, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)
^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*lo
g(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)
^2 + 9*f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(
f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36
*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c
*log(f) - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^a*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)
/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
 f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*
sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*er
f(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f
^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f
^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f^2 +
(4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-
c*log(f) + I*f)) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*
log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1
/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (I*c^2*f^
a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^(a + 2)*e^
(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*l
og(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) +
sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2
+ f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36*d*f^2
+ (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f)
 - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^a*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)
^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(
```

```
c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2
))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*
(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f
) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(
((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^
(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f^2 + (
4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^a*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3
/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c
^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-c*log(
f) + I*f)) + ((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log
(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/
4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^a*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f)
)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2))*log(f)^4 + 10*c^2*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*
f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^4*e^(1/4*b^2
*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2
)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(289) = 578$.
time = 3.04, size = 731, normalized size = 1.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x + b
*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 - 3*(-4*I*c^2*d - I*b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*
f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^
2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3
- 108*I*d*f^2 - 3*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) -
 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9*f^3)*s
qrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x + b*c)*log(
f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*log(f) -
```

```
(b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*log(f)^2)/(c
^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f
^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) +
(2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/
4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d -
I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)
^2 + 9*f^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^3, x)
```

3.100 $\int f^{a+bx+cx^2} \sin(d+ex+fx^2)\,dx$

Optimal. Leaf size=212

$$\frac{ie^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+
2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*
exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2*x*(I
*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\frac{i\sqrt{\pi}\,f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}\,f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((I/4)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*
Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt
[I*f - c*Log[f]] - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}i\int \exp\left(-id+a\log(f)-x(ie-b\log(f))-x^2(if-c\log(f))\right)dx\ \cdots\\
&= \frac{1}{2}\left(i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-ie+b\log(f)+2x\cdots}{4(-if+c\cdots}\right.\\
&= \frac{i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{i\exp\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 2.30, size = 347, normalized size = 1.64

$$-\frac{\sqrt[4]{-1}\,e^{-\frac{1}{4}i\left(\frac{e^2}{f-ic\log(f)}+\frac{b^2\log^2(f)}{f+ic\log(f)}\right)}f^{\frac{f(-be+af)+ac^2\log^2(f)}{f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{ib^2f\log^2(f)}{2\left(f^2+c^2\log^2(f)\right)}}f^{\frac{be}{2f+2ic\log(f)}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-i(b+2cx)\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\,(f+ic\log(f))(\cos(d)+i\sin(d))+e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}}f^{\frac{be}{2f-2ic\log(f)}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}\,(i\cos(d)+\sin(d))\right)}{4\left(f^2+c^2\log^2(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2
))*Sqrt[Pi]*(E^(((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f
+ (2*I)*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*
Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Si
n[d]) + E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*Log[
f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*L
og[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(I*Cos[d] + Sin[d])))/(E^((I
```

```
/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2 + c^2*Lo
g[f]^2))
```

Maple [A]
time = 0.58, size = 216, normalized size = 1.02

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^2 b^2+2 i \ln(f) b e-4 i d \ln(f) c+4 d f-e^2}{4(i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i f}\, x+\frac{i e+b \ln(f)}{2\sqrt{-c \ln(f)-i f}}\right)}{4\sqrt{-c \ln(f)-i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-
e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I
*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-
2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*
erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1005 vs. 2(159) = 318.
time = 0.31, size = 1005, normalized size = 4.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*f^(1/4*c*e^2/(c^2*log(f)^2
 + f^2))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(
c^2*log(f)^2 + f^2)) - I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*
d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2))
)*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*
log(f) - I*f)) + (f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*d*f^2 +
 (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + I*f^
a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f -
2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*
f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(c*log(f) + I*f)))*sqrt(c*log(
f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*
f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
- 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + f^a*f^(1/4*c*e^2/(c^2*
log(f)^2 + f^2))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 -
f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*
e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + (-I*f^a*f^(1/4*c*e^2/(c^2*log(
```

```
f)^2 + f^2))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^
2)/(c^2*log(f)^2 + f^2)) + f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(
4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2
)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(
c*log(f) + I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2*e^(1/4*b
^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2))
*log(f)^2 + f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(
f)/(c^2*log(f)^2 + f^2)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(159) = 318$.
time = 4.08, size = 379, normalized size = 1.79

$$\frac{\sqrt{\pi}\left(-i c \log(f)+f\right)\sqrt{-c \log(f)+i f}\,\operatorname{erf}\left(\frac{\left(2 f^2 x+\left(2 c^2 x+b c\right)\log(f)^2+f e+\left(i b f-i c e\right)\log(f)\right)\sqrt{-c \log(f)+i f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(-\frac{\left(b^2 c-4 a c^2\right)\log(f)^3+4 i d f^2-\left(-4 i c^2 d-i b^2 f+2 i b c e\right)\log(f)^2-i f e^2-\left(4 a f^2-2 b f e+c e^2\right)\log(f)}{4\left(c^2\log(f)^2+f^2\right)}\right)}+\sqrt{\pi}\left(i c \log(f)+f\right)\sqrt{-c \log(f)-i f}\,\operatorname{erf}\left(\frac{\left(2 f^2 x+\left(2 c^2 x+b c\right)\log(f)^2+f e+\left(-i b f+i c e\right)\log(f)\right)\sqrt{-c \log(f)-i f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(-\frac{\left(b^2 c-4 a c^2\right)\log(f)^3-4 i d f^2-\left(4 i c^2 d+i b^2 f-2 i b c e\right)\log(f)^2+i f e^2-\left(4 a f^2-2 b f e+c e^2\right)\log(f)}{4\left(c^2\log(f)^2+f^2\right)}\right)}}{4\left(c^2\log(f)^2+f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + (2
*c^2*x + b*c)*log(f)^2 + f*e + (I*b*f - I*c*e)*log(f))*sqrt(-c*log(f) + I*f
)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 - (
-4*I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 - I*f*e^2 - (4*a*f^2 - 2*b*f*e +
 c*e^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*l
og(f) - I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + f*e + (-I*b*f +
I*c*e)*log(f))*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - 4*I*d*f^2 - (4*I*c^2*d + I*b^2*f - 2*I*b*c*e)*log(f)^
2 + I*f*e^2 - (4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c
^2*log(f)^2 + f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2), x)
```

3.101 $\int f^{a+bx+cx^2} \sin^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=268

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}}}{}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)-1/8*exp(-2*I*d-(2*e+I*b*ln(f))^2/(8*I*f-4*c*ln(f)))*f^a*erf(1
/2*(2*I*e-b*ln(f)+2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I
*f-c*ln(f))^(1/2)-1/8*exp(2*I*d+(2*e-I*b*ln(f))^2/(8*I*f+4*c*ln(f)))*f^a*er
fi(1/2*(2*I*e+b*ln(f)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/
(2*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4560, 2266, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\,\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}\,f^a\exp\left(\frac{(2e-ib\log(f))^2}{4c\log(f)+8if}+2id\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)+2ie}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int\left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+c\ldots}\right.$$

$$=-\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\,d\ldots$$

$$=-\left(\frac{1}{4}\int\exp\left(-2id+a\log(f)-x(2ie-b\log(f))-x^2(2if-c\log(f)\right)\right.\ldots$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{1}{4}\left(\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right.\right.\ldots$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\sqrt{\pi}\ldots}{8\sqrt{2if-c\ldots}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1120 vs. 2(268) = 536.
time = 6.68, size = 1120, normalized size = 4.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

[Out] (f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erf[((-1)^(
3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f
]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2 + (4*
I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erf[((-1)^(3/4)
*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]
*Log[f]^2*Sqrt[2*f - I*c*Log[f]] + (2*(-1)^(3/4)*c*f*Cos[2*d]*Erf[((-1)^(1/
4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]]
)]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^
2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Cos[2*d]*Erf[((-1)^(1/4)
*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]
*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^
2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2 + (4*I)
*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erf[((-1)^(3/4)*(2*e + 4
*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]*S
qrt[2*f - I*c*Log[f]]*Sin[2*d] - (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^2 + (4*I)*b
*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erf[((-1)^(3/4)*(2*e + 4*f*x
 - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]^2*Sqr
t[2*f - I*c*Log[f]]*Sin[2*d] + (2*(-1)^(1/4)*c*f*Erf[((-1)^(1/4)*(2*e + 4*f
*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[f]*Sqr
t[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^2*Lo
g[f]^2))/(2*f + I*c*Log[f])) - ((-1)^(3/4)*c^2*Erf[((-1)^(1/4)*(2*e + 4*f*x
 + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[f]^2*Sqr
t[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^2*Lo
g[f]^2))/(2*f + I*c*Log[f]))))/(8*c*Log[f]*(2*f - I*c*Log[f])*(2*f + I*c*Lo
g[f]))

Maple [A]
time = 0.77, size = 263, normalized size = 0.98

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c+16 d f-4 e^{2}}{4(-2 i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{2 i f-c \ln(f)}+\frac{b \ln(f)-2 i e}{2 \sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\sqrt{\pi}\, f^{a} {\mathrm e}^{-\ln(f)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-4
*e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+
1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)
^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)

```
-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(f)-2
*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 1481, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*cos(1/2*(16*d*f^2 + (4*
c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2))*e^(1/4
*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)) + f^a*e^(1/4*b^2*log(f
)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2))*sin(1/2*(16*d*f^2 + (4*c^2*d + b
^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*
log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*log(f) - 2
*I*f)) + (-I*f^a*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 -
 4*f*e^2)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*l
og(f)^2 + 4*f^2)) + f^a*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 +
4*f^2))*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)
/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b*log(f) + 2*I*
e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*lo
g(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((
f^a*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^
2*log(f)^2 + 4*f^2))*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f
^2)) - I*f^a*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2))*sin
(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f
)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*
log(f) + 2*I*f)/(c*log(f) - 2*I*f)) + (f^a*cos(1/2*(16*d*f^2 + (4*c^2*d + b
^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*b^2*log(
f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*e^(1/4*b^2*log(f)/c + c
*e^2*log(f)/(c^2*log(f)^2 + 4*f^2))*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f -
2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f)
+ 2*I*f)*x + b*log(f) + 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))
*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) + 2*sqrt(pi)*
((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^
2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugat
e(1/sqrt(-c*log(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1
/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^2*log(f)^2 + 4
*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2)
+ 2*b*f*e*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*c*x*log(f) + b*log(f))
/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*
```

```
b*f*e*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 + 4*f^2*e^
(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^2*log(f)^2 +
 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f)))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(203) = 406$.
time = 2.18, size = 474, normalized size = 1.77

$$\frac{\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}+2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)+2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^{2}x+\left(2\,c^{2}x+bc\right)\log(f)^{2}+4\,fe-2\left(-i\,bf+i\,ce\right)\log(f)\right)\sqrt{-c\log\left(f\right)+2i\,f}}{2\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}+32i\,df^{2}+2\left(4i\,c^{2}d+i\,b^{2}f-2i\,bce\right)\log(f)^{2}-8i\,fe^{2}-4\left(4\,af^{2}-2\,bfe+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)}+\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^{2}x+\left(2\,c^{2}x+bc\right)\log(f)^{2}+4\,fe-2\left(i\,bf-i\,ce\right)\log(f)\right)\sqrt{-c\log\left(f\right)-2i\,f}}{2\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}-32i\,df^{2}+2\left(-4i\,c^{2}d-i\,b^{2}f+2i\,bce\right)\log(f)^{2}+8i\,fe^{2}-4\left(4\,af^{2}-2\,bfe+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)}-\frac{2\sqrt{\pi}\left(c^{2}\log(f)^{2}+4\,f^{2}\right)\sqrt{-c\log\left(f\right)}\,\operatorname{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log\left(f\right)}}{2\,c}\right)}{f^{\frac{b^{2}-4\,ac}{4\,c}}}}{8\left(c^{3}\log\left(f\right)^{3}+4\,cf^{2}\log\left(f\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(1
/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*f*e - 2*(-I*b*f + I*c*e)*log(f))
*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)
*log(f)^3 + 32*I*d*f^2 + 2*(4*I*c^2*d + I*b^2*f - 2*I*b*c*e)*log(f)^2 - 8*I
*f*e^2 - 4*(4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + sq
rt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1/2*(8*f
^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*f*e - 2*(I*b*f - I*c*e)*log(f))*sqrt(-c
*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^
3 - 32*I*d*f^2 + 2*(-4*I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 + 8*I*f*e^2
- 4*(4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) - 2*sqrt(pi
)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f)
)/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^2, x)
```

3.102 $\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=430

$$\frac{3ie^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}$$

```
[Out] 3/16*I*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)
+2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*
I*exp(-3*I*d-1/4*(3*e+I*b*ln(f))^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e-b*ln
(f)+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1
/2)-3/16*I*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*l
n(f)+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1
/16*I*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+
b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f)
)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\frac{3i\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a\exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}f^a\exp\left(3id-\frac{(b\log(f)+3ie)^2}{4(c\log(f)+3if)}\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt
[Pi]*Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])
/Sqrt[I*f - c*Log[f]] - ((I/16)*E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I
)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*L
og[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/1
6)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
 c*Log[f]] + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(
2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx = \int \left(-\frac{1}{8}ie^{-3i(d+ex+fx^2)}f^{a+bx+cx^2}+\frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i(d\right.\right.$$

$$= -\left(\frac{1}{8}i\int e^{-3i(d+ex+fx^2)}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int \exp\left(6id+6iex+6ifx^2\right.$$

$$= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)-x(3ie-b\log(f))-x^2(3if-c\log(f)\right.\right.$$

$$= \frac{1}{8}\left(3i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2x}{4(-if+c}\right.$$

$$= \frac{3i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{i\exp}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3835 vs. 2(430) = 860.
time = 7.24, size = 3835, normalized size = 8.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[
f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f
] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*
(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*L
og[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*
c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f
*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((
I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Cos[d]*Erf
i[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*
Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4)*E^(((I/4)*(-9*e^2 +
(6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-
1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*L
og[f]])]*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*(-9*e^2 + (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*E^(((I/4)*(-9*e^2 + (6*I)
*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4
)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*(-9*e^2 + (6*I
)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]^3*Sqrt[3*f - I*c*Log[f]] + (27*(-1)^(1/4)*f^3*Cos[d]*Erfi[((-1)^(3/
4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*S
qrt[f + I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f
 + I*c*Log[f])) - (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x +
 I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f +
I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*L
og[f])) + (3*(-1)^(1/4)*c^2*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[
f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log
[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f]))
 - (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I
)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/E^(
((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1
)^(1/4)*f^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x
*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-9
*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(3/4)*
c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f
]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-
9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) - (3*(-1)^(1/
```

```
4)*c^2*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/
4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^
(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]])/E^(((I
/4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + 27*(-
1)^(1/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f
]))*f^3*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] + 27*(-1)^(3/4)*c*E^(((I/4
)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f^2*Erfi[((-1
)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]
])]*Log[f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-e^2 +
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(e
+ 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^3*E^(((I/4)*(-e^2 + (2*I)*b*
e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I
*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f -
I*c*Log[f]]*Sin[d] - (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*L
og[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*S
in[d])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])
) - (27*(-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E
^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(
-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]
))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4
)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f ...
```

Maple [**A**]
time = 1.34, size = 430, normalized size = 1.00

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6 i \ln(f) b e-12 i d \ln(f) c+36 d f-9 e^{2}}{4(3 i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln\left(f\right)-3 i f}\, x+\frac{3 i e+b \ln(f)}{2 \sqrt{-c \ln\left(f\right)-3 i f}}\right)}{16 \sqrt{-c \ln\left(f\right)-3 i f}}+\frac{i\sqrt{\pi}\, f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c+36*
d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1
/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-
1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*f-9*e^2)/(-3*I*f+c*ln(f)
))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*(-3*I*e+b*ln(f))/
(3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*
```

```
b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*
f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a
*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f))
)/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln
(f)-I*f)^(1/2))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4349 vs. $2(320) = 640$.
time = 0.36, size = 4349, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^
2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3
/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*l
og(f)/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c
*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*
log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4
*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*
c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f
- 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f
) - 3*I*f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f)
) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)
/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 +
 f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c
^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(36*
d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f
^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*lo
g(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)
^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log
(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2
 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f) + 3*I*e)*sqrt(-c*log
(f) - 3*I*f)/(c*log(f) + 3*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2
)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)
^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/4*c
*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/4*
c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2
*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*
```

```
log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2) +
 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2
*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2
) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b
^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(
f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + ((c
^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2
*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^
(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^
2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*d*
f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) +
 (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)
/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 +
 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log
(f)/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*sin(1/
4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 +
f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f
)/(c*log(f) + I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*s
qrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)
^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*l
og(f)^2 + 9*f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^
2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*lo
g(f)^2 + 9*f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2
- 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*lo
g(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c
^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f
)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*
log(f)^2 + 9*f^2)))*sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^
2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x + b*l
og(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f)) + ((-I*c^2*f^a*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2
+ f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - I*f^(a + 2)*e^
(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 +
 f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(36*d*f^2 + (4*c^
2*d + b^2*f - 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f
^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f
)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9...
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(320) = 640$.
time = 2.85, size = 873, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)
*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*f
*e - 3*(-I*b*f + I*c*e)*log(f))*sqrt(-c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f
^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 + 3*(4*I*c^2*d + I*b
^2*f - 2*I*b*c*e)*log(f)^2 - 27*I*f*e^2 - 9*(4*a*f^2 - 2*b*f*e + c*e^2)*log
(f))/(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2
+ 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2
*x + b*c)*log(f)^2 + f*e + (I*b*f - I*c*e)*log(f))*sqrt(-c*log(f) + I*f)/(c
^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 - (-4*I
*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 - I*f*e^2 - (4*a*f^2 - 2*b*f*e + c*e
^2)*log(f))/(c^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log
(f)^2 - 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x +
(2*c^2*x + b*c)*log(f)^2 + f*e + (-I*b*f + I*c*e)*log(f))*sqrt(-c*log(f) -
I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2
- (4*I*c^2*d + I*b^2*f - 2*I*b*c*e)*log(f)^2 + I*f*e^2 - (4*a*f^2 - 2*b*f*e
 + c*e^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2
*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f
^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*f*e - 3*(I*b*f - I*c*e)*log(f))*sqrt(-c
*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^
3 - 108*I*d*f^2 + 3*(-4*I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 + 27*I*f*e^
2 - 9*(4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + 9*f^2)))/(c^4*log
(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^3, x)
```

3.103 $\int f^{a+bx+cx^2} \sin\left(a+bx+ex^2\right)\,dx$

Optimal. Leaf size=213

$$\frac{ie^{-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}-\frac{ie^{(i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b(i+\log(f))+2}{2\sqrt{ie+}}\right.}{4\sqrt{ie+c\log(f)}}$$

```
[Out] -1/4*I*erf(1/2*(-b*(I-ln(f))-2*x*(I*e-c*ln(f)))/(I*e-c*ln(f))^(1/2))*Pi^(1/
2)/exp((I-ln(f))*(a-b^2*(I-ln(f))/(4*I*e-4*c*ln(f))))/(I*e-c*ln(f))^(1/2)-1
/4*I*exp((I+ln(f))*(a-b^2*(I+ln(f))/(4*I*e+4*c*ln(f))))*erfi(1/2*(b*(I+ln(f
))+2*x*(I*e+c*ln(f)))/(I*e+c*ln(f))^(1/2))*Pi^(1/2)/(I*e+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\frac{i\sqrt{\pi}\exp\left(-\left((-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\right)\mathrm{Erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}-\frac{i\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\mathrm{Erfi}\left(\frac{b(\log(f)+i)+2x(c\log(f)+ie)}{2\sqrt{c\log(f)+ie}}\right)}{4\sqrt{c\log(f)+ie}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

```
[Out] ((I/4)*Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c
*Log[f]])])/(E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])
))*Sqrt[I*e - c*Log[f]]) - ((I/4)*E^((I + Log[f])*(a - (b^2*(I + Log[f]))/(
(4*I)*e + 4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]
))/(2*Sqrt[I*e + c*Log[f]])])/Sqrt[I*e + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx &= \int \left(\frac{1}{2}ie^{-ia-ibx-iex^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-ia-ibx-iex^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+iex^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}i\int \exp\left(-a(i-\log(f))-bx(i-\log(f))-x^2(ie-c\log(f))\right)dx-\frac{1}{2}\\ &= \frac{1}{2}\left(i\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\right)\int \exp\left(\frac{(-b(i-\log\right.\right.\\ &= \frac{i\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right.}{4\sqrt{ie-c\log(f)}}\end{aligned}$$

Mathematica [A]
time = 1.95, size = 324, normalized size = 1.52

$$\frac{e^{-\frac{b^2c\log^3(f)}{2\left(e^2+c^2\log^2(f)\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\sqrt{\pi}\left(-e^{\frac{1}{4}b^2\left(\frac{1}{-ie+c\log(f)}+\frac{\log^2(f)}{ie+c\log(f)}\right)}f^{\frac{ib^2c\log(f)}{e^2+c^2\log^2(f)}}\mathrm{Erfi}\left(\frac{-i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{-ie+c\log(f)}}\right)(e-ic\log(f))\sqrt{-ie+c\log(f)}(\cos(a)-i\sin(a))+e^{\frac{1}{4}b^2\left(\frac{\log^2(f)}{-ie+c\log(f)}+\frac{1}{ie+c\log(f)}\right)}\mathrm{Erfi}\left(\frac{-i(b+2ex)-(b+2cx)\log(f)}{2\sqrt{ie+c\log(f)}}\right)(e+ic\log(f))\sqrt{ie+c\log(f)}(\cos(a)+i\sin(a))\right)}{4\left(e^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

```
[Out] (f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c*Log[f])^
(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c^2*Log[f]
^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e + c*Log[f]
])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a])) + E^((b^2
*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[((-I)*(b +
 2*e*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[f])*Sq
rt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(4*E^((b^2*c*Log[f]^3)/(2*(e^2 + c
^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

Maple [A]
time = 0.91, size = 217, normalized size = 1.02

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{-\ln(f)^{2} b^{2}+4 i \ln(f) a c-2 i \ln(f) b^{2}-4 a e+b^{2}}{4 i e+4 c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i e}\, x+\frac{b \ln(f)+i b}{2 \sqrt{-c \ln(f)-i e}}\right)}{4 \sqrt{-c \ln(f)-i e}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(\cdots}{\cdots}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(-ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2-4*a*e+
b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-(-c*ln(f)-I*e)^(1/2)*x+1/2*(b
*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+
4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b^2)/(c*ln(f)-I*e))/(I*e-c*ln(f))^(1/2)*e
rf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*b)/(I*e-c*ln(f))^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(161) = 322$.
time = 0.30, size = 1012, normalized size = 4.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*e^2)*((f^(1/4*b^2*c/(c^2*log(f)^2 + e
^2))*f^a*cos(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/
(c^2*log(f)^2 + e^2)) - I*f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*sin(-1/4*(
b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2
)))*erf(1/2*(2*(c*log(f) - I*e)*x + b*log(f) - I*b)*sqrt(-c*log(f) + I*e)/(
c*log(f) - I*e)) + (f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*cos(-1/4*(b^2*e
+ (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)) + I
*f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*sin(-1/4*(b^2*e + (2*b^2*c - 4*a*c^
2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(2*(c*log(f)
+ I*e)*x + b*log(f) + I*b)*sqrt(-c*log(f) - I*e)/(c*log(f) + I*e)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + e^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*e^2)*
((I*f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*cos(-1/4*(b^2*e + (2*b^2*c - 4*a
*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2
*log(f)^2 + e^2))*f^a*sin(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^
2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(2*(c*log(f) - I*e)*x + b*log(f
) - I*b)*sqrt(-c*log(f) + I*e)/(c*log(f) - I*e)) + (-I*f^(1/4*b^2*c/(c^2*lo
g(f)^2 + e^2))*f^a*cos(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 -
 4*a*e^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*si
```

```
n(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)
^2 + e^2)))*erf(1/2*(2*(c*log(f) + I*e)*x + b*log(f) + I*b)*sqrt(-c*log(f)
- I*e)/(c*log(f) + I*e)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + e^2)))/(c^2*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + e^2) + 1/2*b^2*e*log(f)/(c^2*log(f)^2
 + e^2))*log(f)^2 + e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + e^2) + 1/2*b^2*e*
log(f)/(c^2*log(f)^2 + e^2) + 2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(161) = 322$.
time = 3.31, size = 384, normalized size = 1.80

$$\frac{\sqrt{\pi}\left(i\,c\log\left(f\right)+e\right)\sqrt{-c\log\left(f\right)-i\,e}\,\operatorname{erf}\left(\frac{\left(\left(2\,c^{2}x+bc\right)\log(f)^{2}+2\,xe^{2}+be+\left(i\,bc-i\,be\right)\log(f)\right)\sqrt{-c\log\left(f\right)-i\,e}}{2\left(c^{2}\log(f)^{2}+e^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}+i\,b^{2}e-\left(-2i\,b^{2}c+4i\,ac^{2}+i\,b^{2}e\right)\log(f)^{2}-4i\,ae^{2}-\left(b^{2}c-2\,b^{2}e+4\,ae^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+e^{2}\right)}\right)}+\sqrt{\pi}\left(-i\,c\log\left(f\right)+e\right)\sqrt{-c\log\left(f\right)+i\,e}\,\operatorname{erf}\left(\frac{\left(\left(2\,c^{2}x+bc\right)\log(f)^{2}+2\,xe^{2}+be+\left(-i\,bc+i\,be\right)\log(f)\right)\sqrt{-c\log\left(f\right)+i\,e}}{2\left(c^{2}\log(f)^{2}+e^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}-i\,b^{2}e-\left(2i\,b^{2}c-4i\,ac^{2}-i\,b^{2}e\right)\log(f)^{2}+4i\,ae^{2}-\left(b^{2}c-2\,b^{2}e+4\,ae^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+e^{2}\right)}\right)}}{4\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + e)*sqrt(-c*log(f) - I*e)*erf(1/2*((2*c^2*x + b*
c)*log(f)^2 + 2*x*e^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*e)
/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - (-2*
I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - 4*I*a*e^2 - (b^2*c - 2*b^2*e + 4*
a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(-I*c*log(f) + e)*sqrt(-c*l
og(f) + I*e)*erf(1/2*((2*c^2*x + b*c)*log(f)^2 + 2*x*e^2 + b*e + (-I*b*c +
I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - I*b^2*e - (2*I*b^2*c - 4*I*a*c^2 - I*b^2*e)*log(f)^2
+ 4*I*a*e^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)))/(c
^2*log(f)^2 + e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x**2+b*x+a),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(a + b*x + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x^2 + b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sin\left(e\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sin(a + b*x + e*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*sin(a + b*x + e*x^2), x)
```

3.104 $\int e^x \cos(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{e^x\cos(a+bx)}{1+b^2}+\frac{be^x\sin(a+bx)}{1+b^2}$$

```
[Out] exp(x)*cos(b*x+a)/(b^2+1)+b*exp(x)*sin(b*x+a)/(b^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {4518}

$$\frac{be^x\sin(a+bx)}{b^2+1}+\frac{e^x\cos(a+bx)}{b^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[a + b*x],x]
[Out] (E^x*Cos[a + b*x])/(1 + b^2) + (b*E^x*Sin[a + b*x])/(1 + b^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cos(a+bx)\,dx=\frac{e^x\cos(a+bx)}{1+b^2}+\frac{be^x\sin(a+bx)}{1+b^2}$$

Mathematica [A]
time = 0.06, size = 26, normalized size = 0.72

$$\frac{e^x(\cos(a+bx)+b\sin(a+bx))}{1+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[a + b*x],x]
[Out] (E^x*(Cos[a + b*x] + b*Sin[a + b*x]))/(1 + b^2)
```

Maple [A]
time = 0.08, size = 35, normalized size = 0.97

method	result	size
default	$\frac{e^{x} \cos(bx+a)}{b^{2}+1}+\frac{b\,e^{x} \sin(bx+a)}{b^{2}+1}$	35
risch	$-\frac{i e^{x} e^{ibx} e^{ia}}{2(b-i)}+\frac{i e^{x} e^{-ibx} e^{-ia}}{2i+2b}$	46
norman	$\frac{\frac{e^{x}}{b^{2}+1}-\frac{e^{x}\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^{2}+1}+\frac{2b\,e^{x} \tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^{2}+1}}{1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)*cos(b*x+a)/(b^2+1)+b*exp(x)*sin(b*x+a)/(b^2+1)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.69

$$\frac{(b\sin(bx+a)+\cos(bx+a))e^{x}}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*sin(b*x + a) + cos(b*x + a))*e^x/(b^2 + 1)
```

Fricas [A]
time = 2.18, size = 28, normalized size = 0.78

$$\frac{be^{x}\sin(bx+a)+\cos(bx+a)\,e^{x}}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*e^x*sin(b*x + a) + cos(b*x + a)*e^x)/(b^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.24, size = 114, normalized size = 3.17

$$\begin{cases} -\frac{ixe^{x}\sin(a-ix)}{2}+\frac{xe^{x}\cos(a-ix)}{2}+\frac{e^{x}\cos(a-ix)}{2} & \text{for}\ b=-i\\ \frac{ixe^{x}\sin(a+ix)}{2}+\frac{xe^{x}\cos(a+ix)}{2}-\frac{ie^{x}\sin(a+ix)}{2} & \text{for}\ b=i\\ \frac{be^{x}\sin(a+bx)}{b^{2}+1}+\frac{e^{x}\cos(a+bx)}{b^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x)
```

```
[Out] Piecewise((-I*x*exp(x)*sin(a - I*x)/2 + x*exp(x)*cos(a - I*x)/2 + exp(x)*co
s(a - I*x)/2, Eq(b, -I)), (I*x*exp(x)*sin(a + I*x)/2 + x*exp(x)*cos(a + I*x
)/2 - I*exp(x)*sin(a + I*x)/2, Eq(b, I)), (b*exp(x)*sin(a + b*x)/(b**2 + 1)
 + exp(x)*cos(a + b*x)/(b**2 + 1), True))
```

Giac [A]
time = 0.42, size = 33, normalized size = 0.92

$$\left(\frac{b\sin\left(bx+a\right)}{b^2+1}+\frac{\cos\left(bx+a\right)}{b^2+1}\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] (b*sin(b*x + a)/(b^2 + 1) + cos(b*x + a)/(b^2 + 1))*e^x
```

Mupad [B]
time = 0.09, size = 25, normalized size = 0.69

$$\frac{\mathrm{e}^x\left(\cos\left(a+b\,x\right)+b\,\sin\left(a+b\,x\right)\right)}{b^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*exp(x),x)
```

```
[Out] (exp(x)*(cos(a + b*x) + b*sin(a + b*x)))/(b^2 + 1)
```

3.105 $\int e^x \cos\left(a+cx^2\right)\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt[4]{-1}\,(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}\,e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*a+1/c))*erf(1/2*(-1)^(1/4)*(1+2*I*c*x)/c^(1/2)
)*Pi^(1/2)/c^(1/2)+1/4*(-1)^(1/4)*erfi(1/2*(-1)^(1/4)*(1-2*I*c*x)/c^(1/2))*
Pi^(1/2)/exp(1/4*I*(4*a+1/c))/c^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4561, 2266, 2235, 2236}

$$\frac{\sqrt[4]{-1}\,\sqrt{\pi}\,e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\,\sqrt{\pi}\,e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}\,(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[a + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*
I)*c*x))/(2*Sqrt[c])])/Sqrt[c] + ((-1)^(1/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
 (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \cos\left(a+cx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-ia+x-icx^2}+\frac{1}{2}e^{ia+x+icx^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia+x-icx^2}\,dx+\frac{1}{2}\int e^{ia+x+icx^2}\,dx\\ &= \frac{1}{2}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\int e^{\frac{i(1-2icx)^2}{4c}}\,dx+\frac{1}{2}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\int e^{-\frac{i(1+2icx)^2}{4c}}\,dx\\ &= -\frac{\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}\,e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}\,(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 109, normalized size = 0.95

$$\frac{\sqrt[4]{-1}\,e^{-\frac{i}{4}/c}\sqrt{\pi}\left(-\mathrm{Erfi}\left(\frac{(-1)^{3/4}(i+2cx)}{2\sqrt{c}}\right)(\cos(a)-i\sin(a))+e^{\frac{i}{2}/c}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-i+2cx)}{2\sqrt{c}}\right)(-i\cos(a)+\sin(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[a + c*x^2],x]
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(Cos[a]
- I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sqrt[c])]*((-I
)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^((I/4)/c))
```

Maple [A]
time = 0.14, size = 86, normalized size = 0.75

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{ic}\,x-\frac{1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{-ic}\,x-\frac{1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2/(I*c
)^(1/2))+1/4*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*
x-1/2/(-I*c)^(1/2))
```

Maxima [A]
time = 0.29, size = 100, normalized size = 0.87

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left((i-1)\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i+1)\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx-1}{2\sqrt{i\,c}}\right)+\left((i+1)\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i-1)\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx+1}{2\sqrt{-i\,c}}\right)\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I - 1)*cos(1/4*(4*a*c + 1)/c) + (I + 1)*sin(1/4*(4
*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt(I*c)) + ((I + 1)*cos(1/4*(4*a*c +
1)/c) + (I - 1)*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*c)))/
sqrt(c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(73) = 146.
time = 2.35, size = 193, normalized size = 1.68

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}\,(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{4i\,ac+i}{4\,c}\right)}\,\mathrm{C}\left(-\frac{\sqrt{2}\,(2\,cx-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-i\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\,\mathrm{S}\left(\frac{\sqrt{2}\,(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-i\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{4i\,ac+i}{4\,c}\right)}\,\mathrm{S}\left(-\frac{\sqrt{2}\,(2\,cx-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_cos(1/2*sqrt(2)
*(2*c*x + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)/c)*
fresnel_cos(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) - I*sqrt(2)*pi*sqrt(c/pi
)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)/c
) - I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)/c)*fresnel_sin(-1/2*sqrt(2
)*(2*c*x - I)*sqrt(c/pi)/c))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\cos\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*cos(a + c*x**2), x)
```

Giac [A]
time = 0.42, size = 127, normalized size = 1.10

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\left(2\,x+\frac{i}{c}\right)\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4i\,ac+i}{4\,c}\right)}}{4\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\left(2\,x-\frac{i}{c}\right)\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4i\,ac-i}{4\,c}\right)}}{4\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + I/c)*(I*c/abs(c) + 1)*sqrt(ab
s(c)))*e^(-1/4*(4*I*a*c + I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) - 1/4*sqrt(
2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x - I/c)*(-I*c/abs(c) + 1)*sqrt(abs(c)))*e^
(-1/4*(-4*I*a*c - I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^x\,\cos\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(a + c*x^2),x)
```

```
[Out] int(exp(x)*cos(a + c*x^2), x)
```

3.106 $\int e^x \cos\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}\,e^{-ia+\frac{i(i+b)^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*a+(1+I*b)^2/c))*erf(1/2*(-1)^(1/4)*(1+I*b+2*I*
c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(1/4)*exp(-I*a+1/4*I*(I+b)^2/c)*erf
i(1/2*(-1)^(1/4)*(1-I*b-2*I*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4561, 2266, 2235, 2236}

$$\frac{\sqrt[4]{-1}\,\sqrt{\pi}\,e^{\frac{i(b+i)^2}{4c}-ia}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\,\sqrt{\pi}\,e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[a + b*x + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(
1 + I*b + (2*I)*c*x))/(2*Sqrt[c])])/Sqrt[c] + ((-1)^(1/4)*E^((-I)*a + ((I/4
)*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c]
)])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \cos\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-ia+(1-ib)x-icx^2}+\frac{1}{2}e^{ia+(1+ib)x+icx^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia+(1-ib)x-icx^2}\,dx+\frac{1}{2}\int e^{ia+(1+ib)x+icx^2}\,dx\\ &= \frac{1}{2}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\int e^{-\frac{i(1+ib+2icx)^2}{4c}}\,dx+\frac{1}{2}e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\int e^{\frac{i(1-ib-2icx)^2}{4c}}\,dx\\ &= -\frac{\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}\,e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\sqrt{\pi}\,\mathrm{e}}{4\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 135, normalized size = 0.94

$$\frac{\sqrt[4]{-1}\,e^{-\frac{i\left(1-2ib+b^2\right)}{4c}}\sqrt{\pi}\left(-e^{\frac{ib^2}{2c}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(i+b+2cx)}{2\sqrt{c}}\right)(\cos(a)-i\sin(a))+e^{\frac{i}{2}/c}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-i+b+2cx)}{2\sqrt{c}}\right)(-i\cos(a)+\sin(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[a + b*x + c*x^2],x]
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-(E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b + 2*c*x))
/(2*Sqrt[c])]*(Cos[a] - I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b +
 2*c*x))/(2*Sqrt[c])]*((-I)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^(((I/4)*(1 - (2
*I)*b + b^2))/c))
```

Maple [A]
time = 0.14, size = 117, normalized size = 0.81

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{i\left(4ac-b^2-2ib+1\right)}{4c}}\mathrm{erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{i\left(4ac-b^2+2ib+1\right)}{4c}}\mathrm{erf}\left(-\sqrt{-ic}\,x+\frac{ib+1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*I*(-b^2-2*I*b+4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)
*x-1/2*(1-I*b)/(I*c)^(1/2))-1/4*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)/
(-I*c)^(1/2)*erf(-(-I*c)^(1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.91

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i-1)\cos\left(-\frac{b^2-4\,ac-1}{4\,c}\right)-(i+1)\sin\left(-\frac{b^2-4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,cx+i\,b-1)\sqrt{i\,c}}{2\,c}\right)+\left((i+1)\cos\left(-\frac{b^2-4\,ac-1}{4\,c}\right)+(i-1)\sin\left(-\frac{b^2-4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,cx+i\,b+1)\sqrt{-i\,c}}{2\,c}\right)\right)e^{\left(-\frac{b}{2\,c}\right)}}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I + 1)*si
n(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2*I*c*x + I*b - 1)*sqrt(I*c)/c) + (
(I + 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (I - 1)*sin(-1/4*(b^2 - 4*a*c - 1)/
c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(91) = 182$.
time = 3.24, size = 229, normalized size = 1.59

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{i\,b^2-4i\,ac-2\,b-i}{4\,c}\right)}\operatorname{C}\left(\frac{\sqrt{2}\,(2\,cx+b+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{-i\,b^2+4i\,ac-2\,b+i}{4\,c}\right)}\operatorname{C}\left(-\frac{\sqrt{2}\,(2\,cx+b-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-i\,\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{i\,b^2-4i\,ac-2\,b-i}{4\,c}\right)}\operatorname{S}\left(\frac{\sqrt{2}\,(2\,cx+b+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-i\,\sqrt{2}\,\pi\sqrt{\frac{c}{\pi}}\,e^{\left(\frac{-i\,b^2+4i\,ac-2\,b+i}{4\,c}\right)}\operatorname{S}\left(-\frac{\sqrt{2}\,(2\,cx+b-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_co
s(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*
(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I)*sq
rt(c/pi)/c) - I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)
*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - I*sqrt(2)*pi*sqrt(
c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)*(2*c*
x + b - I)*sqrt(c/pi)/c))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\cos\left(a+bx+cx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*cos(a + b*x + c*x**2), x)
```

Giac [A]
time = 0.42, size = 147, normalized size = 1.02

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\left(2\,x+\frac{b-i}{c}\right)\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{i\,b^2-4i\,ac+2\,b-i}{4\,c}\right)}}{4\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\left(2\,x+\frac{b+i}{c}\right)\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-i\,b^2+4i\,ac+2\,b+i}{4\,c}\right)}}{4\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b - I)/c)*(-I*c/abs(c) + 1)*
sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((-I*c/abs(c) + 1)*sqr
t(abs(c))) - 1/4*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b + I)/c)*(I*c/a
bs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((I*c/abs(
c) + 1)*sqrt(abs(c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^x\,\cos\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(a + b*x + c*x^2),x)
```

```
[Out] int(exp(x)*cos(a + b*x + c*x^2), x)
```

3.107 $\int e^{x^2} \cos(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(I*a+1/4*b^2)*erfi(1
/2*I*b+x)*Pi^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4561, 2266, 2235}

$$\frac{1}{4}\sqrt{\pi}\,e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}\,e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cos(a+bx)\,dx &= \int\left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia-ibx+x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+x^2}\,dx\\ &= \frac{1}{2}e^{-ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\ &= \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 1.06

$$\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(-ib+2x)\right)+\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(ib+2x)\right)-\left(\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\mathrm{Erf}\left(\frac{b}{2}+ix\right)\right)\sin(a)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*x
)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

Maple [A]
time = 0.00, size = 54, normalized size = 0.70

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/
4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.68

$$-\frac{1}{4}\sqrt{\pi}\left((i\cos(a)+\sin(a))\,\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}+(i\cos(a)-\sin(a))\,\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a)
 - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A]
time = 2.25, size = 46, normalized size = 0.60

$$\frac{1}{4}\sqrt{\pi}\left(-i\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2+ia\right)}-i\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2-ia\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*e
^(1/4*b^2 - I*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*e^(x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\mathrm{e}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*exp(x^2),x)
```

```
[Out] int(cos(a + b*x)*exp(x^2), x)
```

3.108 $\int e^{x^2} \cos\left(a + cx^2\right) \, dx$

Optimal. Leaf size=83

$$\frac{e^{-ia}\sqrt{\pi}\,\text{Erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}}+\frac{e^{ia}\sqrt{\pi}\,\text{Erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}$$

```
[Out] 1/4*erfi(x*(1-I*c)^(1/2))*Pi^(1/2)/exp(I*a)/(1-I*c)^(1/2)+1/4*exp(I*a)*erfi
(x*(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4561, 2235}

$$\frac{\sqrt{\pi}\,e^{-ia}\text{Erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}}+\frac{\sqrt{\pi}\,e^{ia}\text{Erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(4*Sqrt[1 - I*c]*E^(I*a)) + (E^(I*a)*Sqrt[
Pi]*Erfi[Sqrt[1 + I*c]*x])/(4*Sqrt[1 + I*c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cos\left(a+cx^2\right)\,dx&=\int\left(\frac{1}{2}e^{-ia+(1-ic)x^2}+\frac{1}{2}e^{ia+(1+ic)x^2}\right)\,dx\\&=\frac{1}{2}\int e^{-ia+(1-ic)x^2}\,dx+\frac{1}{2}\int e^{ia+(1+ic)x^2}\,dx\\&=\frac{e^{-ia}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}}+\frac{e^{ia}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}\end{aligned}$$

Mathematica [**A**]
time = 0.22, size = 107, normalized size = 1.29

$$\frac{\sqrt[4]{-1}\,\sqrt{\pi}\left(-\left((-i+c)\sqrt{i+c}\,\mathrm{Erfi}\left((-1)^{3/4}\sqrt{i+c}\,x\right)(\cos(a)-i\sin(a))\right)+(1-ic)\sqrt{-i+c}\,\mathrm{Erfi}\left(\sqrt[4]{-1}\,\sqrt{-i+c}\,x\right)(\cos(a)+i\sin(a))\right)}{4\,(1+c^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*Erfi[(-1)^(3/4)*Sqrt[I + c]*x]
*(Cos[a] - I*Sin[a])) + (1 - I*c)*Sqrt[-I + c]*Erfi[(-1)^(1/4)*Sqrt[-I + c]
*x]*(Cos[a] + I*Sin[a])))/(4*(1 + c^2))
```

Maple [**A**]
time = 0.11, size = 60, normalized size = 0.72

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(\sqrt{ic-1}\,x\right)}{4\sqrt{ic-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(\sqrt{-ic-1}\,x\right)}{4\sqrt{-ic-1}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-I*a)/(-1+I*c)^(1/2)*erf((-1+I*c)^(1/2)*x)+1/4*Pi^(1/2)*ex
p(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(53) = 106.
time = 0.29, size = 133, normalized size = 1.60

$$-\frac{\sqrt{\pi}\,\sqrt{2\,c^2+2}\left((i\,\cos(a)+\sin(a))\,\mathrm{erf}\left(\sqrt{i\,c-1}\,x\right)+(-i\,\cos(a)+\sin(a))\,\mathrm{erf}\left(\sqrt{-i\,c-1}\,x\right)\right)\sqrt{\sqrt{c^2+1}+1}-\sqrt{\pi}\,\sqrt{2\,c^2+2}\left((\cos(a)-i\,\sin(a))\,\mathrm{erf}\left(\sqrt{i\,c-1}\,x\right)+(\cos(a)+i\,\sin(a))\,\mathrm{erf}\left(\sqrt{-i\,c-1}\,x\right)\right)\sqrt{\sqrt{c^2+1}-1}}{8\,(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((I*cos(a) + sin(a))*erf(sqrt(I*c - 1)*x) +
(-I*cos(a) + sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(
pi)*sqrt(2*c^2 + 2)*((cos(a) - I*sin(a))*erf(sqrt(I*c - 1)*x) + (cos(a) + I
*sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Fricas [**A**]
time = 2.63, size = 70, normalized size = 0.84

$$\frac{\sqrt{\pi}\,(i\,c-1)\sqrt{-i\,c-1}\ \mathrm{erf}\left(\sqrt{-i\,c-1}\,x\right)e^{(i\,a)}+\sqrt{\pi}\,\sqrt{i\,c-1}\,(-i\,c-1)\,\mathrm{erf}\left(\sqrt{i\,c-1}\,x\right)e^{(-i\,a)}}{4\,(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c - 1)*sqrt(-I*c - 1)*erf(sqrt(-I*c - 1)*x)*e^(I*a) + sqrt
(pi)*sqrt(I*c - 1)*(-I*c - 1)*erf(sqrt(I*c - 1)*x)*e^(-I*a))/(c^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cos\left(a+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cos(c*x^2 + a)*e^(x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{x^2}\,\cos\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(a + c*x^2),x)
```

```
[Out] int(exp(x^2)*cos(a + c*x^2), x)
```

3.109 $\int e^{x^2} \cos\left(a+bx+cx^2\right)\, dx$

Optimal. Leaf size=151

$$-\frac{e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}+\frac{e^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

```
[Out] -1/4*erfi(1/2*(I*b-2*(1-I*c)*x)/(1-I*c)^(1/2))*Pi^(1/2)/exp(I*(a-b^2/(4*I+4
*c)))/(1-I*c)^(1/2)+1/4*exp(I*a+1/4*b^2/(1+I*c))*erfi(1/2*(I*b+2*(1+I*c)*x)
/(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4561, 2266, 2235}

$$\frac{\sqrt{\pi}\,e^{ia+\frac{b^2}{4(1+ic)}}\mathrm{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}-\frac{\sqrt{\pi}\,e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\mathrm{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + b*x + c*x^2],x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 - I*c
]*E^(I*(a - b^2/(4*I + 4*c)))) + (E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[Pi]*Erfi
[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/(4*Sqrt[1 + I*c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-ia-ibx+(1-ic)x^2}+\frac{1}{2}e^{ia+ibx+(1+ic)x^2}\right)dx \\
&= \frac{1}{2}\int e^{-ia-ibx+(1-ic)x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+(1+ic)x^2}\,dx \\
&= \frac{1}{2}e^{ia+\frac{b^2}{4(1+ic)}}\int \exp\left(\frac{(ib+2(1+ic)x)^2}{4(1+ic)}\right)dx+\frac{1}{2}e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\int \exp\left(\frac{(-ib+2\ldots}{4(1\ldots}\right. \\
&= -\frac{e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\text{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}+\frac{e^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\text{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}
\end{aligned}
$$

Mathematica [A]
time = 0.62, size = 166, normalized size = 1.10

$$
\frac{\sqrt[4]{-1}\,e^{\frac{ib^2}{4i-4c}}\sqrt{\pi}\left(-\left((-i+c)\sqrt{i+c}\,e^{\frac{ib^2c}{2+2c^2}}\text{Erfi}\left(\frac{(-1)^{3/4}(b+2(i+c)x)}{2\sqrt{i+c}}\right)(\cos(a)-i\sin(a))\right)+\sqrt{-i+c}\,(i+c)\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(b+2(-i+c)x)}{2\sqrt{-i+c}}\right)(-i\cos(a)+\sin(a))\right)}{4\left(1+c^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*E^((I*
b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]*(C
os[a] - I*Sin[a])) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I + c)*
x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(4*(1 + c^2))
```

Maple [A]
time = 0.16, size = 127, normalized size = 0.84

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac+4ia-b^2}{4ic-4}}\,\mathrm{erf}\left(\sqrt{ic-1}\,x+\frac{ib}{2\sqrt{ic-1}}\right)}{4\sqrt{ic-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-4ia-b^2}{4(ic+1)}}\,\mathrm{erf}\left(-\sqrt{-ic-1}\,x+\frac{ib}{2\sqrt{-ic-1}}\right)}{4\sqrt{-ic-1}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*(4*a*c+4*I*a-b^2)/(-1+I*c))/(-1+I*c)^(1/2)*erf((-1+I*c
)^(1/2)*x+1/2*I*b/(-1+I*c)^(1/2))-1/4*Pi^(1/2)*exp(-1/4*(4*a*c-4*I*a-b^2)/(
1+I*c))/(-I*c-1)^(1/2)*erf(-(-I*c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(101) = 202.
time = 0.28, size = 474, normalized size = 3.14

$$
\frac{\sqrt{\pi}\sqrt{2c^2+2}\left(\left(-i\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}-e^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\text{erf}\left(-\frac{2(-ic+1)x-ib}{2\sqrt{ic-1}}\right)+\left(-i\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}+e^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\text{erf}\left(-\frac{2(-ic-1)x-ib}{2\sqrt{-ic-1}}\right)\right)\sqrt{\sqrt{c^2+1}+1}+\sqrt{\pi}\sqrt{2c^2+2}\left(\left(\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}-ie^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\text{erf}\left(-\frac{2(-ic+1)x-ib}{2\sqrt{ic-1}}\right)-\left(\cos\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)e^{\left(\frac{b^2}{4(c^2+1)}\right)}+ie^{\left(\frac{b^2}{4(c^2+1)}\right)}\sin\left(-\frac{b^2c-4ac^2-4a}{4(c^2+1)}\right)\right)\text{erf}\left(-\frac{2(-ic-1)x-ib}{2\sqrt{-ic-1}}\right)\right)\sqrt{\sqrt{c^2+1}-1}}{8\left(c^2+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((-I*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 +
1))*e^(1/4*b^2/(c^2 + 1)) - e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2
 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) + (-I*co
s(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + e^(1/4*b^
2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c
 - 1)*x - I*b)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) + 1) + sqrt(pi)*sqrt(2*c
^2 + 2)*((cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1))
 - I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf
(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) - (cos(-1/4*(b^2*c - 4*a*c^2 -
4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b
^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c - 1)*x - I*b)/sqrt(-I*c
 - 1)))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Fricas [A]
time = 2.62, size = 164, normalized size = 1.09

$$\frac{\sqrt{\pi}\,(i\,c+1)\sqrt{i\,c-1}\,\operatorname{erf}\left(-\frac{(bc+2\,(c^2+1)x-i\,b)\,\sqrt{i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\,(c^2+1)}\right)}+\sqrt{\pi}\,(i\,c-1)\sqrt{-i\,c-1}\,\operatorname{erf}\left(\frac{(bc+2\,(c^2+1)x+i\,b)\,\sqrt{-i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{-i\,b^2c+4i\,ac^2+b^2+4i\,a}{4\,(c^2+1)}\right)}}{4\,(c^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c + 1)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*
sqrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 +
1)) + sqrt(pi)*(I*c - 1)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)
*sqrt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^2
 + 1)))/(c^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(c*x^2 + b*x + a)*e^(x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{x^2}\,\cos\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(a + b*x + c*x^2),x)
```

```
[Out] int(exp(x^2)*cos(a + b*x + c*x^2), x)
```

3.110 $\int f^{a+bx} \cos(d+fx^2)\,dx$

Optimal. Leaf size=142

$$-\frac{1}{4}\sqrt[4]{-1}\, e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)} f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\, e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)} f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erfi}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4
)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(1/4)*f^(-1/2+a)*erfi(1/2*(-
1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A]
time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4561, 2325, 2266, 2235, 2236}

$$-\frac{1}{4}\sqrt[4]{-1}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]) - ((-1)^(1/4)*f^(-1/2
+ a)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^(
(I/4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cos\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-id-ifx^2}f^{a+bx}+\frac{1}{2}e^{id+ifx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-id-ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{2}\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx\\ &= \frac{1}{2}\left(e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(2i}{}}\\ &= -\frac{1}{4}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\,e^{-\frac{1}{4}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 133, normalized size = 0.94

$$\frac{1}{4}\sqrt[4]{-1}\,e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f]))
/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^
(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I
/4)*b^2*Log[f]^2)/f))
```

Maple [A]
time = 0.17, size = 114, normalized size = 0.80

method	result	size

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4df\right)}{4f}} \operatorname{erf}\left(-\sqrt{if}\, x+\frac{\ln(f) b}{2\sqrt{if}}\right)}{4\sqrt{if}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4df\right)}{4f}} \operatorname{erf}\left(-\sqrt{-if}\, x+\frac{\ln(f) b}{2\sqrt{-if}}\right)}{4\sqrt{-if}}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^
(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*
d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

Maxima [A]
time = 0.27, size = 147, normalized size = 1.04

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left((i-1)\,f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+4\,df}{4\,f}\right)+(i+1)\,f^{a}\sin\left(\frac{b^{2}\log(f)^{2}+4\,df}{4\,f}\right)\right)\operatorname{erf}\left(\frac{2i\,fx-b\log(f)}{2\,\sqrt{i\,f}}\right)+\left((i+1)\,f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+4\,df}{4\,f}\right)+(i-1)\,f^{a}\sin\left(\frac{b^{2}\log(f)^{2}+4\,df}{4\,f}\right)\right)\operatorname{erf}\left(\frac{2i\,fx+b\log(f)}{2\,\sqrt{-i\,f}}\right)\right)}{8\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I
+ 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x - b*log(f))/sq
rt(I*f)) + ((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I - 1)*f^a*sin
(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x + b*log(f))/sqrt(-I*f)))/s
qrt(f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(98) = 196$.
time = 2.07, size = 265, normalized size = 1.87

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i\,b^{2}\log(f)^{2}+4\,af\log(f)-4i\,df}{4\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i\,b^{2}\log(f)^{2}+4\,af\log(f)+4i\,df}{4\,f}\right)}\,\mathrm{C}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)-i\,\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i\,b^{2}\log(f)^{2}+4\,af\log(f)-4i\,df}{4\,f}\right)}\,\mathrm{S}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)-i\,\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i\,b^{2}\log(f)^{2}+4\,af\log(f)+4i\,df}{4\,f}\right)}\,\mathrm{S}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f
)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*p
i*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_co
s(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)
*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(
2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b
^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x -
I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cos \left(d+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(98) = 196.
time = 0.46, size = 300, normalized size = 2.11

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\,\sqrt{2}\left(4\,x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(-\frac{i\,f}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{8\,f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4\,f}-\frac{i\,\pi^2b^2}{8\,f}-\frac{\pi b^2\log(|f|)}{4\,f}+\frac{i\,b^2\log(|f|)^2}{4\,f}-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{1}{2}i\,\pi a+a\log(|f|)+i\,d\right)}}{4\left(-\frac{i\,f}{|f|}+1\right)\sqrt{|f|}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\,\sqrt{2}\left(4\,x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(\frac{i\,f}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{8\,f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4\,f}+\frac{i\,\pi^2b^2}{8\,f}+\frac{\pi b^2\log(|f|)}{4\,f}-\frac{i\,b^2\log(|f|)^2}{4\,f}-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{1}{2}i\,\pi a+a\log(|f|)-i\,d\right)}}{4\left(\frac{i\,f}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*l
og(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f +
 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f)
)/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(ab
s(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*sqrt(2)*sqrt(pi)*erf(-1
/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) +
 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(
f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^
2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((I*f/abs(f) +
1)*sqrt(abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cos\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cos(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*cos(d + f*x^2), x)
```

3.111 $\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=157

$$\left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+\dots$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)-(1/16+1/16*I)*exp(2*I*d+1/8*I*b^2*ln(f)^2/f)*f^(-1/2+
a)*erf((1/4+1/4*I)*(4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-(1/16+1/16*I)*f^(-1/
2+a)*erfi((1/4+1/4*I)*(4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/8*I*(16*d+b
^2*ln(f)^2/f))
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4561, 2225, 2325, 2266, 2235, 2236}

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\text{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] - ((1/16 + I/16)*f^(-1/2 +
 a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(
16*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\cos^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2id-2ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2id+2ifx^2+a\log(f)+bx\log(f)}\,d
\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2id+\frac{ib^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{i(4ifx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^
\\
&= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)-\left(\frac{1}{16}+\right.
\end{aligned}
$$

Mathematica [**A**]
time = 1.13, size = 158, normalized size = 1.01

$$
\frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)}+\frac{(1-i)e^{-\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)-i\sin(d))^2}{\sqrt{f}}+\frac{(1+i)e^{\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(4+4i)fx+(1-i)b\log(f)}{4\sqrt{f}}\right)(-i\cos(2d)+\sin(2d))}{\sqrt{f}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) + ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)/f
)*Sqrt[f]) + ((1 + I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I)*f
```

```
*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*((-I)*Cos[2*d] + Sin[2*d]))/Sqrt[f]))/1
6
```

Maple [A]
time = 0.38, size = 139, normalized size = 0.89

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{i f}}\right)}{16 \sqrt{i f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \mathrm{erf}\left(-\sqrt{-2 i f}\, x+\frac{}{2\sqrt{}}\right.}{8 \sqrt{-2 i f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*e
rf(-2^(1/2)*(I*f)^(1/2)*x+1/4*b*ln(f)*2^(1/2)/(I*f)^(1/2))-1/8*Pi^(1/2)*f^a
*exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2
*b*ln(f)/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [A]
time = 0.49, size = 186, normalized size = 1.18

$$-\frac{4^{\frac{1}{4}} \sqrt{2}\, \sqrt{\pi} \left(\left(\left(i-1\right) b f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+16\, d f}{8\, f}\right) \log\left(f\right)+\left(i+1\right) b f^{a} \log\left(f\right) \sin\left(\frac{b^{2} \log(f)^{2}+16\, d f}{8\, f}\right)\right) \mathrm{erf}\left(\frac{4 i\, f x-b \log(f)}{2\, \sqrt{2 i\, f}}\right)+\left(\left(i+1\right) b f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+16\, d f}{8\, f}\right) \log\left(f\right)+\left(i-1\right) b f^{a} \log\left(f\right) \sin\left(\frac{b^{2} \log(f)^{2}+16\, d f}{8\, f}\right)\right) \mathrm{erf}\left(\frac{4 i\, f x+b \log(f)}{2\, \sqrt{-2 i\, f}}\right)\right) f^{\frac{3}{2}}-16\, f^{b x} f^{a+2}}{32\, b f^{2} \log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*
d*f)/f)*log(f) + (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f)/f))*e
rf(1/2*(4*I*f*x - b*log(f))/sqrt(2*I*f)) + ((I + 1)*b*f^a*cos(1/8*(b^2*log(
f)^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*
d*f)/f))*erf(1/2*(4*I*f*x + b*log(f))/sqrt(-2*I*f)))*f^(3/2) - 16*f^(b*x)*f
^(a + 2))/(b*f^2*log(f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(103) = 206.
time = 2.30, size = 270, normalized size = 1.72

$$\frac{\pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\, b^{2} \log(f)^{2}+8\, a f \log(f)-16 i\, d f}{8\, f}\right)} \mathrm{C}\left(\frac{\left(4\, f x+i\, b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log\left(f\right)-\pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\, b^{2} \log(f)^{2}+8\, a f \log(f)+16 i\, d f}{8\, f}\right)} \mathrm{C}\left(-\frac{\left(4\, f x-i\, b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log\left(f\right)-i\, \pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\, b^{2} \log(f)^{2}+8\, a f \log(f)-16 i\, d f}{8\, f}\right)} \mathrm{S}\left(\frac{\left(4\, f x+i\, b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log\left(f\right)-i\, \pi b \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\, b^{2} \log(f)^{2}+8\, a f \log(f)+16 i\, d f}{8\, f}\right)} \mathrm{S}\left(-\frac{\left(4\, f x-i\, b \log(f)\right) \sqrt{\frac{f}{\pi}}}{2\, f}\right) \log\left(f\right)+4\, f f^{b x+a}}{8\, b f \log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*
fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - pi*b*sqrt(f/pi)
```

```
*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2*(4*f
*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*lo
g(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f))*s
qrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(
f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f
) + 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx} \cos^2\left(d+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(103) = 206.
time = 0.46, size = 521, normalized size = 3.32

$$\left(\frac{2\,b\cos\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(f)+\frac{1}{2}\,\pi a\right)\log\left(|f|\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\mathrm{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\mathrm{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(f)+\frac{1}{2}\,\pi a\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\mathrm{sgn}\left(f\right)-\pi b\right)^2}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\mathrm{sgn}(f)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\mathrm{sgn}(f)-\frac{1}{2}i\,\pi a\right)}}{2i\,\pi b\mathrm{sgn}\left(f\right)-2i\,\pi b+4\,b\log\left(|f|\right)}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\mathrm{sgn}(f)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{1}{2}i\,\pi a\right)}}{-2i\,\pi b\mathrm{sgn}\left(f\right)+2i\,\pi b+4\,b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\,\sqrt{f}\left(8\,x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(-\frac{i\,f}{|f|}+1\right)\right)e^{\left(\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{16\,f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{8\,f}-\frac{i\,\pi^2b^2}{16\,f}-\frac{\pi b^2\log(|f|)}{8\,f}+\frac{i\,b^2\log(|f|)^2}{8\,f}-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{1}{2}i\,\pi a+a\log(|f|)+2i\,d\right)}}{8\,\sqrt{f}\left(-\frac{i\,f}{|f|}+1\right)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\,\sqrt{f}\left(8\,x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(\frac{i\,f}{|f|}+1\right)\right)e^{\left(-\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{16\,f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{8\,f}+\frac{i\,\pi^2b^2}{16\,f}+\frac{\pi b^2\log(|f|)}{8\,f}-\frac{i\,b^2\log(|f|)^2}{8\,f}-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{1}{2}i\,\pi a+a\log(|f|)-2i\,d\right)}}{8\,\sqrt{f}\left(\frac{i\,f}{|f|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) - 1/8*sqr
t(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(
-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs(f))*sgn(
f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*log(abs(f))
^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*d)/(sqrt(f)*(-I
*f/abs(f) + 1)) - 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (pi*b*sgn(f) - pi*b
+ 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b^2*sgn(f)/f - 1/
8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*b^2*log(abs(f))/
f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(
f)) - 2*I*d)/(sqrt(f)*(I*f/abs(f) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\cos\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cos(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*cos(d + f*x^2)^2, x)
```

3.112 $\int f^{a+bx} \cos^3(d+fx^2) \, dx$

Optimal. Leaf size=298

$$-\frac{3}{16}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{Er}$$

```
[Out] (-1/96-1/96*I)*exp(3*I*d+1/12*I*b^2*ln(f)^2/f)*f^(-1/2+a)*erf((1/12+1/12*I)
*(6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-(1/96+1/96*I)*f^(-1/2+
a)*erfi((1/12+1/12*I)*(6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)/e
xp(1/12*I*(36*d+b^2*ln(f)^2/f))-3/16*(-1)^(1/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/
f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*
(-1)^(1/4)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/
2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A]
time = 0.23, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4561, 2325, 2266, 2235, 2236}

$$-\frac{3}{16}\sqrt[4]{-1}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}\,f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+6ifx)}{\sqrt{6}\,\sqrt{f}}\right)-\frac{3}{16}\sqrt[4]{-1}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}\,f^{a-\frac{1}{2}}e^{-\frac{1}{12}i\left(\frac{b^2\log^2(f)}{f}+36d\right)}\mathrm{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+6ifx)}{\sqrt{6}\,\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I/16)*E^((3
*I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*(
(6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(1/4)*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4
*d + (b^2*Log[f]^2)/f))) - ((1/16 + I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/
2 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2
*Log[f]^2)/f))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx &= \int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx}+\frac{3}{8}e^{id+ifx^2}f^{a+bx}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}e^{3id+3ifx^2}f^{a+bx}\right.\\
&= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+bx}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-id-ifx^2}f^{a+bx}\,dx+\frac{3}{8}\\
&= \frac{1}{8}\int e^{-3id-3ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{8}\int e^{3id+3ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-}\\
&= \frac{1}{8}\left(e^{3id+\frac{ib^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{i(6ifx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\left(3e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2i}{}}\\
&= -\frac{3}{16}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt[4]{-1}\,(2ifx+b\log(f))}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\right.
\end{aligned}$$

Mathematica [A]
time = 0.96, size = 267, normalized size = 0.90

$$\frac{1}{48}\sqrt[4]{-1}\,e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-9\text{Erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+9e^{\frac{ib^2\log^2(f)}{2f}}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))+\sqrt{3}\,e^{\frac{ib^2\log^2(f)}{6f}}\left(-\text{Erfi}\left(\frac{(-1)^{3/4}(6fx+ib\log(f))}{2\sqrt{3}\sqrt{f}}\right)(\cos(3d)-i\sin(3d))+e^{\frac{ib^2\log^2(f)}{6f}}\text{Erfi}\left(\frac{(6+6i)fx+(1-i)b\log(f)}{2\sqrt{6}\sqrt{f}}\right)(-i\cos(3d)+\sin(3d))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f])
)/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + 9*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d]) + Sqrt[3]
```

```
*E^(((I/6)*b^2*Log[f]^2)/f)*(-(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2*Sq
rt[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d])) + E^(((I/6)*b^2*Log[f]^2)/f)*Erfi[
((6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Si
n[3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

Maple [A]
time = 0.60, size = 235, normalized size = 0.79

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36 d f\right)}{12 f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\, \sqrt{i f}\, x+\frac{\ln(f) b \sqrt{3}}{6 \sqrt{i f}}\right)}{48 \sqrt{i f}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4 d f\right)}{4 f}} \mathrm{erf}\left(-\sqrt{i f}\, x+\frac{\ln(f) b}{2 \sqrt{i f}}\right.}{16 \sqrt{i f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*
erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*Pi^(1/2)*f
^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(
f)*b/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)
^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(
1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*ln(
f)*b/(-3*I*f)^(1/2))
```

Maxima [A]
time = 0.49, size = 302, normalized size = 1.01

$$-\frac{9^{\frac{1}{4}} \sqrt{2} \sqrt{\pi} \left(\left(\left(i-1\right) f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+36 d f}{12 f}\right)+\left(i+1\right) f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+36 d f}{12 f}\right)\right) \mathrm{erf}\left(\frac{6 i f x-b \log(f)}{2 \sqrt{3 i f}}\right)+\left(\left(i+1\right) f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+36 d f}{12 f}\right)+\left(i-1\right) f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+36 d f}{12 f}\right)\right) \mathrm{erf}\left(\frac{6 i f x+b \log(f)}{2 \sqrt{-3 i f}}\right)\right) f^{\frac{3}{2}}-9 \sqrt{2} \sqrt{\pi} \left(\left(-\left(i-1\right) f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)-\left(i+1\right) f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)\right) \mathrm{erf}\left(\frac{2 i f x-b \log(f)}{2 \sqrt{i f}}\right)+\left(-\left(i+1\right) f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)-\left(i-1\right) f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+4 d f}{4 f}\right)\right) \mathrm{erf}\left(\frac{2 i f x+b \log(f)}{2 \sqrt{-i f}}\right)\right) f^{\frac{3}{2}}}{96 f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d
*f)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x
- b*log(f))/sqrt(3*I*f)) + ((I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*f)/f)
 + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x + b*lo
g(f))/sqrt(-3*I*f)))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/4*(b
^2*log(f)^2 + 4*d*f)/f) - (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*er
f(1/2*(2*I*f*x - b*log(f))/sqrt(I*f)) + (-(I + 1)*f^a*cos(1/4*(b^2*log(f)^2
 + 4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*
f*x + b*log(f))/sqrt(-I*f)))*f^(3/2))/f^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(196) = 392$.
time = 2.90, size = 525, normalized size = 1.76

$$\frac{\sqrt{6}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+12 a f \log(f)-36 i d f}{12 f}\right)} \mathrm{C}\left(\frac{\sqrt{6} (6 f x+i b \log(f)) \sqrt{\frac{f}{\pi}}}{6 f}\right)-\sqrt{6}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+12 a f \log(f)+36 i d f}{12 f}\right)} \mathrm{C}\left(-\frac{\sqrt{6} (6 f x-i b \log(f)) \sqrt{\frac{f}{\pi}}}{6 f}\right)+9 \sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+4 a f \log(f)-4 i d f}{4 f}\right)} \mathrm{C}\left(\frac{\sqrt{2} (2 f x+i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)-9 \sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+4 a f \log(f)+4 i d f}{4 f}\right)} \mathrm{C}\left(-\frac{\sqrt{2} (2 f x-i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)-i \sqrt{6}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+12 a f \log(f)-36 i d f}{12 f}\right)} \mathrm{S}\left(\frac{\sqrt{6} (6 f x+i b \log(f)) \sqrt{\frac{f}{\pi}}}{6 f}\right)-i \sqrt{6}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+12 a f \log(f)+36 i d f}{12 f}\right)} \mathrm{S}\left(-\frac{\sqrt{6} (6 f x-i b \log(f)) \sqrt{\frac{f}{\pi}}}{6 f}\right)-9 i \sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2} \log(f)^{2}+4 a f \log(f)-4 i d f}{4 f}\right)} \mathrm{S}\left(\frac{\sqrt{2} (2 f x+i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)-9 i \sqrt{2}\, \pi \sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2} \log(f)^{2}+4 a f \log(f)+4 i d f}{4 f}\right)} \mathrm{S}\left(-\frac{\sqrt{2} (2 f x-i b \log(f)) \sqrt{\frac{f}{\pi}}}{2 f}\right)}{48 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I
*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(
6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fre
snel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqr
t(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_cos(1/
2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1
/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*sqrt(2)*(2
*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*
log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*
b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 +
 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f))
*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*
log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi
)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*
I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2)**3, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(196) = 392.
time = 0.49, size = 595, normalized size = 2.00

$$-\frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x-\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\operatorname{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a+a\log(|f|)+id\right)}}{16\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}-\frac{\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{24}\sqrt{6}\sqrt{f}\left(12x-\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\right)e^{\left(\frac{i\pi^2b^2\operatorname{sgn}(f)}{24f}+\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{12f}-\frac{i\pi^2b^2}{24f}-\frac{\pi b^2\log(|f|)}{12f}+\frac{ib^2\log(|f|)^2}{12f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a+a\log(|f|)+3id\right)}}{48\sqrt{f}\left(-\frac{if}{|f|}+1\right)}-\frac{\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{24}\sqrt{6}\sqrt{f}\left(12x+\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(\frac{if}{|f|}+1\right)\right)e^{\left(-\frac{i\pi^2b^2\operatorname{sgn}(f)}{24f}-\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{12f}+\frac{i\pi^2b^2}{24f}+\frac{\pi b^2\log(|f|)}{12f}-\frac{ib^2\log(|f|)^2}{12f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a+a\log(|f|)-3id\right)}}{48\sqrt{f}\left(\frac{if}{|f|}+1\right)}-\frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x+\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\operatorname{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^2\log(|f|)}{4f}-\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a+a\log(|f|)-id\right)}}{16\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
+ 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f
))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(a
bs(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqrt(pi)*erf(
-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(
-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(f))*sgn
```

```
(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*log(abs(
f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/(sqrt(f)*
(-I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(12*x
+ (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/24*I*
pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2/f +
1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f)
+ 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(I*f/abs(f) + 1)) - 3/16*sqr
t(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f
)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*
b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/
4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) -
I*d)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\cos\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cos(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cos(d + f*x^2)^3, x)
```

3.113 $\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=162

$$-\frac{1}{4}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\,e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^
(1/4)*(I*e+2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(1/4)*exp(-I*d+1/4*I
*(e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^
(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4561, 2325, 2266, 2235, 2236}

$$-\frac{1}{4}\sqrt[4]{-1}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}\,(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[P
i]*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]) - ((-1)^(1/4
)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1
)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+bx}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+bx}\right)\,dx\\ &= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int \exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)\,dx+\frac{1}{2}\int \exp\left(id+i\right.\\ &= \frac{1}{2}\left(e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\\ &= -\frac{1}{4}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)\end{aligned}$$

Mathematica [A]
time = 0.40, size = 163, normalized size = 1.01

$$\frac{1}{4}\sqrt[4]{-1}\,e^{-\frac{i\left(e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(-e^{\frac{ie^2}{2f}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^(((I/2)*e^2)/f)*Erfi[((-1
)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((
I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f]
)]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I/4)*(e^2 + b^2*Log[f]^2))/f))
```

Maple [A]
time = 0.19, size = 150, normalized size = 0.93

method	result

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}-2 i \ln(f) b e-e^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{i f}\, x+\frac{b \ln(f)-i e}{2 \sqrt{i f}}\right)}{4 \sqrt{i f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+2 i \ln(f) b e-e^{2}+4 d f\right)}{4 f}} \operatorname{erf}\left(-\sqrt{-i}\right.}{4 \sqrt{-i f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+4*d*f)/f)/(I*f)
^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*e
xp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^
(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

Maxima [A]
time = 0.28, size = 189, normalized size = 1.17

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left(-(i-1)\, f^{a} \cos\left(\frac{b^{2}\log(f)^{2}+4\,df-e^{2}}{4\,f}\right)-(i+1)\, f^{a} \sin\left(\frac{b^{2}\log(f)^{2}+4\,df-e^{2}}{4\,f}\right)\right) \operatorname{erf}\left(\frac{i\,(2i\,fx-b\log(f)+i\,e)\sqrt{i\,f}}{2\,f}\right)+\left((i+1)\, f^{a} \cos\left(\frac{b^{2}\log(f)^{2}+4\,df-e^{2}}{4\,f}\right)+(i-1)\, f^{a} \sin\left(\frac{b^{2}\log(f)^{2}+4\,df-e^{2}}{4\,f}\right)\right) \operatorname{erf}\left(\frac{i\,(2i\,fx+b\log(f)+i\,e)\sqrt{-i\,f}}{2\,f}\right)\right)}{8\sqrt{f}\, f^{\frac{be}{2f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f
) - (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))*erf(1/2*I*(2*I*f*x
 - b*log(f) + I*e)*sqrt(I*f)/f) + ((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*
f - e^2)/f) + (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))*erf(1/2*
I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))/(sqrt(f)*f^(1/2*b*e/f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(113) = 226.
time = 2.40, size = 321, normalized size = 1.98

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\,b^{2}\log(f)^{2}-4i\,df+2\,(2\,af-be)\log(f)+i\,e^{2}}{4\,f}\right)} \mathrm{C}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\,b^{2}\log(f)^{2}+4i\,df+2\,(2\,af-be)\log(f)-i\,e^{2}}{4\,f}\right)} \mathrm{C}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-i\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i\,b^{2}\log(f)^{2}-4i\,df+2\,(2\,af-be)\log(f)+i\,e^{2}}{4\,f}\right)} \mathrm{S}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-i\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i\,b^{2}\log(f)^{2}+4i\,df+2\,(2\,af-be)\log(f)-i\,e^{2}}{4\,f}\right)} \mathrm{S}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 - 4*I*d*f + 2*(2*a*f - b
*e)*log(f) + I*e^2)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqr
t(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*I*d*f + 2*(2*
a*f - b*e)*log(f) - I*e^2)/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f)
+ e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 - 4*I*
d*f + 2*(2*a*f - b*e)*log(f) + I*e^2)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I
*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)
^2 + 4*I*d*f + 2*(2*a*f - b*e)*log(f) - I*e^2)/f)*fresnel_sin(-1/2*sqrt(2)*
(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(109) = 218$.
time = 0.46, size = 378, normalized size = 2.33

$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x-\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)-2e}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\operatorname{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{i\pi be\operatorname{sgn}(f)}{4f}+\frac{1}{2}i\pi a-\frac{i\pi be}{4f}+a\log(|f|)-\frac{be\log(|f|)}{2f}+id-\frac{ie^2}{4f}\right)}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x+\frac{\pi b\operatorname{sgn}(f)-\pi b+2ib\log(|f|)+2e}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\operatorname{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\operatorname{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^2\log(|f|)}{4f}-\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{i\pi be\operatorname{sgn}(f)}{4f}+\frac{1}{2}i\pi a-\frac{i\pi be}{4f}+a\log(|f|)-\frac{be\log(|f|)}{2f}-id+\frac{ie^2}{4f}\right)}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*l
og(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(
f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(
abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sg
n(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/
f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*sqrt(2)*sqrt(
pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/
f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*
log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*
b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*
a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^
2/f)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cos\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cos(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*cos(d + e*x + f*x^2), x)
```

3.114 $\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=179

$$\left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-2id+\frac{i(2e+ib\log}{8f}}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)-(1/16+1/16*I)*exp(2*I*d+1/8*I*(2*I*e+b*ln(f))^2/f)*f^
(-1/2+a)*erf((1/4+1/4*I)*(2*I*e+4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-(1/16+1/
16*I)*exp(-2*I*d+1/8*I*(2*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/4+1/4*I)*(2*
I*e+4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4561, 2225, 2325, 2266, 2235, 2236}

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\mathrm{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*
Sqrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] - (1/1
6 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[P
i]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*
x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx}\right)dx \\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx \\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int \exp\left(-2id-2ifx^2+a\log(f)-x(2ie-b\log(f))\right)dx+ \\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\exp\left(-2id+a\log(f)-\frac{i(-2ie+b\log(f))^2}{8f}\right)\int e^{\frac{i(-2ie-4ifx-}{8f}} \\
&= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(}{\sqrt{f}}\right.
\end{aligned}$$

Mathematica [A]
time = 1.16, size = 245, normalized size = 1.37

$$\frac{e^{-\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{a-\frac{be+f}{2f}}\left(8e^{\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+\sqrt[4]{-1}\,be^{\frac{ib^2\log^2(f)}{4f}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2e+4fx-ib\log(f))}{\sqrt{f}}\right)\log(f)(-i\cos(2d)+\sin(2d))-\sqrt[4]{-1}\,be^{\frac{ie^2}{f}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2e+4fx+ib\log(f))}{\sqrt{f}}\right)\log(f)(i\cos(2d)+\sin(2d))\right)}{16b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 + b
*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erfi[(
(1/4 + I/4)*(2*e + 4*f*x - I*b*Log[f]))/Sqrt[f]]*Log[f]*((-I)*Cos[2*d] + Si
n[2*d]) - (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e + 4*f
```

```
*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^(((I/8)
*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

Maple [A]
time = 0.40, size = 175, normalized size = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{i\left(\ln(f)^{2} b^{2}-4 i \ln(f) b e-4 e^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{(b \ln(f)-2 i e)\sqrt{2}}{4\sqrt{i f}}\right)}{16\sqrt{i f}}-\frac{\sqrt{\pi}\, f^{a} e^{\frac{i\left(\ln(f)^{2} b^{2}+4 i \ln(f) b e-\right.}{8 f}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2-4*I*ln(f)*b*e-4*e^2+16*d*f)/f)*2
^(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)/(
I*f)^(1/2))-1/8*Pi^(1/2)*f^a*exp(1/8*I*(ln(f)^2*b^2+4*I*ln(f)*b*e-4*e^2+16*
d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I*f)^(
1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [A]
time = 0.49, size = 240, normalized size = 1.34

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\, b f^{a} \cos\left(\frac{b^{2}\log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right)\log(f)-(i+1)\, b f^{a}\log(f)\sin\left(\frac{b^{2}\log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right)\right)\operatorname{erf}\left(\frac{i\,(4 i\, f x-b\log(f)+2 i\, e)\sqrt{2 i\, f}}{4\, f}\right)+\left((i+1)\, b f^{a}\cos\left(\frac{b^{2}\log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right)\log(f)+(i-1)\, b f^{a}\log(f)\sin\left(\frac{b^{2}\log(f)^{2}+16\, d f-4\, e^{2}}{8\, f}\right)\right)\operatorname{erf}\left(\frac{i\,(4 i\, f x+b\log(f)+2 i\, e)\sqrt{-2 i\, f}}{4\, f}\right)\right) f^{\frac{3}{2}}-16\, f^{a+2} e^{\left(b x\log(f)+\frac{b e\log(f)}{2\, f}\right)}}{32\, b f^{2} f^{\frac{b e}{2\, f}}\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16
*d*f - 4*e^2)/f)*log(f) - (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d
*f - 4*e^2)/f))*erf(1/4*I*(4*I*f*x - b*log(f) + 2*I*e)*sqrt(2*I*f)/f) + ((I
 + 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*d*f - 4*e^2)/f)*log(f) + (I - 1)*b*f
^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f - 4*e^2)/f))*erf(1/4*I*(4*I*f*x +
b*log(f) + 2*I*e)*sqrt(-2*I*f)/f))*f^(3/2) - 16*f^(a + 2)*e^(b*x*log(f) + 1
/2*b*e*log(f)/f))/(b*f^2*f^(1/2*b*e/f)*log(f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(120) = 240$.
time = 3.04, size = 334, normalized size = 1.87

$$\frac{\pi b\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2}\log(f)^{2}-16 i\, d f+4\,(2\, a f-b e)\log(f)+4 i\, e^{2}}{8\, f}\right)} \operatorname{C}\left(\frac{(4\, f x+i\, b\log(f)+2\, e)\sqrt{\frac{f}{\pi}}}{2\, f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2}\log(f)^{2}+16 i\, d f+4\,(2\, a f-b e)\log(f)-4 i\, e^{2}}{8\, f}\right)} \operatorname{C}\left(-\frac{(4\, f x-i\, b\log(f)+2\, e)\sqrt{\frac{f}{\pi}}}{2\, f}\right)\log(f)-i\pi b\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{-i b^{2}\log(f)^{2}-16 i\, d f+4\,(2\, a f-b e)\log(f)+4 i\, e^{2}}{8\, f}\right)} \operatorname{S}\left(\frac{(4\, f x+i\, b\log(f)+2\, e)\sqrt{\frac{f}{\pi}}}{2\, f}\right)\log(f)-i\pi b\sqrt{\frac{f}{\pi}}\, e^{\left(\frac{i b^{2}\log(f)^{2}+16 i\, d f+4\,(2\, a f-b e)\log(f)-4 i\, e^{2}}{8\, f}\right)} \operatorname{S}\left(-\frac{(4\, f x-i\, b\log(f)+2\, e)\sqrt{\frac{f}{\pi}}}{2\, f}\right)\log(f)+4\, f f^{b x+a}}{8\, b f\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 - 16*I*d*f + 4*(2*a*f - b*e)*l
og(f) + 4*I*e^2)/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi)/f
)*log(f) - pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 16*I*d*f + 4*(2*a*f - b
*e)*log(f) - 4*I*e^2)/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*sqrt(f
/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 - 16*I*d*f + 4*(
2*a*f - b*e)*log(f) + 4*I*e^2)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f) + 2*e
)*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 16*I*d*
f + 4*(2*a*f - b*e)*log(f) - 4*I*e^2)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(
f) + 2*e)*sqrt(f/pi)/f)*log(f) + 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(116) = 232.
time = 0.48, size = 599, normalized size = 3.35

$$\left(\frac{2\,b\cos\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(f)+\frac{1}{2}\,\pi a\right)\log\left(|f|\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\mathrm{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\mathrm{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(f)+\frac{1}{2}\,\pi a\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\mathrm{sgn}\left(f\right)-\pi b\right)^2}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\mathrm{sgn}(f)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\mathrm{sgn}(f)-\frac{1}{2}i\,\pi a\right)}}{2i\,\pi b\mathrm{sgn}\left(f\right)-2i\,\pi b+4\,b\log\left(|f|\right)}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\mathrm{sgn}(f)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{1}{2}i\,\pi a\right)}}{-2i\,\pi b\mathrm{sgn}\left(f\right)+2i\,\pi b+4\,b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\,\sqrt{f}\left(8\,x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)-4\,e}{f}\right)\left(-\frac{i\,f}{|f|}+1\right)\right)e^{\left(\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{16\,f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{8\,f}-\frac{i\,\pi^2b^2}{16\,f}-\frac{\pi b^2\log(|f|)}{8\,f}+\frac{i\,b^2\log(|f|)^2}{8\,f}-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{i\,\pi be\mathrm{sgn}(f)}{4\,f}+\frac{1}{2}i\,\pi a-\frac{i\,\pi be}{4\,f}+a\log(|f|)-\frac{be\log(|f|)}{2\,f}+2i\,d-\frac{i\,e^2}{2\,f}\right)}}{8\,\sqrt{f}\left(-\frac{i\,f}{|f|}+1\right)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\,\sqrt{f}\left(8\,x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)+4\,e}{f}\right)\left(\frac{i\,f}{|f|}+1\right)\right)e^{\left(-\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{16\,f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{8\,f}+\frac{i\,\pi^2b^2}{16\,f}+\frac{\pi b^2\log(|f|)}{8\,f}-\frac{i\,b^2\log(|f|)^2}{8\,f}-\frac{1}{2}i\,\pi a\mathrm{sgn}(f)+\frac{i\,\pi be\mathrm{sgn}(f)}{4\,f}+\frac{1}{2}i\,\pi a-\frac{i\,\pi be}{4\,f}+a\log(|f|)-\frac{be\log(|f|)}{2\,f}-2i\,d+\frac{i\,e^2}{2\,f}\right)}}{8\,\sqrt{f}\left(\frac{i\,f}{|f|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) - 1/8*sqr
t(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) - 4*e
)/f)*(-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs(f)
)*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*log(a
bs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I
*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d - 1/2*I*e^2/f)/(s
qrt(f)*(-I*f/abs(f) + 1)) - 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (pi*b*sgn(
f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b
^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*
```

```
b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*p
i*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(
abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cos(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*cos(d + e*x + f*x^2)^2, x)
```

3.115 $\int f^{a+bx} \cos^3(d+ex+fx^2)\,dx$

Optimal. Leaf size=340

$$-\frac{3}{16}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}}$$

```
[Out] (-1/96-1/96*I)*exp(3*I*d+1/12*I*(3*I*e+b*ln(f))^2/f)*f^(-1/2+a)*erf((1/12+1
/12*I)*(3*I*e+6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-(1/96+1/96
*I)*exp(-3*I*d+1/12*I*(3*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/12+1/12*I)*(3
*I*e+6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-3/16*(-1)^(1/4)*exp
(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(I*e+2*I*f*x+
b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*(-1)^(1/4)*exp(-I*d+1/4*I*(e+I*b*ln(f))^2/f
)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4561, 2325, 2266, 2235, 2236}

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(3e+ib\log(f))^2}{12f}-3id}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]
*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I
/16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (
3*(-1)^(1/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 +
 I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/
6]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx = \int \left(\frac{1}{8}e^{-3i\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx\right.\right.\right.$$

$$= \frac{1}{8}\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx}\,dx+\frac{1}{8}\int \exp\left(6id+6iex+6ifx^2-3i\left(d+ex\right.\right.$$

$$= \frac{1}{8}\int \exp\left(-3id-3ifx^2+a\log(f)-x(3ie-b\log(f))\right)\,dx+\frac{1}{8}\int \exp\left(3\right.$$

$$= \frac{1}{8}\exp\left(-3id+a\log(f)-\frac{i(-3ie+b\log(f))^2}{12f}\right)\int e^{\frac{i(-3ie-6ifx+b\log(f))^2}{12f}}\,dx$$

$$= -\frac{3}{16}\sqrt[4]{-1}\,e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}\,(ie+2ifx+b\log(f))}{2\sqrt{f}}\right.$$

Mathematica [A]
time = 1.63, size = 322, normalized size = 0.95

$$\frac{1}{48}\sqrt[4]{-1}\,e^{-\frac{i\left(3e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(9e^{\frac{i\left(e^2+b^2\log^2(f)\right)}{2f}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))+e^{\frac{ie^2}{f}}\left(-9\mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))-\sqrt{3}\,e^{\frac{i\left(3e^2+b^2\log^2(f)\right)}{6f}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(3e+6fx+ib\log(f))}{2\sqrt{3}\sqrt{f}}\right)(\cos(3d)-i\sin(3d))\right)+\sqrt{3}\,e^{\frac{ib^2\log^2(f)}{3f}}\mathrm{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3e+6fx-ib\log(f))}{\sqrt{6}\sqrt{f}}\right)(-i\cos(3d)+\sin(3d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(9*E^(((I/2)*(e^2 + b^2*Log[f]
^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d
] + Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/
```

```
(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) - Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f]^2)
)/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Cos
[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2 + I/2)
*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d]))
)/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

Maple [A]
time = 0.81, size = 307, normalized size = 0.90

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{i \left(\ln \left(f \right)^{2} b^{2}-6 i \ln \left(f \right) b e -9 e^{2}+36 d f \right)}{12 f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\, \sqrt{i f}\, x +\frac{\left(-3 i e +b \ln \left(f \right)\right) \sqrt{3}}{6 \sqrt{i f}}\right)}{48 \sqrt{i f}}-\frac{3 \sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{i \left(\ln \left(f \right)^{2} b^{2}-2 i \ln \left(f \right) b}{4 f}}}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2-6*I*ln(f)*b*e-9*e^2+36*d*f)/f)*
3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(-3*I*e+b*ln(f))*3^(1/2)
/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+4
*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-3/16
*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f)^(1/
2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*ex
p(1/12*I*(ln(f)^2*b^2+6*I*ln(f)*b*e-9*e^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(-(-
3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))
```

Maxima [A]
time = 0.51, size = 374, normalized size = 1.10

$$-\frac{9^{\frac{1}{4}} \sqrt{2} \sqrt{\pi} \left(\left(-(i-1)\, f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+36 df-9 e^{2}}{12 f}\right)-(i+1)\, f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+36 df-9 e^{2}}{12 f}\right)\right) \mathrm{erf}\left(\frac{i (6 i f x-b \log(f)+3 i e) \sqrt{3 i f}}{6 f}\right)+\left((i+1)\, f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+36 df-9 e^{2}}{12 f}\right)+(i-1)\, f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+36 df-9 e^{2}}{12 f}\right)\right) \mathrm{erf}\left(\frac{i (6 i f x+b \log(f)+3 i e) \sqrt{-3 i f}}{6 f}\right)\right) f^{\frac{3}{2}}-9 \sqrt{2} \sqrt{\pi} \left(\left((i-1)\, f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+4 df-e^{2}}{4 f}\right)+(i+1)\, f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+4 df-e^{2}}{4 f}\right)\right) \mathrm{erf}\left(\frac{i (2 i f x-b \log(f)+i e) \sqrt{i f}}{2 f}\right)+\left(-(i+1)\, f^{a} \cos\left(\frac{b^{2} \log(f)^{2}+4 df-e^{2}}{4 f}\right)-(i-1)\, f^{a} \sin\left(\frac{b^{2} \log(f)^{2}+4 df-e^{2}}{4 f}\right)\right) \mathrm{erf}\left(\frac{i (2 i f x+b \log(f)+i e) \sqrt{-i f}}{2 f}\right)\right) f^{\frac{3}{2}}}{96 f^{2} f^{\frac{be}{2f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*
d*f - 9*e^2)/f) - (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f - 9*e^2)/f))*
erf(1/6*I*(6*I*f*x - b*log(f) + 3*I*e)*sqrt(3*I*f)/f) + ((I + 1)*f^a*cos(1/
12*(b^2*log(f)^2 + 36*d*f - 9*e^2)/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2
+ 36*d*f - 9*e^2)/f))*erf(1/6*I*(6*I*f*x + b*log(f) + 3*I*e)*sqrt(-3*I*f)/f
))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f
 - e^2)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f))*erf(1/2*I
*(2*I*f*x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I + 1)*f^a*cos(1/4*(b^2*log(f
)^2 + 4*d*f - e^2)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f - e^2)/f)
)*erf(1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))*f^(3/2))/(f^2*f^(1/2*
b*e/f))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(228) = 456$.
time = 1.99, size = 645, normalized size = 1.90

$$\frac{\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i\,b^2\log(f)^2-36\,i\,df+6\,(2\,af-be)\log(f)+9\,i\,e^2}{12\,f}\right)}\operatorname{C}\left(\frac{\sqrt{6}\,(6\,fx+i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)-\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i\,b^2\log(f)^2+36\,i\,df+6\,(2\,af-be)\log(f)-9\,i\,e^2}{12\,f}\right)}\operatorname{C}\left(-\frac{\sqrt{6}\,(6\,fx-i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)+9\,\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i\,b^2\log(f)^2-4\,i\,df+2\,(2\,af-be)\log(f)+i\,e^2}{4\,f}\right)}\operatorname{C}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-9\,\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i\,b^2\log(f)^2+4\,i\,df+2\,(2\,af-be)\log(f)-i\,e^2}{4\,f}\right)}\operatorname{C}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-i\,\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i\,b^2\log(f)^2-36\,i\,df+6\,(2\,af-be)\log(f)+9\,i\,e^2}{12\,f}\right)}\operatorname{S}\left(\frac{\sqrt{6}\,(6\,fx+i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)-i\,\sqrt{6}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i\,b^2\log(f)^2+36\,i\,df+6\,(2\,af-be)\log(f)-9\,i\,e^2}{12\,f}\right)}\operatorname{S}\left(-\frac{\sqrt{6}\,(6\,fx-i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)-9\,i\,\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{-i\,b^2\log(f)^2-4\,i\,df+2\,(2\,af-be)\log(f)+i\,e^2}{4\,f}\right)}\operatorname{S}\left(\frac{\sqrt{2}\,(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-9\,i\,\sqrt{2}\,\pi\sqrt{\frac{f}{\pi}}\,e^{\left(\frac{i\,b^2\log(f)^2+4\,i\,df+2\,(2\,af-be)\log(f)-i\,e^2}{4\,f}\right)}\operatorname{S}\left(-\frac{\sqrt{2}\,(2\,fx-i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 - 36*I*d*f + 6*(2*a*f
- b*e)*log(f) + 9*I*e^2)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3
*e)*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 36*I*d*
f + 6*(2*a*f - b*e)*log(f) - 9*I*e^2)/f)*fresnel_cos(-1/6*sqrt(6)*(6*f*x -
I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*lo
g(f)^2 - 4*I*d*f + 2*(2*a*f - b*e)*log(f) + I*e^2)/f)*fresnel_cos(1/2*sqrt(
2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*
(I*b^2*log(f)^2 + 4*I*d*f + 2*(2*a*f - b*e)*log(f) - I*e^2)/f)*fresnel_cos(
-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/p
i)*e^(1/12*(-I*b^2*log(f)^2 - 36*I*d*f + 6*(2*a*f - b*e)*log(f) + 9*I*e^2)/
f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)/f) - I*sqr
t(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 36*I*d*f + 6*(2*a*f - b*e)*log
(f) - 9*I*e^2)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f) + 3*e)*sqrt(
f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 - 4*I*d*f + 2*
(2*a*f - b*e)*log(f) + I*e^2)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f
) + e)*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4
*I*d*f + 2*(2*a*f - b*e)*log(f) - I*e^2)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x
 - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**3, x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(220) = 440$.
time = 0.56, size = 751, normalized size = 2.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn
(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log
(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*s
gn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))
/f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqr
t(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs
(f)) - 6*e)/f)*(-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2
*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/1
2*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I
*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 3*I*d - 3/
4*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(-1/24*sq
rt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 6*e)/f)*(I*
f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f
)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f)
)^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b
*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/4*I*e^2/f)/(sqrt(f
)*(I*f/abs(f) + 1)) - 3/16*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*s
gn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e
^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b
^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sg
n(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f))
- 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cos(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cos(d + e*x + f*x^2)^3, x)
```

3.116 $\int f^{a+cx^2} \cos(d+ex)\,dx$

Optimal. Leaf size=147

$$-\frac{e^{-id+\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\, \mathrm{Erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}} + \frac{e^{id+\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\, \mathrm{Erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(
1/2*(I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4561, 2325, 2266, 2235}

$$\frac{\sqrt{\pi}\, f^a e^{\frac{e^2}{4c\log(f)}+id} \mathrm{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}} - \frac{\sqrt{\pi}\, f^a e^{\frac{e^2}{4c\log(f)}-id} \mathrm{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x],x]
```

```
[Out] -1/4*(E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(I*d + e^2/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*S
qrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cos(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-id-iex}f^{a+cx^2}+\frac{1}{2}e^{id+iex}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-id-iex}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+iex}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{2}\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{2}\left(e^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 116, normalized size = 0.79

$$\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+\mathrm{Erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x],x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])]*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.16, size = 121, normalized size = 0.82

method	result
risch	$\frac{\sqrt{\pi}\, f^a e^{-\frac{4id\ln(f)c-e^2}{4\ln(f)c}}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{ie}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^a e^{\frac{4id\ln(f)c+e^2}{4\ln(f)c}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\ldots\right)}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf
((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4*I
*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(
-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 204, normalized size = 1.39

$$-\frac{\sqrt{\pi}\left(f^{a}(\cos(d)-i\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}(\cos(d)+i\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-f^{a}(\cos(d)+i\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)+ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-f^{a}(\cos(d)-i\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)-ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}\right)\sqrt{-c\log(f)}}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(cos(d) - I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) + 1
/2*I*conjugate(1/sqrt(-c*log(f)))*e)*e^(1/4*e^2/(c*log(f))) + f^a*(cos(d) +
 I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*conjugate(1/sqrt(-c*log
(f)))*e)*e^(1/4*e^2/(c*log(f))) - f^a*(cos(d) + I*sin(d))*erf(1/2*(2*c*x*lo
g(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - f^a*(cos(d) - I*sin(d
))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))))*sq
rt(-c*log(f))/(c*log(f))
```

Fricas [A]
time = 1.85, size = 142, normalized size = 0.97

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}+4icd\log(f)+e^{2}}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}-4icd\log(f)+e^{2}}{4c\log(f)}\right)}}{4c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f))
/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) + s
qrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*log
(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*log(f)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^{2}}\cos(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\cos\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + e*x),x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x), x)
```

3.117 $\int f^{a+cx^2} \cos^2(d+ex)\,dx$

Optimal. Leaf size=171

$$\frac{f^a\sqrt{\pi}\,\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*I*d+e^2/c/ln(f))*f^a*erfi((-I*e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*I*d+e^2/c/ln(f))*f^a*erfi((I*e+c*x*l
n(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(1
/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4561, 2235, 2325, 2266}

$$-\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{c\log(f)}-2id}\text{Erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{c\log(f)}+2id}\text{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos^2(d+ex)\,dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2iex}f^{a+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2id-2iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2id+2iex-}\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2id}\right.\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2id+\frac{}{c\,l}}}{}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 131, normalized size = 0.77

$$\frac{f^a\sqrt{\pi}\left(2\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)+e^{\frac{e^2}{c\log(f)}}\left(\text{Erfi}\left(\frac{-ie+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+\text{Erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(2d)+i\sin(2d))\right)\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + E^(e^2/(c*Log[f]))*(Erfi[((
-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erfi[
(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.22, size = 145, normalized size = 0.85

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{}{\sqrt{-}}\right.}{8 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c
*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c+e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))
+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 236, normalized size = 1.38

$$\frac{\sqrt{\pi}\left(f^{a}(\cos(2d)-i\sin(2d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}(\cos(2d)+i\sin(2d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}-f^{a}(\cos(2d)+i\sin(2d))\operatorname{erf}\left(\frac{cx\log(f)+ie}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}-f^{a}(\cos(2d)-i\sin(2d))\operatorname{erf}\left(\frac{cx\log(f)-ie}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+2f^{a}\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}\right)+2f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)\right)}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*(f^a*(cos(2*d) - I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f)))
 + I*conjugate(1/sqrt(-c*log(f)))*e)*e^(e^2/(c*log(f))) + f^a*(cos(2*d) + I
*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))) - I*conjugate(1/sqrt(-c*log(f))
)*e)*e^(e^2/(c*log(f))) - f^a*(cos(2*d) + I*sin(2*d))*erf((c*x*log(f) + I*e
)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) - I*sin(2*d))*erf((c*
x*log(f) - I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) + 2*f^a*erf(x*conjugate
(sqrt(-c*log(f)))) + 2*f^a*erf(sqrt(-c*log(f))*x))/sqrt(-c*log(f))
```

Fricas [A]
time = 2.13, size = 159, normalized size = 0.93

$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(cx\log(f)+ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^{2}+2icd\log(f)+e^{2}}{c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(cx\log(f)-ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^{2}-2icd\log(f)+e^{2}}{c\log(f)}\right)}}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) + sqrt(pi)*sqrt
(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(
f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf((c*
x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*log(
f) + e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos^2\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cos\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x)^2, x)
```

3.118 $\int f^{a+cx^2} \cos^3(d+ex)\,dx$

Optimal. Leaf size=293

$$-\frac{3e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}}{16\sqrt{c}}$$

```
[Out] 3/16*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*I*d+9/4*e^2/c/ln(f))*f^a*
erfi(1/2*(-3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(
1/2)+3/16*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(I*e+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*I*d+9/4*e^2/c/ln(f))*f^
a*erfi(1/2*(3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^
(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4561, 2325, 2266, 2235}

$$-\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}+3id}\text{Erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (-3*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^((-3*I)*d + (9*e^2)
/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(I*d + e^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt
[Log[f]]) + (E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos^3(d+ex)\,dx &= \int\left(\frac{3}{8}e^{-id-iex}f^{a+cx^2}+\frac{3}{8}e^{id+iex}f^{a+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}e^{3id+3iex}f^{a+cx^2}\right)\,dx\\ &= \frac{1}{8}\int e^{-3id-3iex}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3id+3iex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-id-iex}f^{a+cx^2}\,dx+\frac{3}{8}\int\\ &= \frac{1}{8}\int e^{-3id-3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{8}\int e^{3id+3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{-id-}\\ &= \frac{1}{8}\left(3e^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(3e^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\ &= -\frac{3e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-2cx\log}{2\sqrt{c}\sqrt{\log}}\right.}{16\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 218, normalized size = 0.74

$$\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(3\mathrm{Erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+3\mathrm{Erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+e^{\frac{2e^2}{c\log(f)}}\left(\mathrm{Erfi}\left(\frac{-3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(3d)-i\sin(3d))+\mathrm{Erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(3d)+i\sin(3d))\right)\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(3*Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + 3*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + E^((2*e^2)/(c*Log[f]))*(Erfi[((-3*
I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3*d]) + Er
fi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] + I*Sin[3*d
]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.56, size = 242, normalized size = 0.83

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c-3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\right.}{16 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*
erf((-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4
*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I
*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)
/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c
*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 406, normalized size = 1.39

$$-\frac{\sqrt{\pi}\left(f^{a}(\cos(3d)-i\sin(3d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{3}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}+f^{a}(\cos(3d)+i\sin(3d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{3}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}-f^{a}(\cos(3d)+i\sin(3d))\operatorname{erf}\left(\frac{2cx\log(f)+3ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}-f^{a}(\cos(3d)-i\sin(3d))\operatorname{erf}\left(\frac{2cx\log(f)-3ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{9e^{2}}{4c\log(f)}\right)}+3f^{a}(\cos(d)-i\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+3f^{a}(\cos(d)+i\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}i\overline{\frac{1}{\sqrt{-c\log(f)}}}e\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-3f^{a}(\cos(d)+i\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)+ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}-3f^{a}(\cos(d)-i\sin(d))\operatorname{erf}\left(\frac{2cx\log(f)-ie}{2\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}\right)\sqrt{-c\log(f)}}{32c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*sqrt(pi)*(f^a*(cos(3*d) - I*sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))
) + 3/2*I*conjugate(1/sqrt(-c*log(f)))*e)*e^(9/4*e^2/(c*log(f))) + f^a*(cos
(3*d) + I*sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))) - 3/2*I*conjugate(1/sq
rt(-c*log(f)))*e)*e^(9/4*e^2/(c*log(f))) - f^a*(cos(3*d) + I*sin(3*d))*erf(
1/2*(2*c*x*log(f) + 3*I*e)/sqrt(-c*log(f)))*e^(9/4*e^2/(c*log(f))) - f^a*(c
os(3*d) - I*sin(3*d))*erf(1/2*(2*c*x*log(f) - 3*I*e)/sqrt(-c*log(f)))*e^(9/
4*e^2/(c*log(f))) + 3*f^a*(cos(d) - I*sin(d))*erf(x*conjugate(sqrt(-c*log(f
))) + 1/2*I*conjugate(1/sqrt(-c*log(f)))*e)*e^(1/4*e^2/(c*log(f))) + 3*f^a*
(cos(d) + I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*conjugate(1/sq
rt(-c*log(f)))*e)*e^(1/4*e^2/(c*log(f))) - 3*f^a*(cos(d) + I*sin(d))*erf(1/
2*(2*c*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(cos
(d) - I*sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c
*log(f))))*sqrt(-c*log(f))/(c*log(f))
```

Fricas [A]
time = 3.31, size = 280, normalized size = 0.96

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+3ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}+12icd\log(f)+9e^{2}}{4c\log(f)}\right)}+3\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}+4icd\log(f)+e^{2}}{4c\log(f)}\right)}+3\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}-4icd\log(f)+e^{2}}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-3ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}-12icd\log(f)+9e^{2}}{4c\log(f)}\right)}}{16c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*log(
f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(f)
)) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) +
3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(
pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f
)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/(c*log(f
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \cos^3{(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,{\cos\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + e*x)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x)^3, x)
```

3.119 $\int f^{a+cx^2} \cos(d + fx^2)\, dx$

Optimal. Leaf size=103

$$\frac{e^{-id} f^a \sqrt{\pi}\, \mathrm{Erf}\left(x\sqrt{if - c\log(f)}\right)}{4\sqrt{if - c\log(f)}} + \frac{e^{id} f^a \sqrt{\pi}\, \mathrm{Erfi}\left(x\sqrt{if + c\log(f)}\right)}{4\sqrt{if + c\log(f)}}$$

```
[Out] 1/4*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)+1/
4*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4561, 2325, 2236, 2235}

$$\frac{\sqrt{\pi}\, e^{-id} f^a \mathrm{Erf}\left(x\sqrt{-c\log(f) + if}\right)}{4\sqrt{-c\log(f) + if}} + \frac{\sqrt{\pi}\, e^{id} f^a \mathrm{Erfi}\left(x\sqrt{c\log(f) + if}\right)}{4\sqrt{c\log(f) + if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(4*E^(I*d)*Sqrt[I*f - c*Log[f]])
 + (E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/(4*Sqrt[I*f + c*Log[
f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-id-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+cx^2}\right)\,dx\\ &= \frac{1}{2}\int e^{-id-ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx+\frac{1}{2}\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\ &= \frac{e^{-id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 170, normalized size = 1.65

$$-\frac{(-1)^{3/4} f^a \sqrt{\pi} \left(\text{Erfi}\left(\sqrt[4]{-1}\, x \sqrt{f-i c \log(f)}\right) \sqrt{f-i c \log(f)}\, (f+i c \log(f)) (\cos(d)+i \sin(d))+\sqrt{f+i c \log(f)} \left(f \cos(d) \text{Erf}\left(\frac{(1+i) x \sqrt{f+i c \log(f)}}{\sqrt{2}}\right)-\text{Erfi}\left((-1)^{3/4} x \sqrt{f+i c \log(f)}\right) (c \cos(d) \log(f)+(f-i c \log(f)) \sin(d))\right)\right)}{4 \left(f^2+c^2 \log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt
[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[f]
]*(f*Cos[d]*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]] - Erfi[(-1)^(3/4)
*x*Sqrt[f + I*c*Log[f]]]*(c*Cos[d]*Log[f] + (f - I*c*Log[f])*Sin[d]))))/(f^
2 + c^2*Log[f]^2)
```

Maple [A]
time = 0.16, size = 82, normalized size = 0.80

method	result	size
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-i d}\, \mathrm{erf}\left(x \sqrt{i f-c \ln\left(f\right)}\right)}{4 \sqrt{i f-c \ln\left(f\right)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{i d}\, \mathrm{erf}\left(\sqrt{-c \ln\left(f\right)-i f}\, x\right)}{4 \sqrt{-c \ln\left(f\right)-i f}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2))+1
/4*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)*x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(73) = 146$.
time = 0.28, size = 205, normalized size = 1.99

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(f^a(i\,\cos\left(d\right)+\sin\left(d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+i\,f}\,x\right)+f^a(-i\,\cos\left(d\right)+\sin\left(d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}\,x\right)\right)\sqrt{c\log\left(f\right)+\sqrt{c^2\log\left(f\right)^2+f^2}}-\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(f^a(\cos\left(d\right)-i\,\sin\left(d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+i\,f}\,x\right)+f^a(\cos\left(d\right)+i\,\sin\left(d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}\,x\right)\right)\sqrt{-c\log\left(f\right)+\sqrt{c^2\log\left(f\right)^2+f^2}}}{8\left(c^2\log\left(f\right)^2+f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(f^a*(I*cos(d) + sin(d))*erf(sq
rt(-c*log(f) + I*f)*x) + f^a*(-I*cos(d) + sin(d))*erf(sqrt(-c*log(f) - I*f)
*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)
^2 + 2*f^2)*(f^a*(cos(d) - I*sin(d))*erf(sqrt(-c*log(f) + I*f)*x) + f^a*(co
s(d) + I*sin(d))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*lo
g(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Fricas [**A**]
time = 2.24, size = 109, normalized size = 1.06

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-i\,f\right)\sqrt{-c\log\left(f\right)-i\,f}\;\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}\;x\right)e^{\left(a\log\left(f\right)+i\,d\right)}+\sqrt{\pi}\left(c\log\left(f\right)+i\,f\right)\sqrt{-c\log\left(f\right)+i\,f}\;\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+i\,f}\;x\right)e^{\left(a\log\left(f\right)-i\,d\right)}}{4\left(c^2\log\left(f\right)^2+f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) -
I*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*log(f) + I*f
)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\cos\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + f*x^2), x)
```

3.120 $\int f^{a+cx^2} \cos^2(d+fx^2)\, dx$

Optimal. Leaf size=140

$$\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*er
f(x*(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*I*d)/(2*I*f-c*ln(f))^(1/2)+1/8*ex
p(2*I*d)*f^a*erfi(x*(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4561, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,e^{-2id}f^a\mathrm{Erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}\,e^{2id}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) + (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2id+a\log(f)-x^2(2if-c\log(f))\right.\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id}f^a\ldots}{}
\end{aligned}
$$

Mathematica [A]
time = 0.89, size = 189, normalized size = 1.35

$$\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(-\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{2f+ic\log(f)}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}\,(\cos(2d)-i\sin(2d))+\mathrm{Erfi}\left(\sqrt[4]{-1}\,x\sqrt{2f-ic\log(f)}\right)\sqrt{2f-ic\log(f)}\,(-2if+c\log(f))(\cos(2d)+i\sin(2d))\right)}{4f^2+c^2\log^2(f)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(-(Erfi[(-1)^(3/4)*x*Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*S
qrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d])) + Erfi[(-1)^(1/4)*x*Sqrt[2*f
 - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*((-2*I)*f + c*Log[f])*(Cos[2*d] + I*
Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A]
time = 0.26, size = 107, normalized size = 0.76

method	result
risch	$\frac{\sqrt{\pi}\,f^a e^{-2id}\,\mathrm{erf}\left(x\sqrt{2if-c\ln(f)}\right)}{8\sqrt{2if-c\ln(f)}}+\frac{\sqrt{\pi}\,f^a e^{2id}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-2if}\,x\right)}{8\sqrt{-c\ln(f)-2if}}+\frac{f^a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\ldots\right.}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1
/2))+1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I*f
)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 315, normalized size = 2.25

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+8\,f^2}\left(f^a(i\,\cos(2\,d)+\sin(2\,d))\,\mathrm{erf}\left(\sqrt{-c\log(f)+2i\,f}\,x\right)+f^a(-i\,\cos(2\,d)+\sin(2\,d))\,\mathrm{erf}\left(\sqrt{-c\log(f)-2i\,f}\,x\right)\right)\sqrt{c\log(f)+\sqrt{c^2\log(f)^2+4\,f^2}}\,\sqrt{-c\log(f)}-\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+8\,f^2}\left(f^a(\cos(2\,d)-i\,\sin(2\,d))\,\mathrm{erf}\left(\sqrt{-c\log(f)+2i\,f}\,x\right)+f^a(\cos(2\,d)+i\,\sin(2\,d))\,\mathrm{erf}\left(\sqrt{-c\log(f)-2i\,f}\,x\right)\right)\sqrt{-c\log(f)+\sqrt{c^2\log(f)^2+4\,f^2}}\,\sqrt{-c\log(f)}-2\sqrt{\pi}\left(\left(c^2f^a\log(f)^2+4\,f^{a+2}\right)\mathrm{erf}\left(x\sqrt{-c\log(f)}\right)+\left(c^2f^a\log(f)^2+4\,f^{a+2}\right)\mathrm{erf}\left(\sqrt{-c\log(f)}\,x\right)\right)}{16\left(c^2\log(f)^2+4\,f^2\right)\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(I*cos(2*d) + sin(2*d))*e
rf(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(-I*cos(2*d) + sin(2*d))*erf(sqrt(-c*lo
g(f) - 2*I*f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f
)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(cos(2*d) - I*sin(2*d))*erf
(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(cos(2*d) + I*sin(2*d))*erf(sqrt(-c*log(f
) - 2*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f))
 - 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(sqrt(-c*log
(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)))/((c^2*lo
g(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Fricas [A]
time = 2.27, size = 167, normalized size = 1.19

$$-\frac{2\sqrt{\pi}\left(c^2\log(f)^2+4\,f^2\right)\sqrt{-c\log(f)}\,f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}\,x\right)+\sqrt{\pi}\left(c^2\log(f)^2-2i\,cf\log(f)\right)\sqrt{-c\log(f)-2i\,f}\,\mathrm{erf}\left(\sqrt{-c\log(f)-2i\,f}\,x\right)e^{(a\log(f)+2i\,d)}+\sqrt{\pi}\left(c^2\log(f)^2+2i\,cf\log(f)\right)\sqrt{-c\log(f)+2i\,f}\,\mathrm{erf}\left(\sqrt{-c\log(f)+2i\,f}\,x\right)e^{(a\log(f)-2i\,d)}}{8\left(c^3\log(f)^3+4\,cf^2\log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) + sqrt(pi)*(c^2*log(f)^
2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)*x)*
e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\cos\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + f*x^2)^2, x)
```

3.121 $\int f^{a+cx^2}\cos^3(d+fx^2)\,dx$

Optimal. Leaf size=205

$$\frac{3e^{-id}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{id}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}$$

```
[Out] 3/16*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)+1
/16*f^a*erf(x*(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*I*d)/(3*I*f-c*ln(f))^(1
/2)+3/16*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1
/2)+1/16*exp(3*I*d)*f^a*erfi(x*(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(
f))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4561, 2325, 2236, 2235}

$$\frac{3\sqrt{\pi}e^{-id}f^a\mathrm{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{-3id}f^a\mathrm{Erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}e^{id}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}e^{3id}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+3if}\right)}{16\sqrt{c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(16*E^(I*d)*Sqrt[I*f - c*Log[f
]]) + (f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(16*E^((3*I)*d)*Sqrt[(
3*I)*f - c*Log[f]]) + (3*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])
/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(3*I)*f
+ c*Log[f]]])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx &= \int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}e^{3id+3ifx^2}f^{a+\cdots}\right.\\ &= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-id-ifx^2}f^{a+cx^2}\,dx+\cdots\\ &= \frac{1}{8}\int \exp\left(-3id+a\log(f)-x^2(3if-c\log(f))\right)\,dx+\frac{1}{8}\int \exp\left(3id+a\log(f)\cdots\right.\\ &= \frac{3e^{-id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}+\cdots\end{aligned}$$

Mathematica [A]
time = 2.36, size = 389, normalized size = 1.90

$$\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(3\mathrm{Erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\left(-9if^3+9cf^2\log(f)-ic^2f\log^2(f)+c^3\log^3(f)\right)(\cos(d)+i\sin(d))+(f-ic\log(f))\left(-\left((3f-ic\log(f))\left(9f\mathrm{Erf}\left(\frac{(1+i)x\sqrt{f+ic\log(f)}}{\sqrt{2}}\right)\sqrt{f+ic\log(f)}\sin(d)+3\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{f+ic\log(f)}\right)\sqrt{f+ic\log(f)}(\cos(d)(3f+ic\log(f))+c\log(f)\sin(d))+\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{3f+ic\log(f)}\right)(f+ic\log(f))\sqrt{3f+ic\log(f)}(\cos(3d)-i\sin(3d))\right)\right)+\mathrm{Erfi}\left(\sqrt[4]{-1}x\sqrt{3f-ic\log(f)}\right)\sqrt{3f-ic\log(f)}\left(-3if^2+4cf\log(f)+ic^2\log^2(f)\right)(\cos(3d)+i\sin(3d))\right)\right)}{16\left(9f^4+10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
- I*c*Log[f]]*((-9*I)*f^3 + 9*c*f^2*Log[f] - I*c^2*f*Log[f]^2 + c^3*Log[f]^
3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(-((3*f - I*c*Log[f])*(9*f*Erf[((
1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*Erf
i[(-1)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(Cos[d]*(3*f + I*
c*Log[f]) + c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f
 + I*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(Cos[3*d] - I*Sin[3*d]))) + Erfi[(-1)
^(1/4)*x*Sqrt[3*f - I*c*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*((-3*I)*f^2 + 4*c*f
*Log[f] + I*c^2*Log[f]^2)*(Cos[3*d] + I*Sin[3*d]))))/(16*(9*f^4 + 10*c^2*f^
2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A]
time = 0.56, size = 162, normalized size = 0.79

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-3 i d} \operatorname{erf}\left(x \sqrt{3 i f-c \ln \left(f\right)}\right)}{16 \sqrt{3 i f-c \ln \left(f\right)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-i d} \operatorname{erf}\left(x \sqrt{i f-c \ln \left(f\right)}\right)}{16 \sqrt{i f-c \ln \left(f\right)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f\right.}\right.}{16 \sqrt{-c \ln \left(f\right)-i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1
/2)*x)+1/16*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*
I*f)^(1/2)*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(145) = 290$.
time = 0.30, size = 667, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((-I*c^2*cos(3*d) - c^2*sin(3
*d))*f^a*log(f)^2 + f^(a + 2)*(-I*cos(3*d) - sin(3*d)))*erf(sqrt(-c*log(f)
+ 3*I*f)*x) + ((I*c^2*cos(3*d) - c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(I*
cos(3*d) - sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqrt(c*log(f) + sqrt(
c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((-I*c^2*
cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(-I*cos(d) - sin(d)))*erf(s
qrt(-c*log(f) + I*f)*x) + ((I*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(
a + 2)*(I*cos(d) - sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(c*log(f) + s
qrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*co
s(3*d) - I*c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) - I*sin(3*d)))*
erf(sqrt(-c*log(f) + 3*I*f)*x) + ((c^2*cos(3*d) + I*c^2*sin(3*d))*f^a*log(f
)^2 + f^(a + 2)*(cos(3*d) + I*sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sq
rt(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2
 + 2*f^2)*(((c^2*cos(d) - I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(cos(d)
- I*sin(d)))*erf(sqrt(-c*log(f) + I*f)*x) + ((c^2*cos(d) + I*c^2*sin(d))*f^
a*log(f)^2 + 9*f^(a + 2)*(cos(d) + I*sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))
*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log
(f)^2 + 9*f^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(145) = 290$.

time = 2.02, size = 311, normalized size = 1.52

$$-\frac{\sqrt{\pi}\left(c^{3}\log\left(f\right)^{3}-3i\,c^{2}f\log\left(f\right)^{2}+cf^{2}\log\left(f\right)-3i\,f^{3}\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3i\,f}\,x\right)e^{\left(a\log(f)+3i\,d\right)}+3\,\sqrt{\pi}\left(c^{3}\log\left(f\right)^{3}-i\,c^{2}f\log\left(f\right)^{2}+9\,cf^{2}\log\left(f\right)-9i\,f^{3}\right)\sqrt{-c\log\left(f\right)-i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}\,x\right)e^{\left(a\log(f)+i\,d\right)}+3\,\sqrt{\pi}\left(c^{3}\log\left(f\right)^{3}+i\,c^{2}f\log\left(f\right)^{2}+9\,cf^{2}\log\left(f\right)+9i\,f^{3}\right)\sqrt{-c\log\left(f\right)+i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+i\,f}\,x\right)e^{\left(a\log(f)-i\,d\right)}+\sqrt{\pi}\left(c^{3}\log\left(f\right)^{3}+3i\,c^{2}f\log\left(f\right)^{2}+cf^{2}\log\left(f\right)+3i\,f^{3}\right)\sqrt{-c\log\left(f\right)+3i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+3i\,f}\,x\right)e^{\left(a\log(f)-3i\,d\right)}}{16\left(c^{4}\log\left(f\right)^{4}+10\,c^{2}f^{2}\log\left(f\right)^{2}+9\,f^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*I*
d) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*f^3
)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d) + 3
*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt
(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) + sqrt(pi
)*(c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(
f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(c^4*log(f
)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\cos\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + f*x^2)^3, x)
```

3.122 $\int f^{a+cx^2} \cos\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=183

$$\frac{e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(I*
f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(I*d+e^2/(4*I*f+4*c*l
n(f)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(
I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\mathrm{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\mathrm{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] (E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f -
 c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log[f]]) + (E^(I*d +
 e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))
/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)\,dx+\frac{1}{2}\int \exp\left(id+i\right.\\
&= \frac{1}{2}\left(e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)\,dx+\frac{1}{2}\left(e^{i}\right.\\
&= \frac{e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\operatorname{erfi}\left(\right.}{4\sqrt{if+c\,\mathrm{lo}}}
\end{aligned}$$

Mathematica [A]
time = 0.96, size = 217, normalized size = 1.19

$$\frac{\sqrt[4]{-1}\,e^{\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\left(-e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}}\operatorname{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+2icx\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}\,(\cos(d)-i\sin(d))+\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-2icx\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\,(-if+c\log(f))(\cos(d)+i\sin(d))\right)}{4\left(f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(-(E^(((I/2)*e^2*f)
/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Si
n[d])) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*L
og[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d])))/(4*
(f^2 + c^2*Log[f]^2))
```

Maple [A]
time = 0.19, size = 167, normalized size = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c+4 d f-e^{2}}{4(-i f+c \ln(f))}} \operatorname{erf}\left(x \sqrt{i f-c \ln(f)}+\frac{i e}{2 \sqrt{i f-c \ln(f)}}\right)}{4 \sqrt{i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 i d \ln(f) c-4 d f+e^{2}}{4 i f+4 c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ldots\right.}{4 \sqrt{-c \ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*
ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-1/4*Pi^
(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(
1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(135) = 270.
time = 0.29, size = 749, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*f^(1/4*c*e^2/(c^2*log(f)
^2 + f^2))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2
)) + f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d
*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/s
qrt(-c*log(f) + I*f)) + (-I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*
(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + f^a*f^(1/4*c*e
^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*
log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f
)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 2*f^2)*((f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*c^2*d*log(f)
^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) - I*f^a*f^(1/4*c*e^2/(c^2*log(f
)^2 + f^2))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^
2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + (f^a*f^(
1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2
)/(c^2*log(f)^2 + f^2)) + I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*
(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*l
og(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log
(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(135) = 270.
time = 1.77, size = 301, normalized size = 1.64

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-i\,f\right)\sqrt{-c\log\left(f\right)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^{2}x\log(f)^{2}+2\,f^{2}x+i\,ce\log(f)+fe\right)\sqrt{-c\log\left(f\right)-i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(\frac{4\,ac^{2}\log(f)^{3}+4i\,c^{2}d\log(f)^{2}+4i\,df^{2}-i\,fe^{2}+\left(4\,af^{2}+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)}+\sqrt{\pi}\left(c\log\left(f\right)+i\,f\right)\sqrt{-c\log\left(f\right)+i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^{2}x\log(f)^{2}+2\,f^{2}x-i\,ce\log(f)+fe\right)\sqrt{-c\log\left(f\right)+i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(\frac{4\,ac^{2}\log(f)^{3}-4i\,c^{2}d\log(f)^{2}-4i\,df^{2}+i\,fe^{2}+\left(4\,af^{2}+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)}}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(
f)^2 + 2*f^2*x + I*c*e*log(f) + f*e)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
f^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 + 4*I*d*f^2 - I*f*e^2 +
 (4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) + I*f
)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*log(f)
+ f*e)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3
 - 4*I*c^2*d*log(f)^2 - 4*I*d*f^2 + I*f*e^2 + (4*a*f^2 + c*e^2)*log(f))/(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\cos\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x + f*x^2), x)
```

3.123 $\int f^{a+cx^2} \cos^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=211

$$\frac{f^a\sqrt{\pi}\,\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(-\right.}{8\sqrt{2if+c\log}}$$

[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-2*I*d-e^2/(2*I*f-c*ln(f)))*f^a*erf((I*e+x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^(1/2)+1/8*exp(2*I*d+e^2/(2*I*f+c*ln(f)))*f^a*erfi((I*e+x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4561, 2235, 2325, 2266, 2236}

$$\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\text{Erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\text{Erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]

[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Log[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx}\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int \exp\left(-2id-2iex+a\log(f)-x^2(2i\right.\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\right)\int \exp\left(\frac{(-2ie+2}{4(-}\right.\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+x(2if-c\log(f}{\sqrt{2if-c\log(}}\right.}{8\sqrt{2if-c\log(f)}}
\end{aligned}
$$

Mathematica [A]
time = 2.44, size = 252, normalized size = 1.19

$$
\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(-e^{\frac{e^2}{-2if+c\log(f)}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+icx\log(f))}{\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}\,(\cos(2d)-i\sin(2d))+e^{\frac{e^2}{2if+c\log(f)}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(e+2fx-icx\log(f))}{\sqrt{2f-ic\log(f)}}\right)\sqrt{2f-ic\log(f)}\,(2f+ic\log(f))(-i\cos(2d)+\sin(2d))\right)}{4f^2+c^2\log^2(f)}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(-(E^(e^2/((-2*I)*f + c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*
```

```
c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[
f]]*(Cos[2*d] - I*Sin[2*d])) + E^(e^2/((2*I)*f + c*Log[f]))*Erfi[((-1)^(1/4
)*(e + 2*f*x - I*c*x*Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]
]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/
8
```

Maple [A]
time = 0.35, size = 191, normalized size = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c+4 d f-e^{2}}{-2 i f+c \ln(f)}} \operatorname{erf}\left(x \sqrt{2 i f-c \ln(f)}+\frac{i e}{\sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c-4 d f+e^{2}}{2 i f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-\ldots}\right.}{8 \sqrt{-\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c*
ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^
(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(
1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*Pi^(
1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 851, normalized size = 4.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(c*e^2/(c^2*log(f)^2
+ 4*f^2))*cos(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2))
+ f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*sin(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*
e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(
f) + 2*I*f)) + (-I*f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*cos(2*(c^2*d*log(f)
^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(c*e^2/(c^2*log(f)^2
+ 4*f^2))*sin(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)))
*erf(((c*log(f) + 2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) +
 sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2
 + 8*f^2)*((f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*cos(2*(c^2*d*log(f)^2 + 4*
d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)) - I*f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f
^2))*sin(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(
((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(f) + 2*I*f)) + (f^a*f^(c*e^2/(c^2*
log(f)^2 + 4*f^2))*cos(2*(c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 +
```

```
 4*f^2)) + I*f^a*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*sin(2*(c^2*d*log(f)^2 + 4
*d*f^2 - f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) + 2*I*f)*x + I*e)/s
qrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(
-c*log(f)) + 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(s
qrt(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x))
)/((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(155) = 310$.
time = 2.33, size = 361, normalized size = 1.71

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-2icf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2if}\operatorname{erf}\left(\frac{\left(c^2x\log(f)^2+4f^2x+ice\log(f)+2fe\right)\sqrt{-c\log\left(f\right)-2if}}{c^2\log(f)^2+4f^2}\right)e^{\left(\frac{ac^2\log(f)^3+2ic^2d\log(f)^2+8idf^2-2ife^2+\left(4af^2+ce^2\right)\log(f)}{c^2\log(f)^2+4f^2}\right)}+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2icf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)+2if}\operatorname{erf}\left(\frac{\left(c^2x\log(f)^2+4f^2x-ice\log(f)+2fe\right)\sqrt{-c\log\left(f\right)+2if}}{c^2\log(f)^2+4f^2}\right)e^{\left(\frac{ac^2\log(f)^3-2ic^2d\log(f)^2-8idf^2+2ife^2+\left(4af^2+ce^2\right)\log(f)}{c^2\log(f)^2+4f^2}\right)}}{8\left(c^3\log\left(f\right)^3+4cf^2\log\left(f\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*f*e)*sqrt(-c*log(f) - 2*I*
f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 + 8*I*d*
f^2 - 2*I*f*e^2 + (4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + sqrt(
pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*x*log(
f)^2 + 4*f^2*x - I*c*e*log(f) + 2*f*e)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^
2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 - 8*I*d*f^2 + 2*I*f*e^2
 + (4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^3 + 4*c*f
^2*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^2, x)
```

3.124 $\int f^{a+cx^2} \cos^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=369

$$\frac{3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}+3e^{id+\cdots}$$

```
[Out] 3/16*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(I
*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*exp(-3*I*d-9/4*e^2/(3*
I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2)
)*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+3/16*exp(I*d+e^2/(4*I*f+4*c*ln(f)))*f^a*er
fi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^
(1/2)+1/16*exp(3*I*d+9/4*e^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+2*x*(3*I*
f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}f^ae^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)+4if}+id}\text{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{4(c\log(f)+3if)}+3id}\text{Erfi}\left(\frac{2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f
 - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*Log[f]]) + (E^((-
3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e + 2*x*
((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*L
og[f]]) + (3*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e +
2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
+ (E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*
e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)
*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int\left(\frac{1}{8}e^{-3i(d+ex+fx^2)}f^{a+cx^2}+\frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+\right.\right.\right.$$
$$=\frac{1}{8}\int e^{-3i(d+ex+fx^2)}f^{a+cx^2}\,dx+\frac{1}{8}\int\exp\left(6id+6iex+6ifx^2-3i\left(d+e\right.\right.$$
$$=\frac{1}{8}\int\exp\left(-3id-3iex+a\log(f)-x^2(3if-c\log(f))\right)\,dx+\frac{1}{8}\int\exp\left(\right.$$
$$=\frac{1}{8}\left(3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)\,dx+\frac{1}{8}$$
$$=\frac{3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\ }}{16\sqrt{3i}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2997 vs. 2(369) = 738.
time = 7.01, size = 2997, normalized size = 8.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-27*(-1)^(3/4)*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*
I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e
^2)/(f - I*c*Log[f])) + (27*(-1)^(1/4)*c*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2
*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log
[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) - (3*(-1)^(3/4)*c^2*f*Cos[d]*Erfi[((
-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (3*(-1)^(1/4)*c^
3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*L
og[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) -
(3*(-1)^(3/4)*f^3*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]
))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]])/E^((((9*I)/4)*e^2)/(
3*f - I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f
*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*L
og[f]])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f])) - (3*(-1)^(3/4)*c^2*f*Cos[3*
d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f
])) + ((-1)^(1/4)*c^3*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]])/E^((((9
*I)/4)*e^2)/(3*f - I*c*Log[f])) - 27*(-1)^(1/4)*E^(((I/4)*e^2)/(f + I*c*Log
[f]))*f^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f
 + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]] + 27*(-1)^(3/4)*c*E^(((I/4)*e^2)/(f +
 I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(
2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]] - 3*(-1)^(1/4)*c^2*E^(
((I/4)*e^2)/(f + I*c*Log[f]))*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]] + 3*(-
1)^(3/4)*c^3*E^(((I/4)*e^2)/(f + I*c*Log[f]))*Cos[d]*Erfi[((-1)^(3/4)*(e +
2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*
Log[f]] - 3*(-1)^(1/4)*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f^3*Cos[3*d]*
Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]
])]*Sqrt[3*f + I*c*Log[f]] + (-1)^(3/4)*c*E^((((9*I)/4)*e^2)/(3*f + I*c*Log
[f]))*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sq
rt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]] - 3*(-1)^(1/4)*c^2*E^(
(((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*
x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*
Log[f]] + (-1)^(3/4)*c^3*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*Cos[3*d]*Er
fi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
]*Log[f]^3*Sqrt[3*f + I*c*Log[f]] + (27*(-1)^(1/4)*f^3*Erfi[((-1)^(1/4)*(e
+ 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]
*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Erfi[((-1)
^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqr
t[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (3*(-1)^(1/4)*
c^2*f*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[
f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])
) + (3*(-1)^(3/4)*c^3*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*S
qrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/
```

```
(f - I*c*Log[f])) + 27*(-1)^(3/4)*E^(((I/4)*e^2)/(f + I*c*Log[f]))*f^3*Erfi
[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt
[f + I*c*Log[f]]*Sin[d] + 27*(-1)^(1/4)*c*E^(((I/4)*e^2)/(f + I*c*Log[f]))*
f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]
])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^2*E^(((I/4)*e^2)/(f
 + I*c*Log[f]))*f*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[
f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^3*E
^(((I/4)*e^2)/(f + I*c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log
[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d] + (3*(
-1)^(1/4)*f^3*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*
f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/4)*e^2)/(3*f
- I*c*Log[f])) + ((-1)^(3/4)*c*f^2*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*
x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*
d])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f])) + (3*(-1)^(1/4)*c^2*f*Erfi[((-1)
^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]
^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f]))
+ ((-1)^(3/4)*c^3*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqr
t[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/
4)*e^2)/(3*f - I*c*Log[f])) + 3*(-1)^(3/4)*E^((((9*I)/4)*e^2)/(3*f + I*c*Lo
g[f]))*f^3*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f +
 I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d] + (-1)^(1/4)*c*E^((((9*I)/4)
*e^2)/(3*f + I*c*Log[f]))*f^2*Erfi[((-1)^(3/4)*...
```

Maple [A]
time = 0.81, size = 334, normalized size = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c+12 d f-3 e^{2}\right)}{4(-3 i f+c \ln(f))}} \operatorname{erf}\left(x \sqrt{3 i f-c \ln(f)}+\frac{3 i e}{2 \sqrt{3 i f-c \ln(f)}}\right)}{16 \sqrt{3 i f-c \ln(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c+4 d f-e^{2}}{4(-i f+c \ln(f))}} \operatorname{erf}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(-3*I*f+c*ln(f)))/(
3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f-c*ln(f))^(1
/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(
I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-3
/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)
-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))-1/16*
Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f
)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*f)^(1/2)
)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2140 vs. $2(269) = 538$.

time = 0.33, size = 2140, normalized size = 5.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*f^(9/4*c*e^2/(c^2
*log(f)^2 + 9*f^2))*log(f)^2 + I*f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f
^2)))*cos(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2
)) + (c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*f^(
9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*sin(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9
*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/s
qrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))
*log(f)^2 - I*f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(4*c^
2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*f^(9/
4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*f^(9/4*c*e^2/(c^2*log(
f)^2 + 9*f^2)))*sin(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)
^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I
*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*l
og(f)^2 + 2*f^2)*(((-I*c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2
- 9*I*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*c^2*d*log(f)
^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^a*f^(1/4*c*e^2/(c^2*lo
g(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*s
in(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*
(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + ((I*c^2*f^a*f^(1/4*c*
e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*I*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)
^2 + f^2)))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^
2)) - (c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*f^
(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log
(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 18*f^2)*(((c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)
^2 + f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(4*c^2*d*log(f
)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) - (I*c^2*f^a*f^(9/4*c*e^2
/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + I*f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2
+ 9*f^2)))*sin(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 +
9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/sqrt(-c*log(f) + 3*I*f))
+ ((c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*f^(9/
4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f
*e^2)/(c^2*log(f)^2 + 9*f^2)) - (-I*c^2*f^a*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*
f^2))*log(f)^2 - I*f^(a + 2)*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)))*sin(3/4*
(4*c^2*d*log(f)^2 + 36*d*f^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2
*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f) + s
qrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2
```

```
*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*f^(1/4*c*e^2
/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*l
og(f)^2 + f^2)) + (-I*c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*log(f)^2 -
 9*I*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*sin(1/4*(4*c^2*d*log(f)^
2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x -
 I*e)/sqrt(-c*log(f) + I*f)) + ((c^2*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))
*log(f)^2 + 9*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*c^2*
d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2)) + (I*c^2*f^a*f^(1/4*c*e
^2/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*I*f^(a + 2)*f^(1/4*c*e^2/(c^2*log(f)^
2 + f^2)))*sin(1/4*(4*c^2*d*log(f)^2 + 4*d*f^2 - f*e^2)/(c^2*log(f)^2 + f^2
)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*lo
g(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f
^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(269) = 538.
time = 2.59, size = 707, normalized size = 1.92

$$-\frac{\sqrt{\pi}\left(c^3\log(f)^3 - 3i c^2 f\log(f)^2 + c f^2\log(f) - 3i f^3\right)\sqrt{-c\log(f) - 3i f}\,\operatorname{erf}\left(\frac{\left(2c^2x\log(f)^2 + 18f^2x + 3ice\log(f) + 9fe\right)\sqrt{-c\log(f) - 3if}}{2\left(c^2\log(f)^2 + 9f^2\right)}\right)e^{\left(\frac{4ac^2\log(f)^3 + 12ic^2d\log(f)^2 + 108idf^2 - 27ife^2 + 9\left(4af^2 + ce^2\right)\log(f)}{4\left(c^2\log(f)^2 + 9f^2\right)}\right)} + \sqrt{\pi}\left(c^3\log(f)^3 + 3ic^2f\log(f)^2 + cf^2\log(f) + 3if^3\right)\sqrt{-c\log(f) + 3if}\,\operatorname{erf}\left(\frac{\left(2c^2x\log(f)^2 + 18f^2x - 3ice\log(f) + 9fe\right)\sqrt{-c\log(f) + 3if}}{2\left(c^2\log(f)^2 + 9f^2\right)}\right)e^{\left(\frac{4ac^2\log(f)^3 - 12ic^2d\log(f)^2 - 108idf^2 + 27ife^2 + 9\left(4af^2 + ce^2\right)\log(f)}{4\left(c^2\log(f)^2 + 9f^2\right)}\right)} + 3\sqrt{\pi}\left(c^3\log(f)^3 - ic^2f\log(f)^2 + 9cf^2\log(f) - 9if^3\right)\sqrt{-c\log(f) - if}\,\operatorname{erf}\left(\frac{\left(2c^2x\log(f)^2 + 2f^2x + ice\log(f) + fe\right)\sqrt{-c\log(f) - if}}{2\left(c^2\log(f)^2 + f^2\right)}\right)e^{\left(\frac{4ac^2\log(f)^3 + 4ic^2d\log(f)^2 + 4idf^2 - ife^2 + \left(4af^2 + ce^2\right)\log(f)}{4\left(c^2\log(f)^2 + f^2\right)}\right)} + 3\sqrt{\pi}\left(c^3\log(f)^3 + ic^2f\log(f)^2 + 9cf^2\log(f) + 9if^3\right)\sqrt{-c\log(f) + if}\,\operatorname{erf}\left(\frac{\left(2c^2x\log(f)^2 + 2f^2x - ice\log(f) + fe\right)\sqrt{-c\log(f) + if}}{2\left(c^2\log(f)^2 + f^2\right)}\right)e^{\left(\frac{4ac^2\log(f)^3 - 4ic^2d\log(f)^2 - 4idf^2 + ife^2 + \left(4af^2 + ce^2\right)\log(f)}{4\left(c^2\log(f)^2 + f^2\right)}\right)}}{16\left(c^4\log(f)^4 + 10c^2f^2\log(f)^2 + 9f^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*lo
g(f) + 9*f*e)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c
^2*log(f)^3 + 12*I*c^2*d*log(f)^2 + 108*I*d*f^2 - 27*I*f*e^2 + 9*(4*a*f^2 +
 c*e^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(2*c^2
*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*f*e)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^2 - 108*
I*d*f^2 + 27*I*f*e^2 + 9*(4*a*f^2 + c*e^2)*log(f))/(c^2*log(f)^2 + 9*f^2))
+ 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*f^3)*s
qrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e*log(f) + f
*e)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3 +
4*I*c^2*d*log(f)^2 + 4*I*d*f^2 - I*f*e^2 + (4*a*f^2 + c*e^2)*log(f))/(c^2*l
og(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*log
(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x -
I*c*e*log(f) + f*e)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a
*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 - 4*I*d*f^2 + I*f*e^2 + (4*a*f^2 + c*e^2
)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cos^3\left(d + ex + fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^3, x)
```

3.125 $\int f^{a+bx+cx^2}\cos(d+ex)\,dx$

Optimal. Leaf size=172

$$-\frac{e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(I*d+1/4*(e
-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4561, 2325, 2266, 2235}

$$\frac{\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

```
[Out] -1/4*(E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e -
 b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
 + (E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Lo
g[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx = \int \left(\frac{1}{2}e^{-id-iex}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex}f^{a+bx+cx^2}\right)dx$$
$$= \frac{1}{2}\int e^{-id-iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+iex}f^{a+bx+cx^2}\,dx$$
$$= \frac{1}{2}\int \exp\left(-id+a\log(f)+cx^2\log(f)-x(ie-b\log(f))\right)dx+\frac{1}{2}\int \exp\left(id+\right.$$
$$= \frac{1}{2}\left(e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{2}\left(e^{-id+\frac{(e+}{4}}\right.$$
$$= -\frac{e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie}{}\right.}{4\sqrt{c}\sqrt{\log(}}$$

Mathematica [A]
time = 0.32, size = 151, normalized size = 0.88

$$\frac{e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{ibe}{c}}\mathrm{Erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+\mathrm{Erfi}\left(\frac{ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

```
[Out] (E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((
I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos
[d] - I*Sin[d]) + Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.18, size = 168, normalized size = 0.98

method	result

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-i e}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+2 i \ln(f) b e-4}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c-e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f)
)^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c
-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(
-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 362, normalized size = 2.10

$$-\frac{\sqrt{\pi}\left(f^{a}\left(\cos\left(\frac{2cd-be}{2c}\right)+i\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(x\sqrt{-c\log(f)}-\frac{1}{2}\left(b\log(f)+ie\right)\frac{1}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{2c}\right)-i\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(x\sqrt{-c\log(f)}-\frac{1}{2}\left(b\log(f)-ie\right)\frac{1}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{2c}\right)+i\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+b\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{2c}\right)-i\sin\left(\frac{2cd-be}{2c}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+b\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{4c\log(f)}\right)}\right)\sqrt{-c\log(f)}}{8cf^{\frac{b^{2}}{4c}}\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(cos(1/2*(2*c*d - b*e)/c) + I*sin(1/2*(2*c*d - b*e)/c))*
erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-c
*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(1/2*(2*c*d - b*e)/c) - I*sin(1
/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - I*e
)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(1/2*(2*c*
d - b*e)/c) + I*sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f)
+ I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(1/2*(2
*c*d - b*e)/c) - I*sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(
f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt(-c*log(f
))/(c*f^(1/4*b^2/c)*log(f))
```

Fricas [A]
time = 2.42, size = 178, normalized size = 1.03

$$-\frac{\sqrt{\pi}\,\sqrt{-c\log(f)}\,\operatorname{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^{2}-4ac)\log(f)^{2}+2(2icd-ibe)\log(f)-e^{2}}{4c\log(f)}\right)}+\sqrt{\pi}\,\sqrt{-c\log(f)}\,\operatorname{erf}\left(\frac{((2cx+b)\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^{2}-4ac)\log(f)^{2}+2(-2icd+ibe)\log(f)-e^{2}}{4c\log(f)}\right)}}{4c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-c*l
og(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 2*(2*I*c*d - I*b*e)*lo
g(f) - e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log
```

```
(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 2*
(-2*I*c*d + I*b*e)*log(f) - e^2)/(c*log(f))))/(c*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cos\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d),x)
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\cos\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x),x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x), x)
```

3.126 $\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$

Optimal. Leaf size=231

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{}}{8\sqrt{}}$$

```
[Out] 1/8*exp(-2*I*d+1/4*(2*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*I*e+b*ln(f)+
2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*I*
d-1/4*(2*I*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*I*e+b*ln(f)+2*c*x*ln(f))/c
^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(1
/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4561, 2266, 2235, 2325}

$$\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{4}\int e^{-2id-2iex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx \\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)+cx^2\log(f)-x(2ie-b\log(f))\right)dx+\frac{1}{4}\int \exp\left(2i\right. \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\right) \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2ie}{}\right.}{8\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A]
time = 0.60, size = 204, normalized size = 0.88

$$
\frac{e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(2e^{\frac{ibe}{c}}\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{-2ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+e^{\frac{e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{2ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)+i\sin(2d))\right)}{8\sqrt{c}\sqrt{\log(f)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[Log[f]]
)/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)*e + (
b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + E^(e
^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
]*(Cos[2*d] + I*Sin[2*d])))/(8*Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

Maple [A]
time = 0.28, size = 217, normalized size = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c-4 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-2 i e}{2 \sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) b e-}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c-4*e^2)/
ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*l
n(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(
f)*c-4*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*
ln(f))/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*e
rf(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 399, normalized size = 1.73

$$\frac{\sqrt{\pi}\left(f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)+i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}(b\log(f)+2ie)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)-i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}(b\log(f)-2ie)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)+i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+b\log(f)+2ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+f^{a}\left(\cos\left(\frac{2cd-be}{c}\right)-i\sin\left(\frac{2cd-be}{c}\right)\right)\operatorname{erf}\left(\frac{(2cx\log(f)+b\log(f)-2ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{e^{2}}{c\log(f)}\right)}+2f^{a}\operatorname{erf}\left(-\frac{1}{2}b\overline{\frac{1}{\sqrt{-c\log(f)}}}\log(f)+x\overline{\sqrt{-c\log(f)}}\right)-2f^{a}\operatorname{erf}\left(\frac{2cx\log(f)+b\log(f)}{2\sqrt{-c\log(f)}}\right)\right)}{16\sqrt{-c\log(f)}\,f^{\frac{b^{2}}{4c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*(f^a*(cos((2*c*d - b*e)/c) + I*sin((2*c*d - b*e)/c))*erf(x*co
njugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 2*I*e)*conjugate(1/sqrt(-c*log(f
))))*e^(e^2/(c*log(f))) + f^a*(cos((2*c*d - b*e)/c) - I*sin((2*c*d - b*e)/c
))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - 2*I*e)*conjugate(1/sq
rt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos((2*c*d - b*e)/c) + I*sin((2*c
*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + 2*I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(e^2/(c*log(f))) + f^a*(cos((2*c*d - b*e)/c) - I*sin((2*c*d - b*
e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - 2*I*e)*sqrt(-c*log(f))/(c*log(f))
)*e^(e^2/(c*log(f))) + 2*f^a*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)))*log(f)
 + x*conjugate(sqrt(-c*log(f)))) - 2*f^a*erf(1/2*(2*c*x*log(f) + b*log(f))/
sqrt(-c*log(f))))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [A]
time = 2.39, size = 226, normalized size = 0.98

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)-2ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^{2}-4ac)\log(f)^{2}+4(2icd-ibe)\log(f)-4e^{2}}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)+2ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^{2}-4ac)\log(f)^{2}+4(-2icd+ibe)\log(f)-4e^{2}}{4c\log(f)}\right)}+\frac{2\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{\frac{b^{2}-4ac}{4c}}}}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 2*I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*(2*I*c*d - I*b*e)*
log(f) - 4*e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)
*log(f) + 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^
2 + 4*(-2*I*c*d + I*b*e)*log(f) - 4*e^2)/(c*log(f))) + 2*sqrt(pi)*sqrt(-c*l
og(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c*l
og(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\cos\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x)^2, x)
```

3.127 $\int f^{a+bx+cx^2} \cos^3(d+ex)\,dx$

Optimal. Leaf size=346

$$-\frac{3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3e^{id}\cdots}{\cdots}$$

```
[Out] 3/16*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*x
*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*I*d+1
/4*(3*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*I*e+b*ln(f)+2*c*x*ln(f))/c^(
1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/16*exp(I*d+1/4*(e-I*b*ln(f
))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*P
i^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/c/ln(f))*f
^a*erfi(1/2*(3*I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)
```

Rubi [A]
time $= 0.32$, antiderivative size $= 346$, normalized size of antiderivative $= 1.00$, number of steps used $= 14$, number of rules used $= 4$, integrand size $= 21$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used $= \{4561, 2325, 2266, 2235\}$

$$-\frac{3\sqrt{\pi}\,f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{3id-\frac{(b\log(f)+3ie)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+3ie}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (-3*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - b
*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]
) - (E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3
*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (3*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*S
qrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx &= \int \left(\frac{3}{8}e^{-id-iex}f^{a+bx+cx^2}+\frac{3}{8}e^{id+iex}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+bx+cx^2}+\frac{1}{8}e^{3id+3ie}\right.\\ &= \frac{1}{8}\int e^{-3id-3iex}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3id+3iex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-id-iex}f^{a+bx+c}\\ &= \frac{1}{8}\int \exp\left(-3id+a\log(f)+cx^2\log(f)-x(3ie-b\log(f))\right)\,dx+\frac{1}{8}\int \exp\left(3i\right.\\ &= \frac{1}{8}\left(3e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx+\frac{1}{8}\left(3e^{-id+}\right.\\ &= -\frac{3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{\exp\left(-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}\right.}{16\sqrt{}}\end{aligned}$$

<u>Mathematica</u> [A]
time = 0.97, size = 386, normalized size = 1.12

$$\frac{e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\cos(3d)\mathrm{Erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)+e^{\frac{2e^2}{c\log(f)}}\cos(3d)\mathrm{Erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)+3e^{\frac{2ibe}{c}}\mathrm{Erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+3e^{\frac{ibe}{c}}\mathrm{Erfi}\left(\frac{ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))-ie^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\sin(3d)+ie^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\sin(3d)\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((
e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2*c*x)
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + E^((2*e^2)/(c*Log[f]))*Cos[3*d]*Erfi[(
(3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^(((2*I)*b*e)/
```

```
c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] - I
*Sin[d]) + 3*E^((I*b*e)/c)*Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[
Log[f]])]*(Cos[d] + I*Sin[d]) - I*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))
*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*Sin[3*d] +
I*E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqr
t[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A]
time = 0.62, size = 334, normalized size = 0.97

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6 i \ln(f) b e+12 i d \ln(f) c-9 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{-3 i e+b \ln(f)}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b}{4 \ln(f}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c-9*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(-3*I*e+b*ln(f))/(-
c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d
*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f
)-I*e)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*
b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/
2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6
*I*ln(f)*b*e-12*I*d*ln(f)*c-9*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f)
)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 696, normalized size = 2.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*sqrt(pi)*(f^a*(cos(3/2*(2*c*d - b*e)/c) + I*sin(3/2*(2*c*d - b*e)/c))
*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 3*I*e)*conjugate(1/sqrt
(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(cos(3/2*(2*c*d - b*e)/c) - I*si
n(3/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) -
3*I*e)*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(cos(3/2*
(2*c*d - b*e)/c) + I*sin(3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*lo
g(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f))) + f^a*(cos
(3/2*(2*c*d - b*e)/c) - I*sin(3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) +
 b*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f))) + 3*f
```

```
^a*(cos(1/2*(2*c*d - b*e)/c) + I*sin(1/2*(2*c*d - b*e)/c))*erf(x*conjugate(
sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/
4*e^2/(c*log(f))) + 3*f^a*(cos(1/2*(2*c*d - b*e)/c) - I*sin(1/2*(2*c*d - b*
e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - I*e)*conjugate(1/
sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + 3*f^a*(cos(1/2*(2*c*d - b*e)/c)
+ I*sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + I*e)*sqrt(
-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + 3*f^a*(cos(1/2*(2*c*d - b*e
)/c) - I*sin(1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - I*e)*
sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt(-c*log(f))/(c*f^(1
/4*b^2/c)*log(f))
```

Fricas [A]
time = 2.66, size = 348, normalized size = 1.01

$$-\frac{\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)-3i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)^{2}+6\,\left(2i\,cd-i\,be\right)\log\left(f\right)-9\,e^{2}}{4\,c\log\left(f\right)}\right)}+3\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)-i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)^{2}+2\,\left(2i\,cd-i\,be\right)\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)}+\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)+3i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)^{2}+6\,\left(-2i\,cd+i\,be\right)\log\left(f\right)-9\,e^{2}}{4\,c\log\left(f\right)}\right)}+3\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(-\frac{\left(b^{2}-4\,ac\right)\log\left(f\right)^{2}+2\,\left(-2i\,cd+i\,be\right)\log\left(f\right)-e^{2}}{4\,c\log\left(f\right)}\right)}}{16\,c\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(-
c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 6*(2*I*c*d - I*b*e)
*log(f) - 9*e^2)/(c*log(f))) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x +
 b)*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)
^2 + 2*(2*I*c*d - I*b*e)*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f
))*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4
*((b^2 - 4*a*c)*log(f)^2 + 6*(-2*I*c*d + I*b*e)*log(f) - 9*e^2)/(c*log(f)))
 + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 + 2*(-2*I*c*d + I*b*e)*lo
g(f) - e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}}\cos^{3}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^3, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\cos\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x)^3, x)
```

3.128 $\int f^{a+bx+cx^2} \cos(d+fx^2)\, dx$

Optimal. Leaf size=189

$$-\frac{e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] -1/4*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*f-
c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(I*d-b^2
*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f+c
*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\,\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*(E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(
b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Lo
g[f]] + (E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[
(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f +
c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-id-ifx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+bx+cx^2}\right)\,dx\\ &= \frac{1}{2}\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)\,dx+\frac{1}{2}\int\exp\left(i\right.\\ &= \frac{1}{2}\left(e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)\,dx+\frac{1}{2}\left(\right.\\ &= -\frac{e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}}{4\sqrt{if+}}\end{aligned}$$

Mathematica [A]
time = 1.04, size = 231, normalized size = 1.22

$$-\frac{(-1)^{3/4}e^{\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}\,(-i\cos(d)-\sin(d))+e^{\frac{ib^2f\log^2(f)}{2(f^2+c^2\log^2(f))}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}\,(2fx-i(b+2cx)\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\,(f+ic\log(f))(\cos(d)+i\sin(d))\right)}{4\left(f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(Er
fi[((-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*(f
 - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*((-I)*Cos[d] - Sin[d]) + E^(((I/2)*b^2*
f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*L
```

```
og[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Co
s[d] + I*Sin[d])))/(f^2 + c^2*Log[f]^2)
```

Maple [A]
time = 0.15, size = 178, normalized size = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i d \ln(f) c+4 d f}{4(-i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{i f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{i f-c \ln(f)}}\right)}{4 \sqrt{i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i d \ln(f) c+4 d f}{4(i f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(-I*f+c*ln(f))
)/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^
(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*l
n(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-I*f)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(145) = 290$.
time = 0.29, size = 648, normalized size = 3.43

$$\frac{\sqrt{\pi}\sqrt{2c^2\log(f)^2+2f^2}\left(\left(if^a\cos\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)+f^a\sin\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)-if)x+b\log(f)}{2\sqrt{-c\log(f)+if}}\right)+\left(-if^a\cos\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)+f^a\sin\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)+if)x+b\log(f)}{2\sqrt{-c\log(f)-if}}\right)\right)\sqrt{c\log(f)+\sqrt{c^2\log(f)^2+f^2}}-\sqrt{\pi}\sqrt{2c^2\log(f)^2+2f^2}\left(\left(f^a\cos\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)-if^a\sin\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)-if)x+b\log(f)}{2\sqrt{-c\log(f)+if}}\right)+\left(f^a\cos\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)+if^a\sin\left(\frac{4df^2+(4c^2d+b^2f)\log(f)^2}{4(c^2\log(f)^2+f^2)}\right)\right)\operatorname{erf}\left(\frac{2(c\log(f)+if)x+b\log(f)}{2\sqrt{-c\log(f)-if}}\right)\right)\sqrt{-c\log(f)+\sqrt{c^2\log(f)^2+f^2}}}{8\left(c^2e^{\left(\frac{b^2c\log(f)^3}{4(c^2\log(f)^2+f^2)}\right)}\log(f)^2+f^2e^{\left(\frac{b^2c\log(f)^3}{4(c^2\log(f)^2+f^2)}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*cos(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x
 + b*log(f))/sqrt(-c*log(f) + I*f)) + (-I*f^a*cos(1/4*(4*d*f^2 + (4*c^2*d +
 b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^2*d +
 b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*
log(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) -
 sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*cos(1/4*(4*d*f^2 + (4*c^2*d +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^a*sin(1/4*(4*d*f^2 + (4*c^2*d
+ b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b
*log(f))/sqrt(-c*log(f) + I*f)) + (f^a*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)
*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^a*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*
f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f
))/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2
*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^2*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + f^2)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(145) = 290.
time = 2.70, size = 311, normalized size = 1.65

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-i\,f\right)\sqrt{-c\log\left(f\right)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^{2}x-i\,bf\log(f)+\left(2\,c^{2}x+bc\right)\log(f)^{2}\right)\sqrt{-c\log\left(f\right)-i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(\frac{4\,af^{2}\log(f)-\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}+4i\,df^{2}+\left(4i\,c^{2}d+i\,b^{2}f\right)\log(f)^{2}}{4\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)}+\sqrt{\pi}\left(c\log\left(f\right)+i\,f\right)\sqrt{-c\log\left(f\right)+i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^{2}x+i\,bf\log(f)+\left(2\,c^{2}x+bc\right)\log(f)^{2}\right)\sqrt{-c\log\left(f\right)+i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(\frac{4\,af^{2}\log(f)-\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}-4i\,df^{2}+\left(-4i\,c^{2}d-i\,b^{2}f\right)\log(f)^{2}}{4\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)}}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*
b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2
+ f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (
4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) +
 I*f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x + b*
c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*lo
g(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*f)*log(
f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\cos\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + f*x^2), x)
```

3.129 $\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=245

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{E}}{8\sqrt{2if}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)-1/8*exp(-2*I*d+b^2*ln(f)^2/(8*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*
ln(f)-2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^
(1/2)+1/8*exp(2*I*d-b^2*ln(f)^2/(8*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*
x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4561, 2266, 2235, 2325, 2236}

$$-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\mathrm{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}$$

$$=\frac{1}{4}\int\exp\left(-2id+a\log(f)+bx\log(f)-x^2(2if-c\log(f))\right)dx+\frac{1}{4}\int \mathrm{ex}$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log\right.$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2}{2\sqrt{2if-}}\right.}{8\sqrt{2if-c\log(f)}}$$

Mathematica [A]
time = 3.30, size = 301, normalized size = 1.23

$$\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\,e^{\frac{b^2\log^2(f)}{8if-4c\log(f)}}\left(-\mathrm{Erfi}\left(\frac{(-1)^{3/4}(4fx+i(b+2cx)\log(f))}{2\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))+e^{\frac{ib^2f\log^2(f)}{4f^2+c^2\log^2(f)}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(4fx-i(b+2cx)\log(f))}{2\sqrt{2f-ic\log(f)}}\right)\sqrt{2f-ic\log(f)}(2f+ic\log(f))(-i\cos(2d)+\sin(2d))\right)}{4f^2+c^2\log^2(f)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
```

```
g[f]))*(-(Erfi[((-1)^(3/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*
c*Log[f]])]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d
])) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(4*f*x
 - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log[f]
]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/
8
```

Maple [A]
time = 0.28, size = 227, normalized size = 0.93

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+8 i d \ln(f) c+16 d f}{4(-2 i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{2 i f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-8 i d \ln(f) c+}{4(2 i f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(-2*I*f+c*ln(
f)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-
c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*f
)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2
*ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))
^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 997, normalized size = 4.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(1/4*b^2/c)*cos(1/2*(
16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1/4
*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f
^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*log(f) + 2*I*f))
+ (-I*f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^
2*log(f)^2 + 4*f^2)) + f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b*
log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2
))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((f^a*f^(1/4*b^2
/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))
 - I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2
*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*lo
```

```
g(f) + 2*I*f)) + (f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*l
og(f)^2)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 +
(4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) +
 2*I*f)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*l
og(f)^2 + 4*f^2))*sqrt(-c*log(f)) + 2*sqrt(pi)*((c^2*f^a*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c
^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)))*log(f) + x*c
onjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2
)))*erf(1/2*(2*c*x*log(f) + b*log(f))/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*c
*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 + 4*f^2*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f)
))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(185) = 370$.
time = 2.72, size = 402, normalized size = 1.64

$$-\frac{\sqrt{\pi}\left(c^2\log(f)^2 - 2i\,cf\log(f)\right)\sqrt{-c\log(f) - 2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x - 2i\,bf\log(f) + (2\,c^2x + bc)\log(f)^2\right)\sqrt{-c\log(f) - 2i\,f}}{2\left(c^2\log(f)^2 + 4\,f^2\right)}\right)e^{\left(\frac{16\,af^2\log(f) - \left(b^2c - 4\,ac^2\right)\log(f)^3 + 32i\,df^2 - 2\left(-4i\,c^2d - i\,b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2 + 4\,f^2\right)}\right)} + \sqrt{\pi}\left(c^2\log(f)^2 + 2i\,cf\log(f)\right)\sqrt{-c\log(f) + 2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x + 2i\,bf\log(f) + (2\,c^2x + bc)\log(f)^2\right)\sqrt{-c\log(f) + 2i\,f}}{2\left(c^2\log(f)^2 + 4\,f^2\right)}\right)e^{\left(\frac{16\,af^2\log(f) - \left(b^2c - 4\,ac^2\right)\log(f)^3 - 32i\,df^2 - 2\left(4i\,c^2d + i\,b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2 + 4\,f^2\right)}\right)} + \frac{2\sqrt{\pi}\left(c^2\log(f)^2 + 4\,f^2\right)\sqrt{-c\log(f)}\,\operatorname{erf}\left(\frac{(2\,cx + b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2 - 4\,ac}{4\,c}}}}{8\left(c^3\log(f)^3 + 4\,cf^2\log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(
1/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) -
2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2)*
log(f)^3 + 32*I*d*f^2 - 2*(-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 +
4*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*
erf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f
) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c
^2)*log(f)^3 - 32*I*d*f^2 - 2*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 4*f^2)) + 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c
*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2
*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cos\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^2, x)
```

3.130 $\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right) dx$

Optimal. Leaf size=378

$$-\frac{3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}} f^a \sqrt{\pi}\, \mathrm{Erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}} f^a \sqrt{\pi}\, \mathrm{Erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{\cdots}}{\cdots}$$

```
[Out] -3/16*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*f
-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*exp(-3*I*
d+b^2*ln(f)^2/(12*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*I*f-c*ln(f)))
/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+3/16*exp(I*d-b^2*ln(
f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f+c*ln(
f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*exp(3*I*d-1/4*b^2*ln(f)^2/(3*I
*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/
2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$-\frac{3\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\mathrm{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\mathrm{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{\sqrt{\pi}\, f^a \exp\left(3id-\frac{b^2\log^2(f)}{4(c\log(f)+3if)}\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}+\frac{3\sqrt{\pi}\, f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (-3*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*
Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*
Log[f]]) - (E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f]))*f^a*Sqrt[
Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])]
)/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
 c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d - (b^2*Log[f]^2)/(4*(
(3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((3*I)*f + c*Log[f]
))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx &= \int\left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx+cx^2}+\frac{1}{8}e\right.\\
&= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-id-ifx^2}f^{a}\\
&= \frac{1}{8}\int\exp\left(-3id+a\log(f)+bx\log(f)-x^2(3if-c\log(f))\right)\,dx+\frac{1}{8}\int\exp\\
&= \frac{1}{8}\left(3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)\,dx+\frac{1}{8}\\
&= -\frac{3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}}{16\sqrt{3i}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3285 vs. 2(378) = 756.
time = 7.00, size = 3285, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3
*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f
 - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Log[f
]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2
*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*
Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f
 - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4
)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-1)^(1/4)
*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[
3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f])
)*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2
*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*
E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4)*(6*
f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^2*
Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*
Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))
/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]] - (27*(-1)^(1/
4)*f^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I
*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log
[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[d]
*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*
Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) + (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (
2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/
E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*f^3*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ ((-1)^(3/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^((
(I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f])) + ((-1)^(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] +
(2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*
b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] -
(2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] +
27*(-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^2*Erfi[((-1)^(1
```

```
/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[
f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(
f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f])
)/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(
3/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*f*x
- I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f
 - I*c*Log[f]]*Sin[d] + (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(2*f*x + I*b*Lo
g[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*Si
n[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (27*(-1)^(1/4)*c*f^2*Erfi
[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d]
)/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (3*(-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f]
)) + 3*(-1)^(1/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Erfi[((-1
)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3
*f - I*c*Log[f]))*f^2*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + 3
*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Erfi[((-1)^(1
/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Lo
g[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(...
```

Maple [A]
time = 0.73, size = 354, normalized size = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+12 i d \ln(f) c+36 d f}{4(-3 i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{3 i f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{3 i f-c \ln(f)}}\right)}{16 \sqrt{3 i f-c \ln(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i d \ln(f)}{4(-i f+c \ln(f}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f)*c+36*d*f)/(-3*I*f+c*l
n(f)))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*I*
f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d
*f)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f
)*b/(I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f
)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+
1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-1
2*I*d*ln(f)*c+36*d*f)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f
)-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*I*f)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2456 vs. 2(289) = 578.
time = 0.34, size = 2456, normalized size = 6.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^
2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*lo
g(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log
(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36*
d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*
log(f) - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^a*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*
log(f)^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f
)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*
sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*er
f(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f
^2)*(((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 -
9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f
^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^a*e^(1/4*b^
2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-
c*log(f) + I*f)) + ((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)
)*log(f)^2 + 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*c
os(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - (c^2*
f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^
(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*l
og(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) -
sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^
2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9
*f^2)) - (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 +
I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36*d*f^2
+ (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f)
 - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^a*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c
^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*l
```

```
og(f)^2 + 9*f^2)) - (-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))
*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*
(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f
) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(
((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a +
 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^
3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(
c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-c*log
(f) + I*f)) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f
)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4
*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (I*c^2*f^a*e^(
1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^(a + 2)*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f)
)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2))*log(f)^4 + 10*c^2*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*
f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^4*e^(1/4*b^2
*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2
)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(289) = 578.
time = 2.95, size = 725, normalized size = 1.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x + b
*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 - 3*(-4*I*c^2*d - I*b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^
2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3
- 108*I*d*f^2 - 3*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) +
 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*f^3)*sq
rt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x + b*c)*log(f
)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*log(f) - (
```

```
b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^
2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*
log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (
2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4
*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I
*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^
2 + 9*f^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**3,x)
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^3, x)
```

3.131 $\int f^{a+bx+cx^2} \cos(d+ex+fx^2)\, dx$

Optimal. Leaf size=208

$$\frac{e^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+2*
x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(
I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2*x*(I*f+c
*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi} f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] (E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I
*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*
f - c*Log[f]]) + (E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]
])])/(4*Sqrt[I*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right) dx &= \int \left(\frac{1}{2} e^{-id-iex-ifx^2} f^{a+bx+cx^2} + \frac{1}{2} e^{id+iex+ifx^2} f^{a+bx+cx^2}\right) dx \\
&= \frac{1}{2} \int e^{-id-iex-ifx^2} f^{a+bx+cx^2}\, dx + \frac{1}{2} \int e^{id+iex+ifx^2} f^{a+bx+cx^2}\, dx \\
&= \frac{1}{2} \int \exp\left(-id + a\log(f) - x(ie - b\log(f)) - x^2(if - c\log(f))\right) dx + \cdots \\
&= \frac{1}{2}\left(\exp\left(-id - \frac{(e+ib\log(f))^2}{4if - 4c\log(f)}\right) f^a\right) \int \exp\left(\frac{(-ie + b\log(f) + 2x(\cdots}{4(-if + c\,\mathrm{l}\cdots}\right. \\
&= \frac{\exp\left(-id - \frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right) f^a \sqrt{\pi}\, \mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} + \frac{\exp\left(i\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 2.21, size = 348, normalized size = 1.67

$$
\frac{\sqrt[4]{-1}\, e^{-\frac{1}{4} i \left(\frac{e^2}{f-ic\log(f)} + \frac{b^2\log^2(f)}{f+ic\log(f)}\right)} f^{\frac{f(-be+af)+ac^2\log^2(f)}{f^2+c^2\log^2(f)}} \sqrt{\pi} \left(-e^{\frac{ie^2 f}{2\left(f^2+c^2\log^2(f)\right)}} f^{\frac{be}{2f-2ic\log(f)}} \mathrm{Erfi}\left(\frac{(-1)^{3/4}(e+2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(d)-i\sin(d)) + e^{\frac{ib^2 f\log^2(f)}{2\left(f^2+c^2\log^2(f)\right)}} f^{\frac{be}{2f+2ic\log(f)}} \mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-i(b+2cx)\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}(f+ic\log(f))(-i\cos(d)+\sin(d))\right)}{4\left(f^2+c^2\log^2(f)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Sq
rt[Pi]*(-(E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*Lo
g[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c
*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Sin[d])) + E^(
((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f + (2*I)*c*Log[f]
))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f - I*c*Log
[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*((-I)*Cos[d] + Sin[d])))/(4*E^
```

```
((I/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2 + c^2
*Log[f]^2))
```

Maple [A]
time = 0.23, size = 214, normalized size = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(f) c+4 d f-e^{2}}{4(-i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{i f-c \ln(f)}+\frac{b \ln(f)-i e}{2 \sqrt{i f-c \ln(f)}}\right)}{4 \sqrt{i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}+}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e
^2)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*l
n(f)-I*e)/(I*f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*l
n(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-
(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(159) = 318$.
time = 0.30, size = 1006, normalized size = 4.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*f^(1/4*c*e^2/(c^2*log(f
)^2 + f^2))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2
)/(c^2*log(f)^2 + f^2)) + f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4
*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)
))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c
*log(f) - I*f)) + (-I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*d*f
^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) +
f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
- 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) +
I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(c*log(f) + I*f)))*sqrt(c*lo
g(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((
f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
- 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) - I*f^a*f^(1/4*c*e^2/(c^
2*log(f)^2 + f^2))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2
- f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) -
I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + (f^a*f^(1/4*c*e^2/(c^2*log(f
```

```
)^2 + f^2))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2
)/(c^2*log(f)^2 + f^2)) + I*f^a*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*sin(1/4*
(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^
2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/
(c*log(f) + I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2)
)*log(f)^2 + f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log
(f)/(c^2*log(f)^2 + f^2)))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(159) = 318$.
time = 3.13, size = 381, normalized size = 1.83

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)+i\,f\right)\sqrt{-c\log\left(f\right)+i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^{2}x+\left(2\,c^{2}x+bc\right)\log\left(f\right)^{2}+fe+\left(i\,bf-i\,ce\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)+i\,f}}{2\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log\left(f\right)^{3}+4i\,df^{2}-\left(-4i\,c^{2}d-i\,b^{2}f+2i\,bce\right)\log\left(f\right)^{2}-i\,fe^{2}-\left(4\,af^{2}-2\,bfe+ce^{2}\right)\log\left(f\right)}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}\right)}+\sqrt{\pi}\left(c\log\left(f\right)-i\,f\right)\sqrt{-c\log\left(f\right)-i\,f}\,\operatorname{erf}\left(\frac{\left(2\,f^{2}x+\left(2\,c^{2}x+bc\right)\log\left(f\right)^{2}+fe+\left(-i\,bf+i\,ce\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-i\,f}}{2\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log\left(f\right)^{3}-4i\,df^{2}-\left(4i\,c^{2}d+i\,b^{2}f-2i\,bce\right)\log\left(f\right)^{2}+i\,fe^{2}-\left(4\,af^{2}-2\,bfe+ce^{2}\right)\log\left(f\right)}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}\right)}}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + (2
*c^2*x + b*c)*log(f)^2 + f*e + (I*b*f - I*c*e)*log(f))*sqrt(-c*log(f) + I*f
)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 - (
-4*I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 - I*f*e^2 - (4*a*f^2 - 2*b*f*e +
 c*e^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*l
og(f) - I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + f*e + (-I*b*f +
I*c*e)*log(f))*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - 4*I*d*f^2 - (4*I*c^2*d + I*b^2*f - 2*I*b*c*e)*log(f)^
2 + I*f*e^2 - (4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c
^2*log(f)^2 + f^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x + f*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cos\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2), x)
```

3.132 $\int f^{a+bx+cx^2} \cos^2(d+ex+fx^2)\, dx$

Optimal. Leaf size=268

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}}}{}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)+1/8*exp(-2*I*d-(2*e+I*b*ln(f))^2/(8*I*f-4*c*ln(f)))*f^a*erf(1
/2*(2*I*e-b*ln(f)+2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I
*f-c*ln(f))^(1/2)+1/8*exp(2*I*d+(2*e-I*b*ln(f))^2/(8*I*f+4*c*ln(f)))*f^a*er
fi(1/2*(2*I*e+b*ln(f)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/
(2*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4561, 2266, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}\,f^a\exp\left(\frac{(2e-ib\log(f))^2}{4c\log(f)+8if}+2id\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)+2ie}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+c}\right.\\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)-x(2ie-b\log(f))-x^2(2if-c\log(f))\right)\,d\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right.\right.\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\sqrt{\pi}}{8\sqrt{2if-c}}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1118 vs. 2(268) = 536.
time = 6.70, size = 1118, normalized size = 4.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) - 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erfi[((-1)^
(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[
f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^2 + (4
*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erfi[((-1)^(1/
4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]]
)]*Log[f]^2*Sqrt[2*f - I*c*Log[f]] - (2*(-1)^(1/4)*c*f*Cos[2*d]*Erfi[((-1)^
(3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[
f]])]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] +
 b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(3/4)*c^2*Cos[2*d]*Erfi[((-1)^(
3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f]
+ b^2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*
e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log
[f]*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2 + (4
*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*e +
 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]
^2*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (2*(-1)^(3/4)*c*f*Erfi[((-1)^(3/4)*(2*
e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log
[f]*Sqrt[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] +
 b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Erfi[((-1)^(3/4)*(2*e
 + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[
f]^2*Sqrt[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f]
+ b^2*Log[f]^2))/(2*f + I*c*Log[f]))))/(8*c*Log[f]*(2*f - I*c*Log[f])*(2*f
+ I*c*Log[f]))
```

Maple [A]
time = 0.36, size = 263, normalized size = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^2 b^2-4i\ln(f)be+8id\ln(f)c+16df-4e^2}{4(-2if+c\ln(f))}} \operatorname{erf}\left(-x\sqrt{2if-c\ln(f)}+\frac{b\ln(f)-2ie}{2\sqrt{2if-c\ln(f)}}\right)}{8\sqrt{2if-c\ln(f)}} - \frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln \cdots}{}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-
4*e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)
+1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f
)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln(f
```

```
)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(f)-
2*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(
f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 1481, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*cos(1/2*(16*d*f^2 + (4
*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2))*e^(1/
4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)) + f^a*e^(1/4*b^2*log(
f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2))*sin(1/2*(16*d*f^2 + (4*c^2*d +
b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c
*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*log(f) -
2*I*f)) + (-I*f^a*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2
- 4*f*e^2)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*
log(f)^2 + 4*f^2)) + f^a*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 +
 4*f^2))*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2
)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b*log(f) + 2*I
*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*l
og(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(
(f^a*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c
^2*log(f)^2 + 4*f^2))*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*
f^2)) - I*f^a*e^(1/4*b^2*log(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2))*si
n(1/2*(16*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(
f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c
*log(f) + 2*I*f)/(c*log(f) - 2*I*f)) + (f^a*cos(1/2*(16*d*f^2 + (4*c^2*d +
b^2*f - 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*b^2*log
(f)/c + c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*e^(1/4*b^2*log(f)/c +
c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2))*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f -
 2*b*c*e)*log(f)^2 - 4*f*e^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f)
 + 2*I*f)*x + b*log(f) + 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f))
)*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - 2*sqrt(pi)
*((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c
^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log
(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjuga
te(1/sqrt(-c*log(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(
1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^2*log(f)^2 +
4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2)
 + 2*b*f*e*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*c*x*log(f) + b*log(f)
)/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2
```

```
*b*f*e*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 + 4*f^2*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*f*e*log(f)/(c^2*log(f)^2
+ 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f)))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(203) = 406$.
time = 2.70, size = 474, normalized size = 1.77

$$-\frac{\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}+2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)+2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^{2}x+\left(2\,c^{2}x+bc\right)\log(f)^{2}+4\,fe-2\left(-i\,bf+i\,ce\right)\log(f)\right)\sqrt{-c\log\left(f\right)+2i\,f}}{2\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}+32i\,df^{2}+2\left(4i\,c^{2}d+i\,b^{2}f-2i\,bce\right)\log(f)^{2}-8i\,fe^{2}-4\left(4\,af^{2}-2\,bfe+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)}+\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^{2}x+\left(2\,c^{2}x+bc\right)\log(f)^{2}+4\,fe-2\left(i\,bf-i\,ce\right)\log(f)\right)\sqrt{-c\log\left(f\right)-2i\,f}}{2\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log(f)^{3}-32i\,df^{2}+2\left(-4i\,c^{2}d-i\,b^{2}f+2i\,bce\right)\log(f)^{2}+8i\,fe^{2}-4\left(4\,af^{2}-2\,bfe+ce^{2}\right)\log(f)}{4\left(c^{2}\log(f)^{2}+4\,f^{2}\right)}\right)}+\frac{2\sqrt{\pi}\left(c^{2}\log(f)^{2}+4\,f^{2}\right)\sqrt{-c\log\left(f\right)}\,\operatorname{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log\left(f\right)}}{2\,c}\right)}{f^{\frac{b^{2}-4\,ac}{4\,c}}}}{8\left(c^{3}\log\left(f\right)^{3}+4\,cf^{2}\log\left(f\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(
1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*f*e - 2*(-I*b*f + I*c*e)*log(f)
)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2
)*log(f)^3 + 32*I*d*f^2 + 2*(4*I*c^2*d + I*b^2*f - 2*I*b*c*e)*log(f)^2 - 8*
I*f*e^2 - 4*(4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + s
qrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1/2*(8*
f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*f*e - 2*(I*b*f - I*c*e)*log(f))*sqrt(-
c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)
^3 - 32*I*d*f^2 + 2*(-4*I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 + 8*I*f*e^2
 - 4*(4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + 2*sqrt(p
i)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c x^2+b x+a} \cos \left(f x^2+e x+d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^2, x)
```

3.133 $\int f^{a+bx+cx^2} \cos^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=422

$$\frac{3e^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}} f^a\sqrt{\pi}\,\text{Erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}} f^a\sqrt{\pi}\,\text{Erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}$$

```
[Out] 3/16*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+2
*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*ex
p(-3*I*d-1/4*(3*e+I*b*ln(f))^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e-b*ln(f)+
2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+
3/16*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2
*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*ex
p(3*I*d-1/4*(3*I*e+b*ln(f))^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+b*ln(f)+
2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\frac{3\sqrt{\pi} f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi} f^a \exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi} f^a \exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi} f^a \exp\left(3id-\frac{(b\log(f)+3ie)^2}{4(c\log(f)+3if)}\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[
(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt
[I*f - c*Log[f]]) + (E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I)*f - c*Log
[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*Log[f]))/(2*
Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d + (e
- I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] +
 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
 + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[
Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f +
c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{8}e^{-3i\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+e\right.\right.\right.\\ &= \frac{1}{8}\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int\exp\left(6id+6iex+6ifx^2-3i\left(d\right.\right.\\ &= \frac{1}{8}\int\exp\left(-3id+a\log(f)-x(3ie-b\log(f))-x^2(3if-c\log(f))\right)\,d\\ &= \frac{1}{8}\left(3\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-ie+b\log(f)+2x}{4(-if+c}\right.\\ &= \frac{3\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{\exp\left(\right.}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3829 vs. $2(422) = 844$.
time = 6.82, size = 3829, normalized size = 9.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[
f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f
] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*
(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*L
og[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*
c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f
*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((
I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Cos[d]*Erf
i[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*
Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*E^(((I/4)*(-9*e^2 +
(6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-
1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*L
og[f]])]*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*(-9*e^2 + (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*E^(((I/4)*(-9*e^2 + (6*I)
*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4
)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*(-9*e^2 + (6*I
)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]^3*Sqrt[3*f - I*c*Log[f]] - (27*(-1)^(1/4)*f^3*Cos[d]*Erfi[((-1)^(3/
4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*S
qrt[f + I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f
 + I*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x +
 I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f +
I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*L
og[f])) - (3*(-1)^(1/4)*c^2*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[
f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log
[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f]))
 + (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I
)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/E^(
((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1
)^(1/4)*f^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x
*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-9
*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(3/4)*
c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f
]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-
9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^2*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/
```

```
4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^
(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]])/E^(((I
/4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + 27*(-
1)^(1/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f
]))*f^3*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] + 27*(-1)^(3/4)*c*E^(((I/4
)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f^2*Erfi[((-1
)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]
])]*Log[f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-e^2 +
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(e
+ 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^3*E^(((I/4)*(-e^2 + (2*I)*b*
e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I
*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f -
I*c*Log[f]]*Sin[d] + (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*L
og[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*S
in[d])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])
) + (27*(-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E
^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + (3*(
-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]
))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4
)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f ...
```

Maple [A]
time = 1.13, size = 426, normalized size = 1.01

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6 i \ln(f) b e+12 i d \ln(f) c+36 d f-9 e^{2}}{4(-3 i f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{3 i f-c \ln(f)}+\frac{-3 i e+b \ln(f)}{2 \sqrt{3 i f-c \ln(f)}}\right)}{16 \sqrt{3 i f-c \ln(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*
f-9*e^2)/(-3*I*f+c*ln(f)))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/
2)+1/2*(-3*I*e+b*ln(f))/(3*I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(
ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*l
n(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2
))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*
f-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*
(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2
```

```
+6*I*ln(f)*b*e-12*I*d*ln(f)*c+36*d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*
f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)
^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4354 vs. 2(320) = 640.
time = 0.36, size = 4354, normalized size = 10.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*
e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e
^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f - 2
*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c
*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/
4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c
*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f -
 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f)
 - 3*I*f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f))
 + ((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(
f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2
 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f
)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*
(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 +
 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*
log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(
f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log
(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2
 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f) + 3*I*e)*sqrt(-c*log
(f) - 3*I*f)/(c*log(f) + 3*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2
)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((-I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/
4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*
log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2) +
 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*
f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^a*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2)
 + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2
```

```
*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c^2*log(f)^2 + 9*f^2
) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b
^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(
f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + ((I
*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)/(c
^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*
I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(f)
/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*cos(1/4*(
4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 + f^2
)) - (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log(
f)/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2
 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 9/2*b*f*e*log
(f)/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)))*sin(1/
4*(4*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - f*e^2)/(c^2*log(f)^2 +
f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f
)/(c*log(f) + I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*s
qrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log
(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)
^2 + 9*f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^2 - 9*
f*e^2)/(c^2*log(f)^2 + 9*f^2)) - (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2
*log(f)^2 + 9*f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f
)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*
log(f)^2 + 9*f^2)))*sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f - 2*b*c*e)*log(f)^
2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x + b*l
og(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f)) + ((c^2*f^a*e^(1
/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f
^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + f^(a + 2)*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^2 + f^2)
 + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d +
 b^2*f - 2*b*c*e)*log(f)^2 - 9*f*e^2)/(c^2*log(f)^2 + 9*f^2)) - (-I*c^2*f^a
*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*f*e*log(f)/(c^2*log(f)^
2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f...
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(320) = 640$.
time = 3.58, size = 867, normalized size = 2.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3
)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*
f*e - 3*(-I*b*f + I*c*e)*log(f))*sqrt(-c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*
f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 + 3*(4*I*c^2*d + I*
b^2*f - 2*I*b*c*e)*log(f)^2 - 27*I*f*e^2 - 9*(4*a*f^2 - 2*b*f*e + c*e^2)*lo
g(f))/(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2
 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^
2*x + b*c)*log(f)^2 + f*e + (I*b*f - I*c*e)*log(f))*sqrt(-c*log(f) + I*f)/(
c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 - (-4*
I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 - I*f*e^2 - (4*a*f^2 - 2*b*f*e + c*
e^2)*log(f))/(c^2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log
(f)^2 + 9*c*f^2*log(f) - 9*I*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x +
(2*c^2*x + b*c)*log(f)^2 + f*e + (-I*b*f + I*c*e)*log(f))*sqrt(-c*log(f) -
I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2
- (4*I*c^2*d + I*b^2*f - 2*I*b*c*e)*log(f)^2 + I*f*e^2 - (4*a*f^2 - 2*b*f*e
 + c*e^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) - 3*I*f^3)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^
2*x + (2*c^2*x + b*c)*log(f)^2 + 9*f*e - 3*(I*b*f - I*c*e)*log(f))*sqrt(-c*
log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3
 - 108*I*d*f^2 + 3*(-4*I*c^2*d - I*b^2*f + 2*I*b*c*e)*log(f)^2 + 27*I*f*e^2
 - 9*(4*a*f^2 - 2*b*f*e + c*e^2)*log(f))/(c^2*log(f)^2 + 9*f^2)))/(c^4*log(
f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^3, x)
```

3.134 $\int f^{a+bx+cx^2} \cos\left(a+bx+ex^2\right)\,dx$

Optimal. Leaf size=209

$$\frac{e^{-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}+\frac{e^{(i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b(i+\log(f))+2x(}{2\sqrt{ie+c}}\right.}{4\sqrt{ie+c\log(f)}}$$

```
[Out] -1/4*erf(1/2*(-b*(I-ln(f))-2*x*(I*e-c*ln(f)))/(I*e-c*ln(f))^(1/2))*Pi^(1/2)
/exp((I-ln(f))*(a-b^2*(I-ln(f))/(4*I*e-4*c*ln(f))))/(I*e-c*ln(f))^(1/2)+1/4
*exp((I+ln(f))*(a-b^2*(I+ln(f))/(4*I*e+4*c*ln(f))))*erfi(1/2*(b*(I+ln(f))+2
*x*(I*e+c*ln(f)))/(I*e+c*ln(f))^(1/2))*Pi^(1/2)/(I*e+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\exp\left(-\left((-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\right)\mathrm{Erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}+\frac{\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\mathrm{Erfi}\left(\frac{b(\log(f)+i)+2x(c\log(f)+ie)}{2\sqrt{c\log(f)+ie}}\right)}{4\sqrt{c\log(f)+ie}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c*Log[f
]])])/(4*E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])))*S
qrt[I*e - c*Log[f]]) + (E^((I + Log[f])*(a - (b^2*(I + Log[f]))/((4*I)*e +
4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]))/(2*Sqrt
[I*e + c*Log[f]])])/(4*Sqrt[I*e + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx &= \int \left(\frac{1}{2}e^{-ia-ibx-iex^2}f^{a+bx+cx^2}+\frac{1}{2}e^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{2}\int e^{-ia-ibx-iex^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{ia+ibx+iex^2}f^{a+bx+cx^2}\,dx \\
&= \frac{1}{2}\int \exp\left(-a(i-\log(f))-bx(i-\log(f))-x^2(ie-c\log(f))\right)dx+\frac{1}{2} \\
&= \frac{1}{2}\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\int \exp\left(\frac{(-b(i-\log(f)}{4(-}\right. \\
&= \frac{\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}
\end{aligned}
$$

Mathematica [A]
time = 1.28, size = 325, normalized size = 1.56

$$
-\frac{ie^{-\frac{b^2c\log^3(f)}{2\left(e^2+c^2\log^2(f)\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\sqrt{\pi}\left(-e^{\frac{1}{4}b^2\left(\frac{1}{-ie+c\log(f)}+\frac{\log^2(f)}{ie+c\log(f)}\right)}f^{\frac{ib^2c\log(f)}{e^2+c^2\log^2(f)}}\mathrm{Erfi}\left(\frac{-i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{-ie+c\log(f)}}\right)(e-ic\log(f))\sqrt{-ie+c\log(f)}(\cos(a)-i\sin(a))+e^{\frac{1}{4}b^2\left(\frac{\log^2(f)}{-ie+c\log(f)}+\frac{1}{ie+c\log(f)}\right)}\mathrm{Erfi}\left(\frac{i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{ie+c\log(f)}}\right)(e+ic\log(f))\sqrt{ie+c\log(f)}(\cos(a)+i\sin(a))\right)}{4\left(e^2+c^2\log^2(f)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

```
[Out] ((-1/4*I)*f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c
*Log[f])^(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c
^2*Log[f]^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e +
 c*Log[f]])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a]))
+ E^((b^2*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[(
I*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[
f])*Sqrt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(E^((b^2*c*Log[f]^3)/(2*(e^2
 + c^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

Maple [A]
time = 0.20, size = 215, normalized size = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) a c-2 i \ln(f) b^{2}+4 a e-b^{2}}{4(c \ln(f)-i e)}} \operatorname{erf}\left(-\sqrt{i e-c \ln(f)}\, x+\frac{b \ln(f)-i b}{2 \sqrt{i e-c \ln(f)}}\right)}{4 \sqrt{i e-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{-\ln(f)^{2} b^{2}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b
^2)/(c*ln(f)-I*e))/(I*e-c*ln(f))^(1/2)*erf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*ln
(f)-I*b)/(I*e-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(-ln(f)^2*b^2+4*I*ln
(f)*a*c-2*I*ln(f)*b^2-4*a*e+b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-(
-c*ln(f)-I*e)^(1/2)*x+1/2*(b*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1013 vs. 2(161) = 322.
time = 0.31, size = 1013, normalized size = 4.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*e^2)*((I*f^(1/4*b^2*c/(c^2*log(f)^2
+ e^2))*f^a*cos(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^
2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*sin(-1/4*
(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^
2)))*erf(1/2*(2*(c*log(f) - I*e)*x + b*log(f) - I*b)*sqrt(-c*log(f) + I*e)/
(c*log(f) - I*e)) + (-I*f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*cos(-1/4*(b^
2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2))
 + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*sin(-1/4*(b^2*e + (2*b^2*c - 4*a*
c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(2*(c*log(f
) + I*e)*x + b*log(f) + I*b)*sqrt(-c*log(f) - I*e)/(c*log(f) + I*e)))*sqrt(
c*log(f) + sqrt(c^2*log(f)^2 + e^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*e^2
)*((f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*cos(-1/4*(b^2*e + (2*b^2*c - 4*a
*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)) - I*f^(1/4*b^2*c/(c
^2*log(f)^2 + e^2))*f^a*sin(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f
)^2 - 4*a*e^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(2*(c*log(f) - I*e)*x + b*log
(f) - I*b)*sqrt(-c*log(f) + I*e)/(c*log(f) - I*e)) + (f^(1/4*b^2*c/(c^2*log
(f)^2 + e^2))*f^a*cos(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 -
4*a*e^2)/(c^2*log(f)^2 + e^2)) + I*f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*s
```

```
in(-1/4*(b^2*e + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2 - 4*a*e^2)/(c^2*log(f
)^2 + e^2)))*erf(1/2*(2*(c*log(f) + I*e)*x + b*log(f) + I*b)*sqrt(-c*log(f)
 - I*e)/(c*log(f) + I*e)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + e^2)))/(c^2
*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + e^2) + 1/2*b^2*e*log(f)/(c^2*log(f)^
2 + e^2))*log(f)^2 + e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + e^2) + 1/2*b^2*e
*log(f)/(c^2*log(f)^2 + e^2) + 2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(161) = 322$.
time = 3.58, size = 386, normalized size = 1.85

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-i\,e\right)\sqrt{-c\log\left(f\right)-i\,e}\ \mathrm{erf}\left(\frac{\left(\left(2\,c^{2}x+bc\right)\log\left(f\right)^{2}+2\,xe^{2}+be+\left(i\,bc-i\,be\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-i\,e}}{2\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log\left(f\right)^{3}+i\,b^{2}e-\left(-2i\,b^{2}c+4i\,ac^{2}+i\,b^{2}e\right)\log\left(f\right)^{2}-4i\,ae^{2}-\left(b^{2}c-2\,b^{2}e+4\,ae^{2}\right)\log\left(f\right)}{4\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}\right)}+\sqrt{\pi}\left(c\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)+i\,e}\ \mathrm{erf}\left(\frac{\left(\left(2\,c^{2}x+bc\right)\log\left(f\right)^{2}+2\,xe^{2}+be+\left(-i\,bc+i\,be\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)+i\,e}}{2\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log\left(f\right)^{3}-i\,b^{2}e-\left(2i\,b^{2}c-4i\,ac^{2}-i\,b^{2}e\right)\log\left(f\right)^{2}+4i\,ae^{2}-\left(b^{2}c-2\,b^{2}e+4\,ae^{2}\right)\log\left(f\right)}{4\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}\right)}}{4\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*e)*sqrt(-c*log(f) - I*e)*erf(1/2*((2*c^2*x + b
*c)*log(f)^2 + 2*x*e^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*e
)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - (-2
*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - 4*I*a*e^2 - (b^2*c - 2*b^2*e + 4
*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(c*log(f) + I*e)*sqrt(-c*l
og(f) + I*e)*erf(1/2*((2*c^2*x + b*c)*log(f)^2 + 2*x*e^2 + b*e + (-I*b*c +
I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - I*b^2*e - (2*I*b^2*c - 4*I*a*c^2 - I*b^2*e)*log(f)^2
+ 4*I*a*e^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)))/(c
^2*log(f)^2 + e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}}\cos\left(a+bx+ex^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x**2+b*x+a),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(a + b*x + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x^2 + b*x + a), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cos\left(e\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cos(a + b*x + e*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*cos(a + b*x + e*x^2), x)
```

3.135 $\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$

Optimal. Leaf size=245

$$\frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-2ef$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)-2*e*f^2*F^(b*c*x+a*c)*cos(e*x+d)/(e^2+b^2*c^2*l
n(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*b*c*f^2
*F^(b*c*x+a*c)*ln(F)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)-2*e*f^2*F^(b*c*x+a*c)
*cos(e*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*f^2*F^(b*c*x+a*c)*ln(F)*
sin(e*x+d)^2/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]

time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6873, 12, 6874, 2225, 4517, 4519}

$$\frac{bcf^2\log(F)\sin^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\sin(d+ex)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}+\frac{f^2F^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x])
/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^2
+ b^2*c^2*Log[F]^2)) + (2*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2
 + b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x])/
(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x]^2
)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+\sin(d+ex))^2\,dx\\
&= f^2\int F^{ac+bcx}(1+\sin(d+ex))^2\,dx\\
&= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\sin(d+ex)+F^{ac+bcx}\sin^2(d+ex)\right)\,dx\\
&= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\sin^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\sin(\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+e}{e^2+b^2c^2\log^2(F)}\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.78, size = 180, normalized size = 0.73

$$
\frac{f^2F^{c(a+bx)}(1+\sin(d+ex))^2\left(\frac{3}{bc\log(F)}-\frac{4e\cos(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{bc\cos(2(d+ex))\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{4bc\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{2e\sin(2(d+ex))}{4e^2+b^2c^2\log^2(F)}\right)}{2\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(1 + Sin[d + e*x])^2*(3/(b*c*Log[F]) - (4*e*Cos[d + e*
x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*Cos[2*(d + e*x)]*Log[F])/(4*e^2 + b^2*c
^2*Log[F]^2) + (4*b*c*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (2*e*
Sin[2*(d + e*x)])/(4*e^2 + b^2*c^2*Log[F]^2)))/(2*(Cos[(d + e*x)/2] + Sin[(
d + e*x)/2])^4)
```

Maple [A]
time = 0.56, size = 271, normalized size = 1.11

method	result
risch	$\frac{3f^2F^{c(bx+a)}}{2bc\ln(F)}-\frac{2F^{c(bx+a)}e\,f^2\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{2F^{c(bx+a)}\ln(F)bc\,f^2\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}-\frac{\ln(F)cb\,f^2F^{c(bx+a)}\cos(2ex+2d)}{2\left(4e^2+b^2c^2\ln(F)^2\right)}-\frac{e\,f^2F^{c(bx+a}}{4e^2+b^2}$
default	$-\frac{F^{ac}f^2\left(-\frac{3F^{bcx}}{bc\ln(F)}+\frac{\frac{\ln(F)bc\,e^{bcx\ln(F)}}{4e^2+b^2c^2\ln(F)^2}+\frac{4e\,e^{bcx\ln(F)}\tan(ex+d)}{4e^2+b^2c^2\ln(F)^2}-\frac{\ln(F)bc\,e^{bcx\ln(F)}\left(\tan^2(ex+d)\right)}{4e^2+b^2c^2\ln(F)^2}}{1+\tan^2(ex+d)}+\frac{\frac{4e\,e^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}-\frac{4e\,e^{bcx\ln(F)}\left(\tan^2\left(\frac{d}{2}\right.\right.}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2}\right.}{2}$
norman	$\frac{\frac{\left(b^4c^4\ln(F)^4-2\ln(F)^3b^3c^3e+7b^2c^2e^2\ln(F)^2-8bc\ln(F)e^3+6e^4\right)f^2e^{c(bx+a)\ln(F)}}{bc\ln(F)\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4\right)}+\frac{f^2\left(b^4c^4\ln(F)^4+2\ln(F)^3b^3c^3e+7b^2c^2e^2\ln(F)^2+8bc\ln(F)e^3+\right.}{\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*F^(a*c)*f^2*(-3*F^(b*c*x)/b/c/ln(F)+(1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b
*c*exp(b*c*x*ln(F))+4/(4*e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))*tan(e*x+d)
-1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln(F))*tan(e*x+d)^2)/(1+tan(
e*x+d)^2)+(4/(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))-4/(e^2+b^2*c^2*ln(F)^
2)*e*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)^2-8*b*c*ln(F)/(e^2+b^2*c^2*ln(F)^2
)*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x))/(1+tan(1/2*d+1/2*e*x)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(246) = 492.
time = 0.30, size = 576, normalized size = 2.35

$$-\frac{\left(\left(F^{ac}b^2c^2\cos(2d)\log(F)^2+2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe)+\left(F^{ac}b^2c^2\cos(2d)\log(F)^2-2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe+4d)-\left(F^{ac}b^2c^2\log(F)^2\sin(2d)-2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe)+\left(F^{ac}b^2c^2\log(F)^2\sin(2d)+2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe+4d)-2\left(\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\cos(2d)^2+\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\sin(2d)^2\right)F^{bcx}\right)f^2}{4\left(\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\cos(2d)^2+\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\sin(2d)^2\right)}-\frac{\left(\left(F^{ac}bc\log(F)\sin(d)+F^{ac}\cos(d)e\right)F^{bcx}\cos(xe+2d)-\left(F^{ac}bc\log(F)\sin(d)-F^{ac}\cos(d)e\right)F^{bcx}\cos(xe)-\left(F^{ac}bc\cos(d)\log(F)-F^{ac}e\sin(d)\right)F^{bcx}\sin(xe+2d)-\left(F^{ac}bc\cos(d)\log(F)+F^{ac}e\sin(d)\right)F^{bcx}\sin(xe)\right)f^2}{\left(b^2c^2\log(F)^2+e^2\right)\cos(d)^2+\left(b^2c^2\log(F)^2+e^2\right)\sin(d)^2}+\frac{F^{bcx+ac}f^2}{bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*x*e) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*
e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*x*e + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*s
in(2*d) - 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(2*x*e) + (F^(a*c)*
b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(
```

```
2*x*e + 4*d) - 2*((F^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*cos(2*d)^2 + (
F^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*sin(2*d)^2)*F^(b*c*x))*f^2/((b^3*
c^3*log(F)^3 + 4*b*c*e^2*log(F))*cos(2*d)^2 + (b^3*c^3*log(F)^3 + 4*b*c*e^2
*log(F))*sin(2*d)^2) - ((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b
*c*x)*cos(x*e + 2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*
c*x)*cos(x*e) - (F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*si
n(x*e + 2*d) - (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin
(x*e))*f^2/((b^2*c^2*log(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*si
n(d)^2) + F^(b*c*x + a*c)*f^2/(b*c*log(F))
```

Fricas [**A**]
time = 2.89, size = 257, normalized size = 1.05

$$-\frac{\left(2\,b^3c^3f^2\cos\left(xe+d\right)e\log\left(F\right)^3+8\,bcf^2\cos\left(xe+d\right)e^3\log\left(F\right)+\left(b^4c^4f^2\cos\left(xe+d\right)^2-2\,b^4c^4f^2\right)\log\left(F\right)^4-6\,f^2e^4+\left(b^2c^2f^2\cos\left(xe+d\right)^2e^2-8\,b^2c^2f^2e^2\right)\log\left(F\right)^2-2\,\left(b^4c^4f^2\log\left(F\right)^4-b^3c^3f^2\cos\left(xe+d\right)e\log\left(F\right)^3+4\,b^2c^2f^2e^2\log\left(F\right)^2-bcf^2\cos\left(xe+d\right)e^3\log\left(F\right)\right)\sin\left(xe+d\right)\right)F^{bcx+ac}}{b^5c^5\log\left(F\right)^5+5\,b^3c^3e^2\log\left(F\right)^3+4\,bce^4\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3*f^2*cos(x*e + d)*e*log(F)^3 + 8*b*c*f^2*cos(x*e + d)*e^3*log(F)
 + (b^4*c^4*f^2*cos(x*e + d)^2 - 2*b^4*c^4*f^2)*log(F)^4 - 6*f^2*e^4 + (b^2
*c^2*f^2*cos(x*e + d)^2*e^2 - 8*b^2*c^2*f^2*e^2)*log(F)^2 - 2*(b^4*c^4*f^2*
log(F)^4 - b^3*c^3*f^2*cos(x*e + d)*e*log(F)^3 + 4*b^2*c^2*f^2*e^2*log(F)^2
 - b*c*f^2*cos(x*e + d)*e^3*log(F))*sin(x*e + d))*F^(b*c*x + a*c)/(b^5*c^5*
log(F)^5 + 5*b^3*c^3*e^2*log(F)^3 + 4*b*c*e^4*log(F))
```

Sympy [**C**] Result contains complex when optimal does not.
time = 23.71, size = 8277, normalized size = 33.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f
**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(F, 1)), (b*
*4*c**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2
*I*e/(b*c)))**4*sin(d + e*x)**2/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b*
*3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c))))
 + 2*b**4*c**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log
(exp(-2*I*e/(b*c)))**4*sin(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 +
5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c
)))) + b**4*c**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*l
og(exp(-2*I*e/(b*c)))**4/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3
*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) - 2*b*
*3*c**3*e*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(
-2*I*e/(b*c)))**3*sin(d + e*x)*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)
```

```
))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*
I*e/(b*c)))) - 2*b**3*c**3*e*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c)
)**(b*c*x)*log(exp(-2*I*e/(b*c)))**3*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e
/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(e
xp(-2*I*e/(b*c)))) + 3*b**2*c**2*e**2*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*
I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*sin(d + e*x)**2/(b**5*c**5*lo
g(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*
c*e**4*log(exp(-2*I*e/(b*c)))) + 8*b**2*c**2*e**2*f**2*exp(-2*I*e/(b*c))**(
a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*sin(d + e*x)/(b**
5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))*
*3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*b**2*c**2*e**2*f**2*exp(-2*I*e/
(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*cos(d +
e*x)**2/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*
I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 5*b**2*c**2*e**2*f**2
*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))
**2/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/
(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) - 2*b*c*e**3*f**2*exp(-2*I*
e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))*sin(d + e
*x)*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*lo
g(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) - 8*b*c*e**3*f
**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c
)))*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*lo
g(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*e**4*f**2*
exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*sin(d + e*x)**2/(b**5*c
**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3
+ 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)
*exp(-2*I*e/(b*c))**(b*c*x)*cos(d + e*x)**2/(b**5*c**5*log(exp(-2*I*e/(b*c)
))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*
I*e/(b*c)))) + 4*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c
*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/
(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))), Eq(F, exp(-2*I*e/(b*c)))),
 (b**4*c**4*f**2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I
*e/(b*c)))**4*sin(d + e*x)**2/(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c
**3*e**2*log(exp(-I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 2*b**
4*c**4*f**2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b
*c)))**4*sin(d + e*x)/(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2
*log(exp(-I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + b**4*c**4*f**
2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**4/(
b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))**
3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) - 2*b**3*c**3*e*f**2*exp(-I*e/(b*c))**
(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**3*sin(d + e*x)*cos(d +
 e*x)/(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b
*c)))**3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) - 2*b**3*c**3*e*f**2*exp(-I*e/(
b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**3*cos(d + e*x)/
(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))*
```

```
*3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 3*b**2*c**2*e**2*f**2*exp(-I*e/(b*c
))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*sin(d + e*x)**2/
(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))*
*3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 8*b**2*c**2*e**2*f**2*exp(-I*e/(b*c
))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*sin(d + e*x)/(b*
*5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))**3
+ 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 2*b**2*c**2*e**2*f**2*exp(-I*e/(b*c))*
*(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*cos(d + e*x)**2/(b*
*5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e...
```

Giac [C] Result contains complex when optimal does not.
time = 0.46, size = 1738, normalized size = 7.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*
sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi*b
*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/
(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(
abs(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*p
i*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*
c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi
*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*(2*b*c*f^2*cos(-
1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(ab
s(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(
F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F
) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*log(abs(F))*sin(1/2*pi*
b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*
b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F)
- pi*b*c + 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn
(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 2*(2*b*c*f^2*log(a
bs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2)
 - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*
x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
I*(-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
```

```
 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*lo
g(abs(F)) + 16*I*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/
2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4
*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) - (-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(
abs(F)) + 2*I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I
*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
 2*b*c*log(abs(F)) - 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (I*f
^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*
pi*a*c - I*e*x - I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*I
*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F
) + 1/2*I*pi*a*c + I*e*x + I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(ab
s(F)) + 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*f^2*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2
*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*
e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
 + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c
*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f^2*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(ab
s(F))) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 3.42, size = 248, normalized size = 1.01

$$-\frac{F^{a\,c+b\,c\,x}\,f^2\,\left(\frac{b^4\,c^4\,{\ln\left(F\right)}^4\,\cos\left(2\,d+2\,e\,x\right)}{2}-\frac{3\,b^4\,c^4\,{\ln\left(F\right)}^4}{2}-2\,b^4\,c^4\,\sin\left(d+e\,x\right)\,{\ln\left(F\right)}^4-6\,e^4-\frac{15\,b^2\,c^2\,e^2\,{\ln\left(F\right)}^2}{2}+b^3\,c^3\,e\,{\ln\left(F\right)}^3\,\sin\left(2\,d+2\,e\,x\right)-8\,b^2\,c^2\,e^2\,\sin\left(d+e\,x\right)\,{\ln\left(F\right)}^2+b\,c\,e^3\,\ln\left(F\right)\,\sin\left(2\,d+2\,e\,x\right)+\frac{b^2\,c^2\,e^2\,{\ln\left(F\right)}^2\,\cos\left(2\,d+2\,e\,x\right)}{2}+2\,b^3\,c^3\,e\,\cos\left(d+e\,x\right)\,{\ln\left(F\right)}^3+8\,b\,c\,e^3\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^4\,c^4\,{\ln\left(F\right)}^4+5\,b^2\,c^2\,e^2\,{\ln\left(F\right)}^2+4\,e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*sin(d + e*x))^2,x)
```

```
[Out] -(F^(a*c + b*c*x)*f^2*((b^4*c^4*log(F)^4*cos(2*d + 2*e*x))/2 - (3*b^4*c^4*l
og(F)^4)/2 - 2*b^4*c^4*sin(d + e*x)*log(F)^4 - 6*e^4 - (15*b^2*c^2*e^2*log(
F)^2)/2 + b^3*c^3*e*log(F)^3*sin(2*d + 2*e*x) - 8*b^2*c^2*e^2*sin(d + e*x)*
log(F)^2 + b*c*e^3*log(F)*sin(2*d + 2*e*x) + (b^2*c^2*e^2*log(F)^2*cos(2*d
+ 2*e*x))/2 + 2*b^3*c^3*e*cos(d + e*x)*log(F)^3 + 8*b*c*e^3*cos(d + e*x)*lo
g(F)))/(b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2))
```

3.136 $\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx$

Optimal. Leaf size=99

$$\frac{fF^{ac+bcx}}{bc\log(F)}-\frac{efF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcfF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)-e*f*F^(b*c*x+a*c)*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^2
)+b*c*f*F^(b*c*x+a*c)*ln(F)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6873, 12, 6874, 2225, 4517}

$$\frac{bcf\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{ef\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (e*f*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2
+ b^2*c^2*Log[F]^2) + (b*c*f*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx &= \int fF^{ac+bcx}(1+\sin(d+ex))\,dx \\
&= f\int F^{ac+bcx}(1+\sin(d+ex))\,dx \\
&= f\int \left(F^{ac+bcx}+F^{ac+bcx}\sin(d+ex)\right)\,dx \\
&= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\sin(d+ex)\,dx \\
&= \frac{fF^{ac+bcx}}{bc\log(F)}-\frac{efF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcfF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 83, normalized size = 0.84

$$
\frac{fF^{c(a+bx)}\left(e^2-bce\cos(d+ex)\log(F)+b^2c^2\log^2(F)+b^2c^2\log^2(F)\sin(d+ex)\right)}{bc\log(F)\left(e^2+b^2c^2\log^2(F)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(e^2 - b*c*e*Cos[d + e*x]*Log[F] + b^2*c^2*Log[F]^2 + b^
2*c^2*Log[F]^2*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

Maple [A]
time = 0.12, size = 97, normalized size = 0.98

method	result
risch	$\frac{f\,F^{c(bx+a)}}{bc\ln(F)}-\frac{e\,F^{c(bx+a)}f\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{\ln(F)cb\,F^{c(bx+a)}f\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$
norman	$\frac{\frac{f\left(b^2c^2\ln(F)^2-\ln(F)bce+e^2\right)e^{c(bx+a)\ln(F)}}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{f\left(b^2c^2\ln(F)^2+\ln(F)bce+e^2\right)e^{c(bx+a)\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{2fbc\ln(F)e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c/ln(F)*f*F^(c*(b*x+a))-e*F^(c*(b*x+a))*f/(e^2+b^2*c^2*ln(F)^2)*cos(e*x
+d)+ln(F)*c*b*F^(c*(b*x+a))*f/(e^2+b^2*c^2*ln(F)^2)*sin(e*x+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(100) = 200$.
time = 0.29, size = 221, normalized size = 2.23

$$-\frac{\left(\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)+F^{ac}\cos\left(d\right)e\right)F^{bcx}\cos\left(xe+2\,d\right)-\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)-F^{ac}\cos\left(d\right)e\right)F^{bcx}\cos\left(xe\right)-\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)-F^{ac}e\sin\left(d\right)\right)F^{bcx}\sin\left(xe+2\,d\right)-\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)+F^{ac}e\sin\left(d\right)\right)F^{bcx}\sin\left(xe\right)\right)f}{2\,\left(\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)\cos\left(d\right)^{2}+\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)\sin\left(d\right)^{2}\right)}+\frac{F^{bcx+ac}f}{bc\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*cos(x*e + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*cos(x*e) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(x*e + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(x*e))*f/((b^2*c^2
*log(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(d)^2) + F^(b*c*x +
 a*c)*f/(b*c*log(F))
```

Fricas [A]
time = 2.65, size = 84, normalized size = 0.85

$$\frac{\left(b^{2}c^{2}f\log\left(F\right)^{2}\sin\left(xe+d\right)+b^{2}c^{2}f\log\left(F\right)^{2}-bcf\cos\left(xe+d\right)e\log\left(F\right)+fe^{2}\right)F^{bcx+ac}}{b^{3}c^{3}\log\left(F\right)^{3}+bce^{2}\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] (b^2*c^2*f*log(F)^2*sin(x*e + d) + b^2*c^2*f*log(F)^2 - b*c*f*cos(x*e + d)*
e*log(F) + f*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

Sympy [C] Result contains complex when optimal does not.
time = 2.49, size = 920, normalized size = 9.29

$$\begin{cases} fx - \frac{f\cos\left(d+ex\right)}{e} & \text{for}\: F = 1 \\ \frac{b^{2}c^{2}f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)^{2}\sin\left(d+ex\right)}{b^{3}c^{3}\log\left(e^{-\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{-\frac{ie}{bc}}\right)} + \frac{b^{2}c^{2}f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)^{2}}{b^{3}c^{3}\log\left(e^{-\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{-\frac{ie}{bc}}\right)} - \frac{bcef\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)\cos\left(d+ex\right)}{b^{3}c^{3}\log\left(e^{-\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{-\frac{ie}{bc}}\right)} + \frac{e^{2}f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}}{b^{3}c^{3}\log\left(e^{-\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{-\frac{ie}{bc}}\right)} & \text{for}\: F = e^{-\frac{ie}{bc}} \\ \frac{b^{2}c^{2}f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)^{2}\sin\left(d+ex\right)}{b^{3}c^{3}\log\left(e^{\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{\frac{ie}{bc}}\right)} + \frac{b^{2}c^{2}f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)^{2}}{b^{3}c^{3}\log\left(e^{\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{\frac{ie}{bc}}\right)} - \frac{bcef\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)\cos\left(d+ex\right)}{b^{3}c^{3}\log\left(e^{\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{\frac{ie}{bc}}\right)} + \frac{e^{2}f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}}{b^{3}c^{3}\log\left(e^{\frac{ie}{bc}}\right)^{3}+bce^{2}\log\left(e^{\frac{ie}{bc}}\right)} & \text{for}\: F = e^{\frac{ie}{bc}} \\ F^{ac}\left(fx - \frac{f\cos\left(d+ex\right)}{e}\right) & \text{for}\: b = 0 \\ fx - \frac{f\cos\left(d+ex\right)}{e} & \text{for}\: c = 0 \\ \frac{F^{ac}F^{bcx}b^{2}c^{2}f\log\left(F\right)^{2}\sin\left(d+ex\right)}{b^{3}c^{3}\log\left(F\right)^{3}+bce^{2}\log\left(F\right)} + \frac{F^{ac}F^{bcx}b^{2}c^{2}f\log\left(F\right)^{2}}{b^{3}c^{3}\log\left(F\right)^{3}+bce^{2}\log\left(F\right)} - \frac{F^{ac}F^{bcx}bcef\log\left(F\right)\cos\left(d+ex\right)}{b^{3}c^{3}\log\left(F\right)^{3}+bce^{2}\log\left(F\right)} + \frac{F^{ac}F^{bcx}e^{2}f}{b^{3}c^{3}\log\left(F\right)^{3}+bce^{2}\log\left(F\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d)),x)
```

```
[Out] Piecewise((f*x - f*cos(d + e*x)/e, Eq(F, 1)), (b**2*c**2*f*exp(-I*e/(b*c))*
*(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*sin(d + e*x)/(b**3*
c**3*log(exp(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c)))) + b**2*c**2*f
*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2/(b
**3*c**3*log(exp(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c)))) - b*c*e*f
*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))*cos(d
 + e*x)/(b**3*c**3*log(exp(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c))))
 + e**2*f*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)/(b**3*c**3*log(ex
p(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c)))), Eq(F, exp(-I*e/(b*c))))
, (b**2*c**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*log(exp(I*e/(b
*c)))**2*sin(d + e*x)/(b**3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(exp(
I*e/(b*c)))) + b**2*c**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*lo
g(exp(I*e/(b*c)))**2/(b**3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(exp(I
*e/(b*c)))) - b*c*e*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*log(exp
(I*e/(b*c)))*cos(d + e*x)/(b**3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(
exp(I*e/(b*c)))) + e**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)/(b*
*3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(exp(I*e/(b*c)))), Eq(F, exp(I
*e/(b*c)))), (F**(a*c)*(f*x - f*cos(d + e*x)/e), Eq(b, 0)), (f*x - f*cos(d
+ e*x)/e, Eq(c, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*sin(d + e*x
)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f
*log(F)**2/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)*b*
c*e*f*log(F)*cos(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c
)*F**(b*c*x)*e**2*f/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

<u>**Giac**</u> [**C**] Result contains complex when optimal does not.
time = 0.45, size = 923, normalized size = 9.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*f*log(abs(F))*s
in(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*
x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b
*c*sgn(F) - pi*b*c + 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi
*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (2*b*c*f*l
og(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e
)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*
```

```
c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 +
(pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
- (-I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1
/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(
F)) + 4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*
c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4
*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (I*f*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a
*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I
*e) + I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
+ 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(
abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*
pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*sgn(
F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F)
 + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 2.59, size = 84, normalized size = 0.85

$$\frac{F^{a\,c+b\,c\,x}\,f\,\left(e^2+b^2\,c^2\,\ln\left(F\right)^2+b^2\,c^2\,\sin\left(d+e\,x\right)\,\ln\left(F\right)^2-b\,c\,e\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\,\ln\left(F\right)^2+e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*sin(d + e*x)),x)
```

```
[Out] (F^(a*c + b*c*x)*f*(e^2 + b^2*c^2*log(F)^2 + b^2*c^2*sin(d + e*x)*log(F)^2
- b*c*e*cos(d + e*x)*log(F)))/(b*c*log(F)*(e^2 + b^2*c^2*log(F)^2))
```

3.137 $\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx$

Optimal. Leaf size=80

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

```
[Out] -2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(
F)/e],I*exp(I*(e*x+d)))/f/(e-I*b*c*ln(F))
```

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4541, 4535}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Rule 4535

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4541

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*
g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)\,dx}{2f}$$
$$= -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Mathematica [A]
time = 1.06, size = 128, normalized size = 1.60

$$\frac{2F^{c(a+bx)}\left(-i\,{}_2F_1\left(1,-\frac{ibc\log(F)}{e};1-\frac{ibc\log(F)}{e};i\cos(d+ex)-\sin(d+ex)\right)-\frac{1}{\cos(d)+i(1+\sin(d))}+\frac{\sin\left(\frac{ex}{2}\right)}{\left(\cos\left(\frac{d}{2}\right)+\sin\left(\frac{d}{2}\right)\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}\right)}{ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

```
[Out] (2*F^(c*(a + b*x))*((-I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b
*c*Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]] - (Cos[d] + I*(1 + Sin[d]))^(-
1) + Sin[(e*x)/2]/((Cos[d/2] + Sin[d/2])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/
2]))))/(e*f)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{f+f\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 2*(6*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3
+ 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(x*e + d)^2 + 2*(F^(a*c)*b^3*c^3*l
og(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(x*e + d)^2 + (5*F^(a*c)*b
^2*c^2*e*log(F)^2 - 4*F^(a*c)*e^3)*F^(b*c*x)*cos(x*e + d) + (F^(a*c)*b^3*c^
```

```
3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(x*e + d) - (6*F^(a*c)
*b*c*e^(b*c*x*log(F) + 2)*log(F) + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*
e^3)*F^(b*c*x)*cos(x*e + d) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2
*log(F))*F^(b*c*x)*sin(x*e + d))*cos(2*x*e + 2*d) + 2*((F^(a*c)*b^5*c^5*log
(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*cos(2*
x*e + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3
 + 4*F^(a*c)*b*c*e^4*log(F))*f*cos(x*e + d)^2 + 4*(F^(a*c)*b^5*c^5*log(F)^5
 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*cos(x*e + d
)*sin(2*x*e + 2*d) + (F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(
F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*sin(2*x*e + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*
log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*sin
(x*e + d)^2 + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3
+ 4*F^(a*c)*b*c*e^4*log(F))*f*sin(x*e + d) + (F^(a*c)*b^5*c^5*log(F)^5 + 5*
F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f - 2*(2*(F^(a*c)*
b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F
))*f*sin(x*e + d) + (F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F
)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f)*cos(2*x*e + 2*d))*integrate(-(3*b*c*cos(
3*x*e + 3*d)*e^(b*c*x*log(F) + 2)*log(F) - 9*b*c*cos(x*e + d)*e^(b*c*x*log(
F) + 2)*log(F) - 9*b*c*e^(b*c*x*log(F) + 2)*log(F)*sin(2*x*e + 2*d) - 3*(b^
2*c^2*e*log(F)^2 - 2*e^3)*F^(b*c*x)*cos(2*x*e + 2*d) - (b^2*c^2*e*log(F)^2
- 2*e^3)*F^(b*c*x)*sin(3*x*e + 3*d) + 3*(b^2*c^2*e*log(F)^2 - 2*e^3)*F^(b*c
*x)*sin(x*e + d) + (b^2*c^2*e*log(F)^2 - 2*e^3)*F^(b*c*x))/((b^4*c^4*log(F)
^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(3*x*e + 3*d)^2 + 9*(b^4*c^4*log(
F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*x*e + 2*d)^2 + 9*(b^4*c^4*lo
g(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d)^2 + (b^4*c^4*log(F)
^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(3*x*e + 3*d)^2 + 18*(b^4*c^4*log
(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d)*sin(2*x*e + 2*d) + 9
*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d)^2 +
 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d)^2 + 6
*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d) + (b^4*
c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f - 6*((b^4*c^4*log(F)^4 + 5
*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d) + (b^4*c^4*log(F)^4 + 5*b^2*c
^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d))*cos(3*x*e + 3*d) - 6*(3*(b^4*c
^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d) + (b^4*c^4*log
(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*cos(2*x*e + 2*d) + 2*(3*(b^4*c^4
*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*x*e + 2*d) - 3*(b^4*c^4
*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d) - (b^4*c^4*log(F
)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*sin(3*x*e + 3*d)), x) + ((F^(a*c)*
b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(x*e + d) - (F^(a
*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*F^(b*c*x)*sin(x*e + d) + 2*(F^(a*c)
*b^2*c^2*e*log(F)^2 - 2*F^(a*c)*e^3)*F^(b*c*x))*sin(2*x*e + 2*d))/((b^4*c^4
*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*x*e + 2*d)^2 + 4*(b^4*c
^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d)^2 + 4*(b^4*c^4
*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d)*sin(2*x*e + 2*d)
 + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d)^2
```

```
 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d)^2 +
 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d) + (b^
4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f - 2*(2*(b^4*c^4*log(F)^4
 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d) + (b^4*c^4*log(F)^4 + 5*b
^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*cos(2*x*e + 2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)/(f*sin(x*e + d) + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{ac}F^{bcx}}{\sin(d+ex)+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d)),x)
[Out] Integral(F**(a*c)*F**(b*c*x)/(sin(d + e*x) + 1), x)/f
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{f+f\,\sin\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*sin(d + e*x)),x)
[Out] int(F^(c*(a + b*x))/(f + f*sin(d + e*x)), x)
```

3.138 $\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2}-\frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2}-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\right.}{}$$

```
[Out] -1/6*F^(c*(b*x+a))*cot(1/2*d+1/4*Pi+1/2*e*x)*csc(1/2*d+1/4*Pi+1/2*e*x)^2/e/
f^2-1/6*b*c*F^(c*(b*x+a))*csc(1/2*d+1/4*Pi+1/2*e*x)^2*ln(F)/e^2/f^2-2/3*exp
(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(F)/e],
I*exp(I*(e*x+d)))*(e+I*b*c*ln(F))/e^2/f^2
```

Rubi [A]
time = 0.07, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4541, 4533, 4535}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{6e^2f^2}-\frac{\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{6ef^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

```
[Out] -1/6*(F^(c*(a + b*x))*Cot[d/2 + Pi/4 + (e*x)/2]*Csc[d/2 + Pi/4 + (e*x)/2]^2
)/(e*f^2) - (b*c*F^(c*(a + b*x))*Csc[d/2 + Pi/4 + (e*x)/2]^2*Log[F])/(6*e^2
*f^2) - (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*
Log[F])/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))]*(e + I*b*c*Log[F]))/(3*
e^2*f^2)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4535

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
```

```
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4541

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*
g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\sec^4\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)\,dx}{4f^2}$$

$$= -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2} - \frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\text{lo}}{6e^2f^2}$$

$$= -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2} - \frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\text{lo}}{6e^2f^2}$$

Mathematica [**A**]
time = 1.82, size = 240, normalized size = 1.30

$$\frac{F^{c(a+bx)}\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(2e^2\sin\left(\frac{1}{2}(d+ex)\right)-e(e+bc\log(F))\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+2(e^2+b^2c^2\log^2(F))\sin\left(\frac{1}{2}(d+ex)\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^2-(1-i)\left(1-(1-i)\,{}_2F_1\left(1,-\frac{ibc\log(F)}{e};1-\frac{ibc\log(F)}{e};i\cos(d+ex)-\sin(d+ex)\right)\right)(e^2+b^2c^2\log^2(F))\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^3\right)}{3e^3f^2(1+\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*(2*e^2*Sin[(d + e*x)
/2] - e*(e + b*c*Log[F])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + 2*(e^2 + b
^2*c^2*Log[F]^2)*Sin[(d + e*x)/2]*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2 -
 (1 - I)*(1 - (1 - I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*
Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]])*(e^2 + b^2*c^2*Log[F]^2)*(Cos[(d
 + e*x)/2] + Sin[(d + e*x)/2])^3))/(3*e^3*f^2*(1 + Sin[d + e*x])^2)
```

Maple [**F**]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{\left(f+f\sin\left(ex+d\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a
*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*
log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d)^2 + 6*(F^(a*c)
*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*l
og(F))*F^(b*c*x)*sin(2*x*e + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16
*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(x*e + d)^2 - 20*(F^(a*c)*b^4*c^4*e*l
og(F)^4 - 26*F^(a*c)*b^2*c^2*e^3*log(F)^2)*F^(b*c*x)*cos(x*e + d) - 140*(F^
(a*c)*b^3*c^3*e^2*log(F)^3 - 8*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(x*e +
d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)
 - ((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*
c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)
^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*cos(x*e +
d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*
F^(a*c)*e^5)*F^(b*c*x)*sin(2*x*e + 2*d) - 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3
+ 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(x*e + d) + 40*(F^(a*c)*b^3*c^3*e
^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(4*x*e + 4*d) - 4*(2*
(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)
*e^5)*F^(b*c*x)*cos(2*x*e + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^
(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d) + (F^(a*c)*b^5*c^5*log(F)^5 +
25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin
(2*x*e + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(
F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(x*e + d) - 4*(F^(a*c)*b^4*c^4*e*log(F)
^4 - 35*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 24*F^(a*c)*e^5)*F^(b*c*x))*cos(3*x*e
 + 3*d) + (8*(4*F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F)^
2 - 144*F^(a*c)*e^5)*F^(b*c*x)*cos(x*e + d) - 4*(F^(a*c)*b^5*c^5*log(F)^5 +
 55*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 624*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*si
n(x*e + d) - (F^(a*c)*b^5*c^5*log(F)^5 - 215*F^(a*c)*b^3*c^3*e^2*log(F)^3 +
 1344*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(2*x*e + 2*d) + 4*((F^(a*c)*b^8
*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5
*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(4*x*e + 4*d)^2 + 16*(
F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*
b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(3*x*e + 3*
d)^2 + 36*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 2
44*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos
```

```
(2*x*e + 2*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*l
og(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)
^2)*f^2*cos(x*e + d)^2 + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e
^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*lo
g(F)^2)*f^2*sin(4*x*e + 4*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c
)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2
*c^2*e^7*log(F)^2)*f^2*sin(3*x*e + 3*d)^2 + 48*(F^(a*c)*b^8*c^8*e*log(F)^8
+ 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*
F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(x*e + d)*sin(2*x*e + 2*d) + 36*(F^(a*
c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c
^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*x*e + 2*d)^2
+ 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^
(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(x*e
+ d)^2 + 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 +
244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*si
n(x*e + d) + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6
+ 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2
- 2*(6*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*
F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(2*
x*e + 2*d) + 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^
6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^
2*sin(3*x*e + 3*d) - 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3
*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(
F)^2)*f^2*sin(x*e + d) - (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e
^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*lo
g(F)^2)*f^2)*cos(4*x*e + 4*d) - 16*(2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a
*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b
^2*c^2*e^7*log(F)^2)*f^2*cos(x*e + d) + 3*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*
F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*
c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*x*e + 2*d))*cos(3*x*e + 3*d) - 12*(4*(F^
(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^
4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(x*e + d) + (
F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(-F^(b*c*x + a*c)/(f^2*cos(x*e + d)^2 - 2*f^2*sin(x*e + d) - 2*f^2)
, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{ac}F^{bcx}}{\sin^2(d+ex)+2\sin(d+ex)+1}\,dx}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d))**2,x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(sin(d + e*x)**2 + 2*sin(d + e*x) + 1), x)/f**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\left(f+f\,\sin\left(d+e\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*sin(d + e*x))^2,x)
```

```
[Out] int(F^(c*(a + b*x))/(f + f*sin(d + e*x))^2, x)
```

3.139 $\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$

Optimal. Leaf size=245

$$\frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcf^2F^{ac+bcx}\cos^2(d+ex)\log(}{4e^2+b^2c^2\log^2(F)}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)+2*b*c*f^2*F^(b*c*x+a*c)*cos(e*x+d)*ln(F)/(e^2+b
^2*c^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+b
*c*f^2*F^(b*c*x+a*c)*cos(e*x+d)^2*ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*e*f^2*F^(
b*c*x+a*c)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+2*e*f^2*F^(b*c*x+a*c)*cos(e*x+d
)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]

time = 0.23, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6873, 12, 6874, 2225, 4518, 4520}

$$\frac{2ef^2\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf^2\log(F)\cos^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{2ef^2\sin(d+ex)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}+\frac{f^2F^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + (2*b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x
]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F
]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x]^2*Log
[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sin[d + e*x])/(e
^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x]
)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+\cos(d+ex))^2\,dx \\
&= f^2\int F^{ac+bcx}(1+\cos(d+ex))^2\,dx \\
&= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\cos(d+ex)+F^{ac+bcx}\cos^2(d+ex)\right)\,dx \\
&= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\cos^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\cos(\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{bcf^2F^{ac+bcx}\cos^2(d+e}{4e^2+b^2c^2\log^2(} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\mathrm{lo}\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 228, normalized size = 0.93

$$
\frac{f^2F^{c(a+bx)}\left(12e^4+15b^2c^2e^2\log^2(F)+3b^4c^4\log^4(F)+b^2c^2\cos(2(d+ex))\log^2(F)\left(e^2+b^2c^2\log^2(F)\right)+4b^2c^2\cos(d+ex)\log^2(F)\left(4e^2+b^2c^2\log^2(F)\right)+16bce^3\log(F)\sin(d+ex)+4b^3c^3e\log^3(F)\sin(d+ex)+2bce^3\log(F)\sin(2(d+ex))+2b^3c^3e\log^3(F)\sin(2(d+ex))\right)}{2\left(4bce^4\log(F)+5b^3c^3e^2\log^3(F)+b^5c^5\log^5(F)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(12*e^4 + 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4
 + b^2*c^2*Cos[2*(d + e*x)]*Log[F]^2*(e^2 + b^2*c^2*Log[F]^2) + 4*b^2*c^2*C
os[d + e*x]*Log[F]^2*(4*e^2 + b^2*c^2*Log[F]^2) + 16*b*c*e^3*Log[F]*Sin[d +
 e*x] + 4*b^3*c^3*e*Log[F]^3*Sin[d + e*x] + 2*b*c*e^3*Log[F]*Sin[2*(d + e*x
)] + 2*b^3*c^3*e*Log[F]^3*Sin[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F] + 5*b^3*c
^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

Maple [A]
time = 0.43, size = 274, normalized size = 1.12

method	result
risch	$\frac{3f^2F^{c(bx+a)}}{2bc\ln(F)}+\frac{2\ln(F)cbf^2F^{c(bx+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{2F^{c(bx+a)}ef^2\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{\ln(F)cbf^2F^{c(bx+a)}\cos(2ex+2d)}{2b^2c^2\ln(F)^2+8e^2}+\frac{ef^2F^{c(bx\cdots}}{4e^2+\cdots}$
default	$\frac{F^{ac}f^2\left(\frac{3F^{bcx}}{bc\ln(F)}+\frac{\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e^2+b^2c^2\ln(F)^2}+\frac{4bc\ln(F)e^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}-\frac{4bc\ln(F)e^{bcx\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}+\frac{\frac{\ln(F)bc\,e^{bcx\ln(F)}}{4e^2+b^2c^2\ln(F)^2}+\frac{4e\,e^{bcx\ln(F)}\tan\cdots}{4e^2+b^2c^2\ln\cdots}}{1+\mathrm{t}\cdots}\cdots\right)}{2}$
norman	$\frac{\frac{12e^3f^2e^{c(bx+a)\ln(F)}\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4}+\frac{4\left(2b^2c^2\ln(F)^2+5e^2\right)ef^2e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4}+\frac{2f^2\left(2b^4c^4\ln(F)^4+8b^2c^2e^2\ln(F)^2+3e^4\right)e^{c(bx+\cdots}}{bc\ln(F)\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e\cdots\right.}\cdots}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*F^(a*c)*f^2*(3*F^(b*c*x)/b/c/ln(F)+(8/(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x
*ln(F))*tan(1/2*d+1/2*e*x)+4*b*c*ln(F)/(e^2+b^2*c^2*ln(F)^2)*exp(b*c*x*ln(F
))-4*b*c*ln(F)/(e^2+b^2*c^2*ln(F)^2)*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)^2)
/(1+tan(1/2*d+1/2*e*x)^2)+(1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln
(F))+4/(4*e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))*tan(e*x+d)-1/(4*e^2+b^2*c
^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln(F))*tan(e*x+d)^2)/(1+tan(e*x+d)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(246) = 492.
time = 0.30, size = 573, normalized size = 2.34

$$\frac{\left(\left(F^{ac}b^2c^2\cos(2d)\log(F)^2+2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe)+\left(F^{ac}b^2c^2\cos(2d)\log(F)^2-2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2xe+4d)-\left(F^{ac}b^2c^2\log(F)^2\sin(2d)-2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe)+\left(F^{ac}b^2c^2\log(F)^2\sin(2d)+2F^{ac}bc\cos(2d)e\log(F)\right)F^{bcx}\sin(2xe+4d)+2\left(\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\cos(2d)^2+\left(F^{ac}b^2c^2\log(F)^2+4F^{ac}e^2\right)\sin(2d)^2\right)F^{bcx}\right)f^2}{4\left(\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\cos(2d)^2+\left(b^3c^3\log(F)^3+4bce^2\log(F)\right)\sin(2d)^2\right)}+\frac{\left(\left(F^{ac}bc\cos(d)\log(F)-F^{ac}e\sin(d)\right)F^{bcx}\cos(xe+2d)+\left(F^{ac}bc\cos(d)\log(F)+F^{ac}e\sin(d)\right)F^{bcx}\cos(xe)+\left(F^{ac}bc\log(F)\sin(d)+F^{ac}\cos(d)e\right)F^{bcx}\sin(xe+2d)-\left(F^{ac}bc\log(F)\sin(d)-F^{ac}\cos(d)e\right)F^{bcx}\sin(xe)\right)f^2}{\left(b^2c^2\log(F)^2+e^2\right)\cos(d)^2+\left(b^2c^2\log(F)^2+e^2\right)\sin(d)^2}+\frac{F^{bcx+ac}f^2}{bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))*
F^(b*c*x)*cos(2*x*e) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e
*log(F)*sin(2*d))*F^(b*c*x)*cos(2*x*e + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*si
n(2*d) - 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(2*x*e) + (F^(a*c)*b
^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*cos(2*d)*e*log(F))*F^(b*c*x)*sin(2
*x*e + 4*d) + 2*((F^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*cos(2*d)^2 + (F
```

```
^(a*c)*b^2*c^2*log(F)^2 + 4*F^(a*c)*e^2)*sin(2*d)^2)*F^(b*c*x))*f^2/((b^3*c
^3*log(F)^3 + 4*b*c*e^2*log(F))*cos(2*d)^2 + (b^3*c^3*log(F)^3 + 4*b*c*e^2*
log(F))*sin(2*d)^2) + ((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*
c*x)*cos(x*e + 2*d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c
*x)*cos(x*e) + (F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*sin
(x*e + 2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(
x*e))*f^2/((b^2*c^2*log(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin
(d)^2) + F^(b*c*x + a*c)*f^2/(b*c*log(F))
```

Fricas [A]
time = 2.39, size = 243, normalized size = 0.99

$$\frac{\left(\left(b^4c^4f^2\cos\left(xe+d\right)^2+2\,b^4c^4f^2\cos\left(xe+d\right)+b^4c^4f^2\right)\log\left(F\right)^4+6\,f^2e^4+\left(b^2c^2f^2\cos\left(xe+d\right)^2e^2+8\,b^2c^2f^2\cos\left(xe+d\right)e^2+7\,b^2c^2f^2e^2\right)\log\left(F\right)^2+2\,\left(\left(b^3c^3f^2\cos\left(xe+d\right)e+b^3c^3f^2e\right)\log\left(F\right)^3+\left(bcf^2\cos\left(xe+d\right)e^3+4\,bcf^2e^3\right)\log\left(F\right)\right)\sin\left(xe+d\right)\right)F^{bcx+ac}}{b^5c^5\log\left(F\right)^5+5\,b^3c^3e^2\log\left(F\right)^3+4\,bce^4\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] ((b^4*c^4*f^2*cos(x*e + d)^2 + 2*b^4*c^4*f^2*cos(x*e + d) + b^4*c^4*f^2)*lo
g(F)^4 + 6*f^2*e^4 + (b^2*c^2*f^2*cos(x*e + d)^2*e^2 + 8*b^2*c^2*f^2*cos(x*
e + d)*e^2 + 7*b^2*c^2*f^2*e^2)*log(F)^2 + 2*((b^3*c^3*f^2*cos(x*e + d)*e +
 b^3*c^3*f^2*e)*log(F)^3 + (b*c*f^2*cos(x*e + d)*e^3 + 4*b*c*f^2*e^3)*log(F
))*sin(x*e + d))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5 + 5*b^3*c^3*e^2*log(F)^3
 + 4*b*c*e^4*log(F))
```

Sympy [C] Result contains complex when optimal does not.
time = 23.62, size = 8277, normalized size = 33.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d))**2,x)
```

```
[Out] Piecewise((f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f
**2*sin(d + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(F, 1)), (b*
*4*c**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2
*I*e/(b*c)))**4*cos(d + e*x)**2/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b*
*3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c))))
 + 2*b**4*c**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log
(exp(-2*I*e/(b*c)))**4*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 +
5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c
)))) + b**4*c**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*l
og(exp(-2*I*e/(b*c)))**4/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3
*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*b*
*3*c**3*e*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(
-2*I*e/(b*c)))**3*sin(d + e*x)*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)
))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*
```

```
I*e/(b*c)))) + 2*b**3*c**3*e*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c)
)**(b*c*x)*log(exp(-2*I*e/(b*c)))**3*sin(d + e*x)/(b**5*c**5*log(exp(-2*I*e
/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(e
xp(-2*I*e/(b*c)))) + 2*b**2*c**2*e**2*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*
I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*sin(d + e*x)**2/(b**5*c**5*lo
g(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*
c*e**4*log(exp(-2*I*e/(b*c)))) + 3*b**2*c**2*e**2*f**2*exp(-2*I*e/(b*c))**(
a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*cos(d + e*x)**2/(
b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)
))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 8*b**2*c**2*e**2*f**2*exp(-2*I
*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))**2*cos(d
 + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*
I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 5*b**2*c**2*e**2*f**2
*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))
**2/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/
(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*b*c*e**3*f**2*exp(-2*I*
e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c)))*sin(d + e
*x)*cos(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*lo
g(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 8*b*c*e**3*f
**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*log(exp(-2*I*e/(b*c
)))*sin(d + e*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*lo
g(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*e**4*f**2*
exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*sin(d + e*x)**2/(b**5*c
**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3
+ 4*b*c*e**4*log(exp(-2*I*e/(b*c)))) + 2*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)
*exp(-2*I*e/(b*c))**(b*c*x)*cos(d + e*x)**2/(b**5*c**5*log(exp(-2*I*e/(b*c)
))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-2*
I*e/(b*c)))) + 4*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c
*x)/(b**5*c**5*log(exp(-2*I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-2*I*e/
(b*c)))**3 + 4*b*c*e**4*log(exp(-2*I*e/(b*c)))), Eq(F, exp(-2*I*e/(b*c)))),
 (b**4*c**4*f**2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I
*e/(b*c)))**4*cos(d + e*x)**2/(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c
**3*e**2*log(exp(-I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 2*b**
4*c**4*f**2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b
*c)))**4*cos(d + e*x)/(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2
*log(exp(-I*e/(b*c)))**3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + b**4*c**4*f**
2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**4/(
b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))**
3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 2*b**3*c**3*e*f**2*exp(-I*e/(b*c))**
(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**3*sin(d + e*x)*cos(d +
 e*x)/(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b
*c)))**3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 2*b**3*c**3*e*f**2*exp(-I*e/(
b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**3*sin(d + e*x)/
(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))*
*3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 2*b**2*c**2*e**2*f**2*exp(-I*e/(b*c
```

```
))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*sin(d + e*x)**2/
(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))*
*3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 3*b**2*c**2*e**2*f**2*exp(-I*e/(b*c
))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*cos(d + e*x)**2/
(b**5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e**2*log(exp(-I*e/(b*c)))*
*3 + 4*b*c*e**4*log(exp(-I*e/(b*c)))) + 8*b**2*c**2*e**2*f**2*exp(-I*e/(b*c
))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*cos(d + e*x)/(b*
*5*c**5*log(exp(-I*e/(b*c)))**5 + 5*b**3*c**3*e...
```

Giac [C] Result contains complex when optimal does not.
time = 0.50, size = 1736, normalized size = 7.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) -
 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*s
gn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(
4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(a
bs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b
*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log
(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c +
2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^
2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x
*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(
F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*
sgn(F) - pi*b*c - 2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2*b*c*
f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c
 - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F)
- 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*
c*log(abs(F))) + 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F
) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2
 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I
*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1
/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(
```

```
abs(F)) + 16*I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4*I
*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F
))) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(a
bs(F)) + 2*I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*
pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
2*b*c*log(abs(F)) - 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*
f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c - I*e*x - I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*
I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(
F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(a
bs(F)) + 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f^2*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2
*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*
e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
 + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c
*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f^2*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(ab
s(F))) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B]
time = 3.31, size = 247, normalized size = 1.01

$$\frac{F^{a\,c+b\,c\,x}\,f^2\,\left(6\,e^4+\frac{3\,b^4\,c^4\,{\ln\left(F\right)}^4}{2}+2\,b^4\,c^4\,\cos\left(d+e\,x\right)\,{\ln\left(F\right)}^4+\frac{b^4\,c^4\,{\ln\left(F\right)}^4\,\cos\left(2\,d+2\,e\,x\right)}{2}+\frac{15\,b^2\,c^2\,e^2\,{\ln\left(F\right)}^2}{2}+8\,b\,c\,e^3\,\sin\left(d+e\,x\right)\,\ln\left(F\right)+8\,b^2\,c^2\,e^2\,\cos\left(d+e\,x\right)\,{\ln\left(F\right)}^2+b^3\,c^3\,e\,{\ln\left(F\right)}^3\,\sin\left(2\,d+2\,e\,x\right)+b\,c\,e^3\,\ln\left(F\right)\,\sin\left(2\,d+2\,e\,x\right)+\frac{b^2\,c^2\,e^2\,{\ln\left(F\right)}^2\,\cos\left(2\,d+2\,e\,x\right)}{2}+2\,b^3\,c^3\,e\,\sin\left(d+e\,x\right)\,{\ln\left(F\right)}^3\right)}{b\,c\,\ln\left(F\right)\,\left(b^4\,c^4\,{\ln\left(F\right)}^4+5\,b^2\,c^2\,e^2\,{\ln\left(F\right)}^2+4\,e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*cos(d + e*x))^2,x)
```

```
[Out] (F^(a*c + b*c*x)*f^2*(6*e^4 + (3*b^4*c^4*log(F)^4)/2 + 2*b^4*c^4*cos(d + e*
x)*log(F)^4 + (b^4*c^4*log(F)^4*cos(2*d + 2*e*x))/2 + (15*b^2*c^2*e^2*log(F
)^2)/2 + 8*b*c*e^3*sin(d + e*x)*log(F) + 8*b^2*c^2*e^2*cos(d + e*x)*log(F)^
2 + b^3*c^3*e*log(F)^3*sin(2*d + 2*e*x) + b*c*e^3*log(F)*sin(2*d + 2*e*x) +
 (b^2*c^2*e^2*log(F)^2*cos(2*d + 2*e*x))/2 + 2*b^3*c^3*e*sin(d + e*x)*log(F
)^3))/(b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2))
```

3.140 $\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx$

Optimal. Leaf size=98

$$\frac{fF^{ac+bcx}}{bc\log(F)}+\frac{bcfF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)+b*c*f*F^(b*c*x+a*c)*cos(e*x+d)*ln(F)/(e^2+b^2*c^2
*ln(F)^2)+e*f*F^(b*c*x+a*c)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6873, 12, 6874, 2225, 4518}

$$\frac{ef\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (b*c*f*F^(a*c + b*c*x)*Cos[d + e*x]*Log[
F])/(e^2 + b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx &= \int fF^{ac+bcx}(1+\cos(d+ex))\,dx\\ &= f\int F^{ac+bcx}(1+\cos(d+ex))\,dx\\ &= f\int \left(F^{ac+bcx}+F^{ac+bcx}\cos(d+ex)\right)\,dx\\ &= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\cos(d+ex)\,dx\\ &= \frac{fF^{ac+bcx}}{bc\log(F)}+\frac{bcfF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 0.84

$$\frac{fF^{c(a+bx)}\left(e^2+b^2c^2\log^2(F)+b^2c^2\cos(d+ex)\log^2(F)+bce\log(F)\sin(d+ex)\right)}{bc\log(F)\left(e^2+b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
[Out] (f*F^(c*(a + b*x))*(e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[d + e*x]*Log[F]^2
+ b*c*e*Log[F]*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

Maple [A]
time = 0.17, size = 96, normalized size = 0.98

method	result	size
risch	$\frac{f\,F^{c(bx+a)}}{bc\ln(F)}+\frac{\ln(F)cbf\,F^{c(bx+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{ef\,F^{c(bx+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	96
norman	$\frac{\frac{\left(2b^2c^2\ln(F)^2+e^2\right)f\,e^{c(bx+a)\ln(F)}}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{e^2f\,e^{c(bx+a)\ln(F)}\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{2ef\,e^{c(bx+a)\ln(F)}\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	166

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c/ln(F)*f*F^(c*(b*x+a))+ln(F)*c*b*f*F^(c*(b*x+a))/(e^2+b^2*c^2*ln(F)^2)
*cos(e*x+d)+e*f*F^(c*(b*x+a))/(e^2+b^2*c^2*ln(F)^2)*sin(e*x+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(99) = 198$.
time = 0.29, size = 219, normalized size = 2.23

$$\frac{\left(\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)-F^{ac}e\sin\left(d\right)\right)F^{bcx}\cos\left(xe+2\,d\right)+\left(F^{ac}bc\cos\left(d\right)\log\left(F\right)+F^{ac}e\sin\left(d\right)\right)F^{bcx}\cos\left(xe\right)+\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)+F^{ac}\cos\left(d\right)e\right)F^{bcx}\sin\left(xe+2\,d\right)-\left(F^{ac}bc\log\left(F\right)\sin\left(d\right)-F^{ac}\cos\left(d\right)e\right)F^{bcx}\sin\left(xe\right)\right)f}{2\left(\left(b^2c^2\log\left(F\right)^2+e^2\right)\cos\left(d\right)^2+\left(b^2c^2\log\left(F\right)^2+e^2\right)\sin\left(d\right)^2\right)}+\frac{F^{bcx+ac}f}{bc\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(x*e + 2*d
) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(x*e) + (F^
(a*c)*b*c*log(F)*sin(d) + F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(x*e + 2*d) - (F^(
a*c)*b*c*log(F)*sin(d) - F^(a*c)*cos(d)*e)*F^(b*c*x)*sin(x*e))*f/((b^2*c^2*
log(F)^2 + e^2)*cos(d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(d)^2) + F^(b*c*x +
a*c)*f/(b*c*log(F))
```

Fricas [A]
time = 1.68, size = 81, normalized size = 0.83

$$\frac{\left(bcfe\log\left(F\right)\sin\left(xe+d\right)+\left(b^2c^2f\cos\left(xe+d\right)+b^2c^2f\right)\log\left(F\right)^2+fe^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="fricas")
```

```
[Out] (b*c*f*e*log(F)*sin(x*e + d) + (b^2*c^2*f*cos(x*e + d) + b^2*c^2*f)*log(F)^
2 + f*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

Sympy [C] Result contains complex when optimal does not.
time = 2.50, size = 920, normalized size = 9.39

$$\begin{cases} fx+\frac{f\sin\left(d+ex\right)}{e} & \text{for}\: F=1 \\ \frac{b^2c^2f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)^2\cos\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{-\frac{ie}{bc}}\right)}+\frac{b^2c^2f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)^2}{b^3c^3\log\left(e^{-\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{-\frac{ie}{bc}}\right)}+\frac{bcef\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\log\left(e^{-\frac{ie}{bc}}\right)\sin\left(d+ex\right)}{b^3c^3\log\left(e^{-\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{-\frac{ie}{bc}}\right)}+\frac{e^2f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}}{b^3c^3\log\left(e^{-\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{-\frac{ie}{bc}}\right)} & \text{for}\: F=e^{-\frac{ie}{bc}} \\ \frac{b^2c^2f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)^2\cos\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{\frac{ie}{bc}}\right)}+\frac{b^2c^2f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)^2}{b^3c^3\log\left(e^{\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{\frac{ie}{bc}}\right)}+\frac{bcef\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\log\left(e^{\frac{ie}{bc}}\right)\sin\left(d+ex\right)}{b^3c^3\log\left(e^{\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{\frac{ie}{bc}}\right)}+\frac{e^2f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}}{b^3c^3\log\left(e^{\frac{ie}{bc}}\right)^3+bce^2\log\left(e^{\frac{ie}{bc}}\right)} & \text{for}\: F=e^{\frac{ie}{bc}} \\ F^{ac}\left(fx+\frac{f\sin\left(d+ex\right)}{e}\right) & \text{for}\: b=0 \\ fx+\frac{f\sin\left(d+ex\right)}{e} & \text{for}\: c=0 \\ \frac{F^{ac}F^{bcx}b^2c^2f\log\left(F\right)^2\cos\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac}F^{bcx}b^2c^2f\log\left(F\right)^2}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac}F^{bcx}bcef\log\left(F\right)\sin\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac}F^{bcx}e^2f}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d)),x)
```

```
[Out] Piecewise((f*x + f*sin(d + e*x)/e, Eq(F, 1)), (b**2*c**2*f*exp(-I*e/(b*c))*
*(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2*cos(d + e*x)/(b**3*
c**3*log(exp(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c)))) + b**2*c**2*f
*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))**2/(b
**3*c**3*log(exp(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c)))) + b*c*e*f
*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*log(exp(-I*e/(b*c)))*sin(d
 + e*x)/(b**3*c**3*log(exp(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c))))
 + e**2*f*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)/(b**3*c**3*log(ex
p(-I*e/(b*c)))**3 + b*c*e**2*log(exp(-I*e/(b*c)))), Eq(F, exp(-I*e/(b*c))))
, (b**2*c**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*log(exp(I*e/(b
*c)))**2*cos(d + e*x)/(b**3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(exp(
I*e/(b*c)))) + b**2*c**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*lo
g(exp(I*e/(b*c)))**2/(b**3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(exp(I
*e/(b*c)))) + b*c*e*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*log(exp
(I*e/(b*c)))*sin(d + e*x)/(b**3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(
exp(I*e/(b*c)))) + e**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)/(b*
*3*c**3*log(exp(I*e/(b*c)))**3 + b*c*e**2*log(exp(I*e/(b*c)))), Eq(F, exp(I
*e/(b*c)))), (F**(a*c)*(f*x + f*sin(d + e*x)/e), Eq(b, 0)), (f*x + f*sin(d
+ e*x)/e, Eq(c, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*cos(d + e*x
)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f
*log(F)**2/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b*
c*e*f*log(F)*sin(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c
)*F**(b*c*x)*e**2*f/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 920, normalized size = 9.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="giac")
```

```
[Out] (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*f*sin(1/2*pi*b*c*x*sgn(F) -
 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) + (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F
)) + a*c*log(abs(F))) + 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sg
```

```
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
+ I*(I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs
(F)) + 4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c +
4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*
f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*p
i*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) -
4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(
F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*l
og(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/
2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/
(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*s
gn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn
(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))
)
```

Mupad [B]
time = 2.56, size = 83, normalized size = 0.85

$$\frac{F^{a\,c+b\,c\,x}\,f\,\left(e^2+b^2\,c^2\ln\left(F\right)^2+b^2\,c^2\,\cos\left(d+e\,x\right)\,\ln\left(F\right)^2+b\,c\,e\,\sin\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\ln\left(F\right)^2+e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*cos(d + e*x)),x)
```

```
[Out] (F^(a*c + b*c*x)*f*(e^2 + b^2*c^2*log(F)^2 + b^2*c^2*cos(d + e*x)*log(F)^2
+ b*c*e*sin(d + e*x)*log(F)))/(b*c*log(F)*(e^2 + b^2*c^2*log(F)^2))
```

3.141 $\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx$

Optimal. Leaf size=79

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{f(ie+bc\log(F))}$$

```
[Out] 2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(F
)/e],-exp(I*(e*x+d)))/f/(b*c*ln(F)+I*e)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4542, 4536}

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{f(bc\log(F)+ie)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/
e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(I*e + b*c*Log[F]))
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 4542

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx &= \frac{\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{2f}\\ &= \frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{f(ie+bc\log(F))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 1.01

$$-\frac{2ie^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

```
[Out] ((-2*I)*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log
[F])/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{f+f\cos(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="maxima")
```

```
[Out] 2*(6*F^(a*c)*b*c*e^(b*c*x*log(F) + 2)*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3
+ 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(x*e + d)^2 + 2*(F^(a*c)*b^3*c^3*l
og(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(x*e + d)^2 + (F^(a*c)*b^3
*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(x*e + d) - (5*F^(a
*c)*b^2*c^2*e*log(F)^2 - 4*F^(a*c)*e^3)*F^(b*c*x)*sin(x*e + d) + (6*F^(a*c)
*b*c*e^(b*c*x*log(F) + 2)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*
c*e^2*log(F))*F^(b*c*x)*cos(x*e + d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a
*c)*e^3)*F^(b*c*x)*sin(x*e + d))*cos(2*x*e + 2*d) + 2*((F^(a*c)*b^5*c^5*log
(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*cos(2*
x*e + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3
 + 4*F^(a*c)*b*c*e^4*log(F))*f*cos(x*e + d)^2 + (F^(a*c)*b^5*c^5*log(F)^5 +
 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*sin(2*x*e + 2
*d)^2 + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^
(a*c)*b*c*e^4*log(F))*f*sin(2*x*e + 2*d)*sin(x*e + d) + 4*(F^(a*c)*b^5*c^5*
```

```
log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f*sin
(x*e + d)^2 + 4*(F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3
+ 4*F^(a*c)*b*c*e^4*log(F))*f*cos(x*e + d) + (F^(a*c)*b^5*c^5*log(F)^5 + 5*
F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f + 2*(2*(F^(a*c)*
b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 4*F^(a*c)*b*c*e^4*log(F
))*f*cos(x*e + d) + (F^(a*c)*b^5*c^5*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^2*log(F
)^3 + 4*F^(a*c)*b*c*e^4*log(F))*f)*cos(2*x*e + 2*d))*integrate(-(3*b*c*cos(
3*x*e + 3*d)*e^(b*c*x*log(F) + 2)*log(F) + 9*b*c*cos(2*x*e + 2*d)*e^(b*c*x*
log(F) + 2)*log(F) + 9*b*c*cos(x*e + d)*e^(b*c*x*log(F) + 2)*log(F) + 3*b*c
*e^(b*c*x*log(F) + 2)*log(F) - (b^2*c^2*e*log(F)^2 - 2*e^3)*F^(b*c*x)*sin(3
*x*e + 3*d) - 3*(b^2*c^2*e*log(F)^2 - 2*e^3)*F^(b*c*x)*sin(2*x*e + 2*d) - 3
*(b^2*c^2*e*log(F)^2 - 2*e^3)*F^(b*c*x)*sin(x*e + d))/((b^4*c^4*log(F)^4 +
5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(3*x*e + 3*d)^2 + 9*(b^4*c^4*log(F)^4
+ 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*x*e + 2*d)^2 + 9*(b^4*c^4*log(F)^
4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d)^2 + (b^4*c^4*log(F)^4 +
5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(3*x*e + 3*d)^2 + 9*(b^4*c^4*log(F)^4
+ 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d)^2 + 18*(b^4*c^4*log(F)
^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d)*sin(x*e + d) + 9*(b
^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d)^2 + 6*(b^4
*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d) + (b^4*c^4*l
og(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f + 2*(3*(b^4*c^4*log(F)^4 + 5*b^
2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*x*e + 2*d) + 3*(b^4*c^4*log(F)^4 + 5*b^
2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*
e^2*log(F)^2 + 4*e^4)*f)*cos(3*x*e + 3*d) + 6*(3*(b^4*c^4*log(F)^4 + 5*b^2*
c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^
2*log(F)^2 + 4*e^4)*f)*cos(2*x*e + 2*d) + 6*((b^4*c^4*log(F)^4 + 5*b^2*c^2*
e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^
2*log(F)^2 + 4*e^4)*f*sin(x*e + d))*sin(3*x*e + 3*d)), x) + ((F^(a*c)*b^2*c
^2*e*log(F)^2 + 4*F^(a*c)*e^3)*F^(b*c*x)*cos(x*e + d) + (F^(a*c)*b^3*c^3*lo
g(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(x*e + d) - 2*(F^(a*c)*b^2*
c^2*e*log(F)^2 - 2*F^(a*c)*e^3)*F^(b*c*x))*sin(2*x*e + 2*d))/((b^4*c^4*log(
F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*x*e + 2*d)^2 + 4*(b^4*c^4*lo
g(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d)^2 + (b^4*c^4*log(F)
^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d)^2 + 4*(b^4*c^4*log(
F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*x*e + 2*d)*sin(x*e + d) + 4*
(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(x*e + d)^2 + 4*(b
^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d) + (b^4*c^4
*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f + 2*(2*(b^4*c^4*log(F)^4 + 5*
b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(x*e + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^
2*e^2*log(F)^2 + 4*e^4)*f)*cos(2*x*e + 2*d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f*cos(x*e + d) + f), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cos{(d+ex)}+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cos(d + e*x) + 1), x)/f
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{f+f\,\cos\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*cos(d + e*x)),x)
```

```
[Out] int(F^(c*(a + b*x))/(f + f*cos(d + e*x)), x)
```

3.142 $\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2} \, dx$

Optimal. Leaf size=169

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)(ie-bc\log(F))}{3e^2f^2}-\frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\right.}{6e^2f^2}$$

[Out] -2/3*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*l
n(F)/e],-exp(I*(e*x+d)))*(I*e-b*c*ln(F))/e^2/f^2-1/6*b*c*F^(c*(b*x+a))*ln(F
)*sec(1/2*e*x+1/2*d)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sec(1/2*e*x+1/2*d)^2*tan(1
/2*e*x+1/2*d)/e/f^2

Rubi [A]
time = 0.07, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4542, 4533, 4536}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6ef^2}$$

Antiderivative was successfully verified.

[In] Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]

[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*(I*e - b*c*Log[F]))/(3*e^2*f^2)
 - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a
 + b*x))*Sec[d/2 + (e*x)/2]^2*Tan[d/2 + (e*x)/2])/(6*e*f^2)

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 4542

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\sec^4\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{4f^2}$$

$$= -\frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2} + \frac{\Big(1+\cdots}{\cdots}$$

$$= -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)(ie-bc\log(F))}{3e^2f^2}$$

Mathematica [A]
time = 0.28, size = 145, normalized size = 0.86

$$\frac{2F^{c(a+bx)}\cos\left(\frac{1}{2}(d+ex)\right)\left(-bc\cos\left(\frac{1}{2}(d+ex)\right)\log(F)+4e^{i(d+ex)}\cos^3\left(\frac{1}{2}(d+ex)\right)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)(-ie+bc\log(F))+e\sin\left(\frac{1}{2}(d+ex)\right)\right)}{3e^2f^2(1+\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (2*F^(c*(a + b*x))*Cos[(d + e*x)/2]*(-(b*c*Cos[(d + e*x)/2]*Log[F]) + 4*E^(
I*(d + e*x))*Cos[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/e,
2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*((-I)*e + b*c*Log[F]) + e*Sin[(d +
e*x)/2]))/(3*e^2*f^2*(1 + Cos[d + e*x])^2)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{\left(f+f\cos\left(ex+d\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="maxima")

[Out] 4*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d)^2 + 6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*x*e + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(x*e + d)^2 - 140*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 8*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d) + 20*(F^(a*c)*b^4*c^4*e*log(F)^4 - 26*F^(a*c)*b^2*c^2*e^3*log(F)^2)*F^(b*c*x)*sin(x*e + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*sin(2*x*e + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(x*e + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(4*x*e + 4*d) + 4*((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*x*e + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*sin(2*x*e + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(x*e + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(3*x*e + 3*d) + (4*(F^(a*c)*b^5*c^5*log(F)^5 + 55*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 624*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(x*e + d) + 8*(4*F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 144*F^(a*c)*e^5)*F^(b*c*x)*sin(x*e + d) + (F^(a*c)*b^5*c^5*log(F)^5 - 215*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 1344*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(2*x*e + 2*d) + 4*((F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(4*x*e + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(3*x*e + 3*d)^2 + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(2*x*e + 2*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(x*e + d)^2 + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(4*x*e + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(3*x*e + 3*d)^2 + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(

```
a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*x*e + 2*d
)^2 + 48*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 +
244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*x*
e + 2*d)*sin(x*e + d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*lo
g(F))*f^2*sin(x*e + d)^2 + 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5
*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*
log(F))*f^2*cos(x*e + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*lo
g(F))*f^2 + 2*(4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log
(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*
cos(3*x*e + 3*d) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4
*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*
f^2*cos(2*x*e + 2*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5
*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(
F))*f^2*cos(x*e + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e
^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F)
)*f^2)*cos(4*x*e + 4*d) + 8*(6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b
^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^
8*log(F))*f^2*cos(2*x*e + 2*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*
c*e^8*log(F))*f^2*cos(x*e + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)
*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*
e^8*log(F))*f^2)*cos(3*x*e + 3*d) + 12*(4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 2
9*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(
a*c)*b*c*e^8*log(F))*f^2*cos(x*e + d) + (F^(a*c...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f^2*cos(x*e + d)^2 + 2*f^2*cos(x*e + d) + f^2), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cos^{2}(d+ex)+2\cos(d+ex)+1}\,dx}{f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d))**2,x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cos(d + e*x)**2 + 2*cos(d + e*x) + 1), x)/f**
2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\left(f+f\,\cos\left(d+e\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*cos(d + e*x))^2,x)
```

```
[Out] int(F^(c*(a + b*x))/(f + f*cos(d + e*x))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != −1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```